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A theoretical study on the conversion efficiencies of gradient meta-surfaces

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Abstract - It was shown in a recent work (Sun S. et al., Nat. Mater., 11 (2012) 426) that an ideal gradient meta-surface (GM) can convert an incident propagating wave (PW) to an obliquely outgoing PW or even a surface wave (SW) with nearly 100% efficiency. Here, based on non-ideal GM systems, we systematically studied the factors that influence the efficiencies of such conversion processes (both PW-PW and PW-SW). We found that while intra-supercell impedance-mismatch can hardly affect the conversion efficiencies, the scatterings caused by inter-supercell discontinuities can have non-negligible effects on the PW-SW conversion efficiency. We proposed a new GM model that can reduce the scatterings so as to improve the PW-SW conversion efficiency. Finally, we demonstrated that a GM containing only 2 supercells can convert a PW to a SW with very high efficiency, while a grating coupler of the same size does not work at all.

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Introduction. - Controlling electromagnetic (EM) waves at will is always fascinating and can result in numerous applications. Conventional methods to achieve this aim are typically based on manipulating the propagation phase(s) of light(s) traveling inside an optical medium. Such devices often exhibit certain curved shapes and are optically thick [1].

Metamaterials (MTMs), man-made EM composites constructed by functional microstructures in subwavelength scales, can possess almost arbitrary values of permittivity \( \varepsilon \) and permeability \( \mu \). Based on bulk MTMs with homogeneous or slow-varying EM properties, lots of fascinating effects have been demonstrated, such as negative refraction [2–4], super imaging [5–7], and invisibility cloak [8–10]. Although such MTM-based systems possess much stronger abilities to manipulate EM waves [11–18], they are still bulky in sizes which are unsuitable in certain environments.

Very recently, gradient meta-surfaces (GMs) constructed by planar subwavelength components with distinct EM responses, drew a great deal of attention. Different from conventional and bulk-MTM–based devices, these GMs explored another degree of freedom to modulate the local reflection phases at different positions on the surfaces. Lots of unusual wave manipulation effects were discovered based on these systems, such as anomalous reflection/refraction [19–22], flat-lens focusing and axicons [23,24], and generations of optical vortexes [25]. In particular, Sun et al. [26] showed that an ideal GM combining a thin MTM with \( \varepsilon_\parallel(x) = \mu_\parallel(x) = 1 + \kappa x \) and a metal sheet can convert an incident propagating wave (PW) to a bounded surface wave (SW) with nearly 100% efficiency. The momentum mismatch between the incident PW and the generated SW is compensated by the slope \( \xi \) of the linearly changing reflection-phase profile of the GM (i.e., \( \Phi(x) = \Phi_0 + \xi x \)). However, we note that such \( \sim 100\% \) conversion efficiency was only demonstrated in an ideal GM system, which is unfortunately not easy to realize in practise. Practical systems, however, might have non-equal \( \varepsilon_\parallel(x) \) and \( \mu_\parallel(x) \) and profiles and sometimes use supercells to truncate the profiles to avoid using too large values of \( \varepsilon \) and \( \mu \).

In this paper, based on non-ideal but practically realizable GM systems, we systematically study how various
factors influence the efficiencies of PW-PW and PW-SW conversion processes and how to improve such efficiencies. We first study a series of model GMs with different $\varepsilon_{\parallel}(x)$ and $\mu_{\parallel}(x)$ profiles but yielding the same $\Phi(x)$, and separately discuss how the conversion efficiencies are affected by the intra-supercell and inter-supercell scatterings. Then we propose a new type of model GM to enhance the conversion efficiency. After that, we study the conversion efficiencies of realistic GM systems with microstructures fully taken into account, which are then compared with conventional grating couplers in some details. Conclusions are summarized in the end.

**Efficiency issues of model GMs.** – As shown in fig. 1, the model GM that we study consists of a thin MTM slab with thickness $d$ and position-dependent $\varepsilon(x)$ and $\mu(x)$, which is put on top of a perfect electric conductor (PEC). The system is designed to exhibit a linearly changing reflection phase distribution (see fig. 1(c))

$$\Phi(x) = \Phi_0 + \xi x,$$

so that it can redirect a normally incident PW to an obliquely outgoing PW when $\xi < k_0$ ($k_0 = \omega/c$ and $\omega$ is the working frequency), and to a SW when $\xi > k_0$. To study the conversion efficiency for the PW-PW process (see fig. 1(a)), we shine a plane wave normally onto the system and then employ the mode expansion theory developed in [26] to compute the reflection coefficients $\rho_{k_z}$ for each reflected mode defined by its parallel wave vector $k_z$. We then calculate the reflectance for such anomalous (non-specular) reflection, which is the energy flux carried by the reflected mode (with $k_z = \xi$) referenced by that of the input signal, using the following formula:

$$R_{PW-PW} = \frac{|\rho_{k_z}|^2 \cos \theta_r / \cos \theta_i}{|\rho_{k_z}|^2 \sqrt{1 - (\xi/k_0)^2}}.$$  \hspace{1cm} (2)

