Effective-medium models and experiments for extraordinary transmission in metamaterial-loaded waveguides

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We show that a metallic waveguide behaves as an electric (magnetic) plasma for transverse-electric
(transverse-magnetic) polarized electromagnetic (EM) waves at frequencies below cutoff value.
Inserting anisotropic resonance structures of either electric or magnetic type into a waveguide, we
find extraordinary transmissions of EM waves with different polarizations through the waveguide at
frequencies well below the waveguide’s cutoff value, following two different mechanisms. Microwave
experiments, in excellent agreements with finite-different-time-domain simulations, are performed to
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Electromagnetic (EM) wave tunneling through classi-
cally opaque medium has drawn much attention recently.1–3
A metallic waveguide is opaque for EM waves at frequencies
below its cutoff value ωc, and it is usually characterized as an
electric plasma with εeff=1−ω2/ω2.4 Recently, experiments
and simulations5,6 revealed unusual EM wave transmissions
through a waveguide loaded with split ring resonators
(SRRs).7 The physics is that a SRR possesses a negative μ at
some frequencies, so that the system becomes transparent
since both εeff and μeff are <0.5,6 Such loaded-waveguide
systems attracted extensive attention recently8–11 since they
provide an alternative approach to build negative index
metamaterials.12

Here, we show that the physics behind such resonance-
induced transparencies is much richer than the simple pic-
ture presented.5,6 We found that while a waveguide can in-
deed be viewed an electric plasma4–6 for transverse-electric
(TE) waves, it must be viewed as a magnetic plasma for
transverse-magnetic (TM) waves. We performed experiments
and finite-difference-time-domain (FDTD) simulations13 to
study the transmissions of EM waves through waveguides
loaded with different resonance structures. Our results unam-
biguously verified the above pictures.

Consider a metallic waveguide of a rectangular cross-
section a×b, filled uniformly with a uniaxial medium with
εz=diag[e2,εt,εm] and μz=diag[μ2,μt,μm]. Here, z
denotes the wave propagation direction. Fixing the transverse
wave vector as kz=(mπ/a,kx=nπ/b, we solved the Maxwell
equations, k×[μz−1(k×E)]=(ω/c)2[εz−1E], to get the dis-


kz=(ω/c)2εz−1μz−1−(ω/c)2μz−1/εz−1, (1)

where ωc=c√(nπ/a)2+(mπ/b)2 is the cutoff frequency of
the waveguide for a given mode.14 Simple calculations show
that the energy flux S=E×H is given by

S=[E]2kz/(ωμ0μz). (2)

Let us consider a homogeneous medium with effective per-
mitivity ε̃ and permeability μ̃. For a plane EM wave propa-
gating along the z direction (with a wave-vector k) inside the
medium, the dispersion relation and the energy flux are

̃kz=(ω/c)2̃ε̃·μ̃, (3)

S=[E]2kz/(ωμ0μ̃). (3)

Comparison between Eqs. (1) and (2) and Eq. (3) suggests
that the waveguide with inclusions can be viewed as an ef-
cective medium15 with

̃εTE(ω)=εz−1−(ω/c)2μ2−1/(μz−1), (4)

for TE waves. For TM waves, similar calculations14 show that

̃kz=(ω/c)2μz−1−μz−1/(ω/c)2εz−1, (5)

S=H×k/(ωμ0εz−1). (5)

Again, comparing Eq. (5) with Eq. (3), we find that for TM
waves, the effective parameters12 of the waveguide with in-
clusion should be

̃εTM(ω)=εz−1, (6)

̃μTM(ω)=μz−1−(ω/c)2/εz−1. (6)

Equations (4) and (6) are our key results. Conclusions drawn
from analyzing Eqs. (4) and (6) are summarized in Table I.
First, for a hollow waveguide, while Eq. (4) shows that for
TM modes, it is indeed an electric plasma,4–6 Eq. (6) shows

