All-dimensional subwavelength cavities made with metamaterials

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By exploiting the reflection phase properties of metamaterial reflectors, the authors propose a method to break the size restrictions imposed strictly on conventional cavities. They design the all-dimensional subwavelength cavities and perform experiments and simulations to demonstrate their subwavelength functionalities. For the smallest cavity that they fabricated, each dimension is only a quarter of the resonance wavelength. © 2006 American Institute of Physics.

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Resonance cavities play crucial roles in both electromagnetic (EM) theory and applications. Conventional cavities are constructed with metallic reflectors, and the governing mechanism imposes a natural lower limit on the cavity size. Consider the simplest $a \times a \times a$ cubic cavity; the lowest mode is $\lambda_{110} = \sqrt{2}a$, implying that a cavity must be large enough in order to create a resonance mode with a definite wavelength.\textsuperscript{1} Cavities with other shapes have similar restrictions. These restrictions greatly limit the applications of a conventional cavity in compact environments. Several ideas were proposed to break such size restrictions. The simplest one is to fill the cavity with high-index materials. More recent ideas include the insertion of a resonance surface into the cavity;\textsuperscript{2,3} or filling the cavity with both positive- and negative-index materials.\textsuperscript{3} Simulations demonstrated that both ideas lead to a significant reduction of the cavity size.\textsuperscript{2,3}

Here, we demonstrate an alternative approach to beat the cavity size restrictions. Distinct from previous methods,\textsuperscript{2,3} we do not insert any other material to the cavity. The key idea is to replace the conventional metallic reflectors by metamaterial reflectors.\textsuperscript{4-6} In our previous work, we have successfully fabricated a double-plate (i.e., one-dimensional) cavity,\textsuperscript{7} with a thickness much less than $\lambda/2$ (see also Refs. 8 and 9). We extend the idea to three dimensions and fabricate cavities that are much smaller than conventional ones working with the same wavelengths. The good agreements among experiments, finite-difference time-domain (FDTD) simulations, and model analysis unambiguously demonstrated the subwavelength functionalities of the constructed cavity.

Consider a $d_x \times d_y \times d_z$ cuboidal cavity bounded by six metamaterial reflectors, each defined by its normal vector (i.e., $\pm \hat{x}, \pm \hat{y}, \pm \hat{z}$). Each reflector reflects EM waves with a definite phase $\Delta \phi_{\parallel}(f)$ depending on frequency $f$, translated to a boundary condition $E_{||} = E_{\parallel}^m = \exp(i \Delta \phi_{\parallel})$ at the surface.

Here $E_{\parallel}$ and $E_{\parallel}^m$ denote the parallel components of $E$-field for the reflected and incident waves with respect to the surface. For a transverse-magnetic (TM) mode,\textsuperscript{10} the $E_z$ component takes a general form

$$E_z(r) = \frac{1}{8} \sum_{j=1}^{8} C(k_j) e^{i k_j r} e^{i u t},$$

with $k_j = \pm k_x, \pm k_y, \pm k_z$ and

$$k_x^2 + k_y^2 + k_z^2 = (\omega/c)^2,$$

where $c$ is the speed of light. Other field components can be derived from $E_z$ through Maxwell equations.\textsuperscript{1} The expansion parameters $C(k_j)$ are determined by matching boundary conditions. For example, for the reflector located at $y = 0$ (with a normal vector $+\hat{y}$), we can identify the wave component $C(k_x, k_y, k_z)$ as the effective incident beam on the surface and the component $C(k_x, -k_y, k_z)$ as the corresponding reflected beam.\textsuperscript{12} Therefore, boundary conditions require that

$$\frac{C(k_x - k_y, k_z) e^{i(k_x-x_ky+k_z)z}}{C(k_x, k_y, k_z) e^{i(k_x+k_y+k_z)z}} \bigg|_{y=0} = \exp(i \Delta \phi_{\parallel})\cdot$$

Similar arguments lead to

$$\frac{C(k_x, k_y - k_z) e^{i(k_x+k_y-k_z)z}}{C(k_x, k_y, k_z) e^{i(k_x+k_y+k_z)z}} \bigg|_{y=d_z} = \exp(i \Delta \phi_{\parallel})$$

at the reflector located at $y = d_z$ (with a normal vector $-\hat{y}$). Combining the above two equations, we get

$$-2k_x d_y + \Delta \phi_{\parallel} + \Delta \phi_{\perp} = m_z \pi,$$

where $m_z$ is an integer. Similarly, we have

$$-2k_z d_y + \Delta \phi_{\parallel} + \Delta \phi_{\perp} = m_x \pi.$$
$-2kd_z + \Delta \phi_{x^\perp} + \Delta \phi_{z} = m_2 \pi$.  