Note that the reflection angle is $\theta_r = \cos^{-1}[\sqrt{1 - (\xi/k_0)^2}]$ and the incidence angle is zero ($\theta_i = 0^\circ$). For the PW-SW process, we have to study a finite-length system as discussed in [26], so that we cannot use the mode expansion theory but have to rely on numerical simulations. The simulation strategy is schematically depicted in fig. 1(b). Shining a PW normally onto a GM with $\xi > k_0$, the reflected wave will be a SW with parallel wave vector $k_{SW} = \xi$ (see fig. 1(b)). To unambiguously define the PW-SW conversion efficiency, we need to carefully extract all the energy carried by the SW out of the GM, since otherwise the SW signals bounced back at the GM boundary will cause troublesome interference problems. This can be done by setting a perfect absorbing boundary condition at the ending surface of the device, but here we choose to adopt a different approach. We note that the generated SW is not the usual surface plasmon polariton (SPP) being the eigenmode of the systems [26]. To make such PW-SW conversion practically more meaningful, we place a carefully designed material slab that supports the eigen-SPP (with the same parallel wave vector $k_{SPP} = k_{SW}$ at the working frequency\(^1\)) to guide such SW out of the GM, as schematically depicted in fig. 1(b). We then integrate the power flow carried by the SW over a region specified by $z \in [0, 3\lambda/4]$ ($\lambda$ is the working wavelength) on an arbitrarily selected reference plane (see fig. 1(b)), and the ratio between such a power flow and that of the input PW is the desired PW-SW conversion efficiency.

To realize the desired $\Phi(x)$ profile as shown in eq. (1), Sun et al. [26] studied two model systems—one is the ideal case assuming $\varepsilon_{\parallel}(x) = \mu_{\parallel}(x) = 1 + \xi x / 2 k_0 d$ (with $\varepsilon_z = \mu_z = 1$) and another assuming that $\varepsilon_{\parallel} = 1$ and only $\mu_{\parallel}$ varies as a function of $x$. However, it is practically difficult to realize the ideal MTM with matched impedance. Moreover, if we release the impedance-matching condition, there are much more possibilities to realize the same $\Phi(x)$ profile. In this paper, we make our model as general as possible. We assume that

$$\varepsilon_{\parallel}(x) = 1 + \alpha \cdot \xi x / 2 k_0 d,$$

\hspace{1cm} (3)

\(^1\)There are multiple choices to design this slab, and we choose to adopt the one yielding the smallest impedance mismatch with the GM boundary to minimize the scatterings.

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54002-p2
becomes impedance-matched when $\alpha$ is completely impedance-mismatched when $\xi$ is. We next employed the mode expansion theory [26] to calculate the $\rho_{0\nu}$ spectra in different cases defined by $\xi$, $\alpha$ and $L_S$ (one sample is shown in the inset to fig. 2(c)), and then used eq. (2) to calculate the PW-PW conversion efficiencies of different GMs. The obtained results are shown in fig. 2(b) and (c).

We note that the PW-PW conversion efficiency is quite stable against varying $\alpha$, and $R_{PW-PW}$ stays at very high values. Such a counterintuitive behavior can be explained by the following two reasons: 1) the generated PW does not flow on the GM so that it is not very sensitive to the surface impedance mismatch; 2) the impedance mismatch varies continuously rather than abruptly inside a supercell.

The situation is different for the case of varying $L_S$ (see fig. 2(c)), where the outgoing PW can indeed “feel” the additional scatterings contributed by the supercell boundaries. To simplify the problem, we kept the impedance matched inside each supercell (i.e., setting $\alpha = 1$), so that there is only abrupt impedance mismatch on the supercell boundaries. As shown in fig. 2(c), the calculated $R_{PW-PW}$ decreases slowly as $L_S$ decreases in both $\xi = 0.4k_0$ and $\xi = 0.8k_0$ cases. This is understandable since a smaller $L_S$ value implies a stronger super-periodicity scattering, which in turn decreases the conversion efficiency of the anomalous reflection.

We then took the $\xi = 1.14k_0$ case to study the PW-SW conversion process. According to the computational strategy discussed above (see fig. 1(b)), we employed full-wave simulations based on the finite-element method (FEM)$^2$ to compute the PW-SW conversion efficiencies using a series of GMs designed with different $\alpha$ and total length. We adopted $N_P$ to denote the total number of supercells inside the device. The $\alpha$-dependence of the conversion efficiency is depicted in fig. 3(c) with the length of a single supercell fixed as $L_S = 7.2\pi/\xi$, where we find again that the PW-SW conversion efficiency is rather robust against varying the parameter $\alpha$. This encouraging results are very helpful for future practical designs.