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that for TM modes, it behaves as a magnetic plasma. Second, we find that two possible inclusions, namely, a uniaxial electric resonant material (UERM) or a uniaxial magnetic resonant material (UMRM), can be inserted into a hollow waveguide to make it transparent at frequencies below $\omega_c$. Consider the TE mode as an example. If the waveguide is filled with a UERM, Eq. (4) shows that the whole waveguide is described by $\varepsilon^\text{in}(\omega) = \varepsilon^\text{wg}(\omega) - (\omega_c/\omega)^2$ and $\bar{\mu}(\omega) = 1$, indicating that it becomes transparent when $\varepsilon^\text{in}$ is positively large enough. The transparent medium possesses a positive refractive index characterized by simultaneously positive $\varepsilon$ and $\bar{\mu}$. On the other hand, when the inclusion is a URM, the effective parameters are $\varepsilon^\text{eff}(\omega) = 1 - (\omega_c/\omega)^2 = \varepsilon^\text{wg}$ and $\mu^\text{eff}(\omega) = \mu^\text{in}$, so that the waveguide becomes transparent when $\mu^\text{in} < 0$, and the system possesses a negative refractive index. The same arguments hold for the TM polarization.

We performed FDTD simulations to verify all predictions presented in Table I. As shown in Figs. 1(a) and 1(b), the UERM we designed is a metallic cross (CRS) structure and the URM consists of two double-ring SRRs perpendicularly interconnected. We employed FDTD simulations to calculate the normal transmission spectra of EM waves through slabs consisting of periodic arrays of either CRS (periodicity=11.43 mm in $x$-$y$ plane) or SRR structures (periodicity=10 mm in $x$-$y$ plane). By fitting the FDTD transmission spectrum of the CRS array [solid circles in Fig. 2(b)], we find that the CRS plate can be accurately modeled as a 1.235-mm-thick slab of UERM with $\varepsilon^\text{eff} = 1 + 260/(5.52^2 - f^2) + 1300/(17.12^2 - f^2)$, where $f$ is the frequency measured in gigahertz. Similarly, we find that the SRR structure shown in Fig. 1(b) can be viewed as a 3.6-mm-thick slab of URM with $\mu^\text{eff} = 1 + 9/(7.56^2 - f^2) + 28/(19.34^2 - f^2)$ by fitting the FDTD transmission spectrum shown as the dash-dot line in Figs. 3(b) and 5(b).

To verify the predictions presented in Table I, we selected four waveguides with geometries specified in Figs. 2–5. In our FDTD simulations, we set the source and receiver antennas on different sides of the waveguide to measure the signals transmitted through the waveguide. For TE (TM) mode, we place the source/receiver antennas parallel to the $y$-$z$ ($x$-$z$) axis, as shown in Figs. 1(c) and 1(d). We first calculated the transmission spectra through those hollow waveguides and depicted the spectra as dotted lines in Figs. 2(c), 3(b), 4(b), and 5(b), correspondingly. In each spectrum, we find the received signal significantly enhanced (reduced) at frequencies above (below) the cutoff value of the designed

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**TABLE I. Transparency conditions and effective refraction indices of waveguides filled with inclusions for different polarizations.**

<table>
<thead>
<tr>
<th>Inclusion</th>
<th>TE resonance</th>
<th>TM resonance</th>
<th>TE resonance</th>
<th>TM resonance</th>
</tr>
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<tbody>
<tr>
<td>Hollow waveguide</td>
<td>$\varepsilon^\text{wg} = 1 - (\omega_c/\omega)^2$</td>
<td>$\mu^\text{wg} = 1$</td>
<td>$\varepsilon^\text{wg} = 1 - (\omega_c/\omega)^2$</td>
<td>$\mu^\text{wg} = 1$</td>
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Transparency conditions:

1. Case 1: $\varepsilon^\text{in} > (\omega_c/\omega)^2$ and $\mu^\text{in} > 0$
2. Case 2: $\varepsilon^\text{in} > (\omega_c/\omega)^2$ and $\mu^\text{in} < 0$
3. Case 3: $\varepsilon^\text{in} < 0$ and $\mu^\text{in} < 0$
4. Case 4: $\varepsilon^\text{in} < 0$ and $\mu^\text{in} > 0$

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**FIG. 2.** (Color online) For a 50-mm-long waveguide with cross section of $22.86 \times 10.16 \text{ mm}^2$ filled periodically with ten CRS plates: (a) the calculated $\bar{\varepsilon}$ and $\bar{\mu}$ vs frequency for TE wave; (b) FDTD transmission spectrum through a CRS array and measured spectrum through a single CRS resonator in a parallel-plate waveguide; (c) FDTD simulated TE-wave transmission spectra through the loaded and hollow waveguides; and (d) measured TE-wave transmission spectra through the loaded and hollow waveguides. In experiments, the waveguide is 100-mm-long filled with 18 CRS plates periodically.