To study how the supercell scatterings affect the PW-SW conversion efficiency, we studied a series of GMs with matched impedance ($\alpha = 1$) and fixed supercell length $L_S = 2\pi/\xi$ but with different $N_P$. As shown in fig. 3(b), we found that the PW-SW efficiency $R_{PW-SW}$ does decay explicitly as $N_P$ increases, indicating that the abrupt inhomogeneity caused by supercell boundaries indeed scatters SWs significantly. We note that such super-periodicity dependence of conversion efficiency in PW-SW case is more significant than that in the PW-PW case (see fig. 2), which can be attributed to the fact that the generated SW does flow on the GM surface so that it can directly “feel” the surface discontinuity.

An improved model GM with enhanced efficiency. – To enhance the PW-SW conversion efficiency, the most important issue is to reduce the scatterings caused by the inhomogeneity of the GM. For a particular PW-SW converter, the SW generated on such a device

\[\Phi(x) = \cos^{-1}\left[\frac{-\varepsilon_{\parallel} + \mu_{\parallel} \tan^2(\sqrt{\varepsilon_{\parallel} \mu_{\parallel}}k_0d)}{\varepsilon_{\parallel} + \mu_{\parallel} \tan^2(\sqrt{\varepsilon_{\parallel} \mu_{\parallel}}k_0d)}\right], \tag{4}\]

satisfy eq. (1). Here $\alpha$ is a parameter ranging in [0,1] to measure the degree of impedance mismatch: the system is completely impedance-mismatched when $\alpha = 0$ and becomes impedance-matched when $\alpha = 1$.

We first study the PW-PW conversion process. Setting $\xi = 0.4k_0$ and $\xi = 0.8k_0$, we computed the required $\varepsilon_{\parallel}(x)$ and $\mu_{\parallel}(x)$ profiles taking different values of $\alpha$. As an example, we depicted in fig. 2(a) the profiles for the $\xi = 0.4k_0$ cases under two extreme conditions ($\alpha = 0, 1$). Note that we have scaled $\mu_{\parallel}$ by a factor so that they can be shown in the same figure with $\varepsilon_{\parallel}$. Knowing that the extremely high values of $\mu_{\parallel}$ are difficult to realize practically, we then introduced supercells with periodicity $L_S$ to truncate these profiles and retain only the part with smallest parameter values. We next employed the

\[\text{COMSOL Multi-physics 3.5, developed by COMSOL}.^\text{©}\]
thickness of the GM is fixed as \(d\) and the conversion efficiencies as functions of \(d\) with \(\alpha\) and \(N_p\). Here, the thickness of the GM is fixed as \(d = \lambda/20\).

![Fig. 3](image)

**Fig. 3:** (Colour on-line) Computed \(R_{PW-SW}\) for a series of \(\xi = 1.14k_0\) GMs (a) with \(L_y = 7 \cdot 2\pi/\xi\), \(N_p = 1\) and different \(\alpha\), and (b) with \(\alpha = 1\), \(L_y = 2\pi/\xi\) and different \(N_p\). Here, the thickness of the GM is fixed as \(d = \lambda/20\).

![Fig. 4](image)

**Fig. 4:** (Colour on-line) (a) Schematic picture of the EM field distribution of the generated SW on the GM. (b) Geometry of the improved GM model. (c) Computed PW-SW conversion efficiencies as functions of \(N_p\) for two models with \(d_m = \lambda/40\) and \(d_m = \lambda/20\), respectively. Here \(d_h = 0\). (d) Computed PW-SW conversion efficiencies as functions of \(N_p\) for three models with \(d_h = 0\), \(d_h = \lambda/40\) and \(d_h = \lambda/20\), respectively, with \(d_m = \lambda/40\) fixed. Here, we set \(\alpha = 1\) in all cases.

The strongest fields appear in the region near the PEC. Motivated by this observation, we propose here an improved version of model GM with geometry schematically depicted in fig. 4(b). The most important improvement is to reduce the thickness \(d_m\) of the inhomogeneous MTM layer and shift the MTM slab away from the PEC, leaving a homogeneous dielectric gap of thickness \(d_h\) in the region where the SW exhibits the strongest field.

We designed a series of GMs based on such an improved model taking different values of \(d_m\) and \(d_h\) (all yielding the same \(\Phi(x)\) profiles with \(\xi = 1.14k_0\)), and studied how \(R_{PW-SW}\) depends on \(N_p\) in these systems. Figure 4(c) compares the conversion efficiencies yielded by two GM devices with the same \(d_h\) but with different \(d_m\). We find that the device with a thinner inhomogeneous MTM layer can indeed yield a higher PW-SW conversion efficiency, as expected. To see the role of the added dielectric gap, we compared in fig. 4(d) the calculated \(R_{PW-SW}\) for three typical systems with different values of \(d_h\) but with \(d_m\) fixed. Figure 4(d) shows that indeed adding a homogeneous dielectric gap in the strong-field region can significantly reduce the scatterings, so that \(R_{PW-SW}\) drops more slowly for the GMs with larger \(d_h\), particularly when \(N_p\) is larger than 4. However, we note that such a remedy does not help increasing \(R_{PW-SW}\) when \(N_p\) is small. This intriguing observation can be understood by noting another effect of adding the dielectric gap, which is unfavorable to the SW generation since it makes the meta-surface less subwavelength by increasing the total thickness of the capping layer. There is a subtle balance between these two mechanisms, and obviously the second effect wins in small \(N_p\) cases.

**Conversion efficiencies of realistic GM systems.** Having studied the efficiency problems of model GMs, we turn to study the realistic structures, which are more interesting in practise. The building block to construct realistic GMs is shown in fig. 5(a), which
is a sandwich structure consisting of a metallic “H” and a PEC separated by a dielectric spacer [26]. Via carefully varying the side bar length $b$, we can get a series of resonators with desired reflection phases to form a particular GM.

It is straightforward to employ the established computational strategy (see fig. 1(b)) to study $R_{PW-SW}$ in a realistic system, for which the guiding-out device is simply set as the usual mushroom structure as in [26]. However, the mode expansion theory for calculating $R_{PW-PW}$ in model systems is difficult to implement in a realistic system, so that we have to find a new and efficient calculation strategy. A feasible method is introduced in [27]. Taking the normal incidence case as an example, we as shine a finite-size GM sample by a plane wave, the reflected wave becomes a finite-width beam exhibiting $k$ components over a certain range, as shown in the scattering patterns computed by finite-difference time domain (FDTD) simulations$^3$ (see fig. 5(b)). Therefore, we integrate over the concerned angle range to obtain the total anomalous reflection power. The reference signal is obtained by repeating the calculation with the GM replaced for a PEC with the same size. Then, the ratio between the obtained two values is the desired PW-PW conversion efficiency for a realistic GM, i.e.,

$$R_{PW-PW} = \frac{\int_{GM} P(\theta_r) \, d\Omega_r}{\int_{Metal} P(\theta_r) \, d\Omega_r}. \quad (5)$$

Figure 5(c) shows the calculated $R_{PW-PW}$ as functions of $N_p$ for two series of realistic GM structures with different $\xi$. Note we cannot vary the intra-supercell impedance-mismatch in realistic structures so that the inter-supercell scattering is the only factor to be considered. We find from fig. 5(c) that the PW-PW conversion efficiencies are quite robust against varying the super-periodicity scatterings for such realistic GMs even with complex microstructures fully taken into account. The results for PW-SW cases are shown in fig. 5(d). We note that the $N_p$-dependency of $R_{PW-SW}$ in realistic situation is quite similar to that in the improved GM model case, since here indeed a homogeneous dielectric gap is present in the realistic structure.

We note that fig. 5(d) actually highlights one important advantage of the GM system working as a PW-SW converter, which can work even with a very small total length. In contrast, a conventional grating coupler fails to work when its length is too short. As shown in fig. 6, we explicitly compared the PW-SW conversion performances for a GM converter and a grating coupler, both having the same lengths (i.e., 2 supercells). Employing the same computational method as described in the second section, we found that the PW-SW conversion efficiency is 78% for the GM device but is only 5.2% for the grating coupler. As shown in fig. 6(b) and (c), the generated SPP field is much higher in the GM case than in the grating coupler case. In fact, while every micro element inside the GM can contribute to the PW-SW conversion process, only the supercell boundaries can contribute to the PW-SW conversion in the grating coupler case. As a result, the conversion efficiency of a GM converter is much higher than a grating coupler, especially when the device has a short length. Such an important character makes the GM system particularly suitable for applications in miniaturized situations where a grating coupler is not suitable.

Conclusions. – To summarize, we have systematically studied the efficiency issues for anomalous reflections by both model GMs and realistic GMs, for both PW-PW and PW-SW conversion processes. We identified that the super periodicity scattering may affect the PW-SW conversion efficiency significantly, but such an effect can be weakened by introducing an improved model GM. Moreover, a comparison with the grating coupler highlights an important advantage of the GM as a PW-SW converter, which can work with high efficiency even with a short length.

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REFERENCES


$^3$CONCERTO 7.0, developed by Vector Fields Software©.