**FIG. 3.** (Color online) For a 20-mm-long waveguide with cross section of $10 \times 10 \text{ mm}^2$ filled with five SRR layers periodically: (a) the calculated $\bar{\varepsilon}$ and $\bar{\mu}$ vs frequency for TE wave; (b) FDTD simulated TE-wave transmission spectra through the loaded and hollow waveguides, and the normal transmission spectrum through a slab of SRR array (dash-dot line).
waveguide, justifying the simulation scheme we adopted. We then periodically inserted some resonant units into those waveguides and calculated the transmission spectra through such loaded waveguides. The FDTD spectra were shown in Figs. 2(c), 3(b), 4(b), and 5(b) as solid lines, corresponding precisely to the four cases presented in Table I. In all situations, we find significantly enhanced transmissions within some frequency windows well below the waveguides’ cutoff frequencies. Qualitatively, the positions of those enhanced transmission bands (ETBs) agree well with the theoretical predictions (Table I). For example, the transparency condition in case 1 dictates the ETB to appear on the left of the resonance frequency, which is indeed the case as shown in Fig. 2(c).

These intriguing phenomena can be quantitatively understood by using Eqs. (4) and (6). The effective parameters of the inclusions filling the waveguides are calculated by $\varepsilon_{\text{eff}} = (1 - p) + p \varepsilon_{\text{CRS}}$ and $\mu_{\text{eff}} = 1$ for cases 1 and 3 and by $\varepsilon_{\text{eff}} = 1$ and $\mu_{\text{eff}} = (1 - p) + p \mu_{\text{ETB}}$ for cases 2 and 4. Here, $p$ is the volume fraction of the inserted materials in different cases. Setting $\varepsilon_{\text{eff}}$ and $\mu_{\text{eff}}$ into Eqs. (4) and (6) to obtain $\bar{\varepsilon}$ and $\bar{\mu}$ of the loaded waveguides, we plotted the calculated $\bar{\varepsilon}$ and $\bar{\mu}$ versus frequency in Figs. 2(a), 3(a), 4(a), and 5(a), correspondingly. In each case, we find a pass band with $\bar{\varepsilon} \cdot \bar{\mu} > 0$ (the shaded region) below the waveguide’s cutoff value, which coincides perfectly with the ETB shown in the FDTD transmission spectrum.

As experimental confirmation on case 2 was available, we focus on cases 1 and 3 to experimentally verify our predictions. We fabricated the CRS plates and waveguides based on theoretical designs and performed microwave measurements following the scheme depicted in Fig. 1. The source and receiver antennas are both 5 mm monopoles, connected to a vector network analyzer Agilent-8722ES. The measured transmission spectra through a 100-mm-long hollow waveguide and through the waveguide loaded with 18 pieces of CRS plates were depicted in Figs. 2(d) and 4(c) for TE and TM polarizations, respectively. The spectrum of a single CRS plate placed in a parallel-plate waveguide is depicted in Fig. 2(b) to indicate the resonance frequency position. For each polarization, the measured transmission spectrum through the loaded waveguide clearly shows an ETB, whose position coincides well with the FDTD results as well as the theoretical analysis. In short, we demonstrate that a waveguide behaves as an electric (magnetic) plasma for the TE (TM) polarized wave, and inserting resonant materials yields high EM wave transmissions through an opaque waveguide following two mechanisms.

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14. We note that similar dispersion relations were derived in Ref. 11.
15. These effective-medium parameters can also be derived by comparing the impedances of the loaded waveguides and a homogenous medium.
16. Compared with previous works Refs. 5 and 6, our results are in agreement with their general conclusions (see case 2 in Table I), but go beyond their results.