(4)

Considering the transverse-electric (TE) mode\cite{11} leads to the same set of equations. Collecting Eqs. (1)–(4), we get the condition

$$\delta_{(m_j)}(\omega) = c \sqrt{\sum_{ijk} \frac{1}{4d_z^2} (\Delta \phi_{x^\perp}(\omega) + \Delta \phi_{z} (\omega) - 2 \pi m_j)^2}$$

$$- \omega = 0$$  

(5)

to determine the eigenmodes of such a cavity, with \{m_x, m_y, m_z\} being the indices to label the mode. Inserting $\Delta \phi_{x^\perp}=\pi$ into Eq. (5), we obtain a series of resonance frequencies $\omega_{jk}=c\sqrt{(i \pi / d_z)^2 + (j \pi / d_z)^2 + (k \pi / d_z)^2}$, with $i=1$, $m_x=1$, $m_y=1$, and $k=1$, recovering those of a conventional cavity\cite{10} processing a well-defined size restriction. In a metamaterial cavity, the resonance behavior means that $\Delta \phi_{x^\perp}(\omega)$ can take a wide range of values and thus offers us the possibility to break such size restrictions.

To illustrate the above ideas, we fabricated a cubic metamaterial cavity with side length=21 mm. As shown in Fig. 1(a), the left and right walls are two copper plates (normal to the z axis), the back and bottom walls are two identical metamaterial reflectors (normal to the x and y axes),\cite{12} and the front and upper ones are two identical metallic meshes, each printed on a 1.6-mm-thick printed circuit board (PCB) substrates ($\varepsilon_r=2.2$). The metallic mesh has a lattice constant of 3.5 mm, and the width of each metal line is 0.4 mm [see Fig. 1(b)]. Although many types of metamaterial configurations\cite{4-7} can be adopted (as long as they can offer appropriate reflection phases), we take the simplest mushroom structure (without vias)\cite{19} as an example to demonstrate the concept. In the mushroom structure (see Fig. 1), the period of the pattern is 7 mm, the width of each air gap is 0.5 mm, and the inner dielectric layer is the same PCB layer. We dig a small hole near the corner of the cavity and put a 4.5-mm-long monopole antenna inside [see Fig. 1(a)], the working frequency of the 4.5-mm-long monopole is 12 GHz. The measured return loss (S11) spectrum of the antenna is shown in Fig. 2(a) as open circles. The dips in the S11 spectrum are the unambiguous signatures of the existences of cavity modes, and we identify the modes at $f=8.0$, 9.18, 9.63, and 10.2 GHz.\cite{13} We note that the lowest mode of our cavity is 8.0 GHz, obviously lower than that (10.1 GHz) of a conventional cavity with the same size.

We performed FDTD simulations to understand the observed phenomena. In our simulations, we discretized the realistic structure by a basic cell sized 0.5 x 0.5 x 0.5 mm$^3$ and adopted perfect metal boundary conditions at each metal surface. Finer meshes were adopted whenever necessary. The calculated S11 spectrum is shown in Fig. 2(a) as a solid line. Fairly good agreement is found compared with the experimental result.

We employed Eq. (5) to identify the indices of these modes. We first calculated the reflection phases $\Delta \phi_{x^\perp}(f)$ for every reflector under normal incidence, and then inserted $\Delta \phi_{x^\perp}(f)$ into Eq. (5) to calculate the function $\delta_{(m_j)}$ with different mode indices. The $\delta_{(m_j)}$ curves depicted in Fig. 2(b) indicated that the resonance frequencies are 7.45, 8.75, and 8.69 GHz, for the lowest three modes indexed, respectively, by (110), (101) [degenerate with (011)], and (111). However, these results are in apparent disagreement with the experimental and the FDTD simulation ones. Such discrepancies stimulate us to reexamine the validity of the approximation to calculate $\Delta \phi_{x^\perp}(f)$ under normal incidence. While $\Delta \phi_{x^\perp}(f)$ has no dependence on the incidence angle for a metal, and a weak dependence for a metallic mesh, it does exhibit a strong incidence-angle dependence for the metamaterial reflector, as illustrated in Fig. 3(a) where $\Delta \phi(f) \sim f$.
curves are shown for different incidence angles. For the lowest (110) mode, symmetry requires strictly that $k_z = k_y$, so that the reflection at the metamaterial surface corresponds to an incidence angle of $45^\circ$. For (101) and (111) modes, we have no such symmetry properties. We incorporate the incidence-angle corrections in an iterative way. We first use the normal incidence $\Delta \phi_n(f)$ to solve Eqs. (1)–(5) and estimate the corresponding incidence angles (with respect to the metamaterial reflectors) from the values of $k_z, k_y, k_x$ obtained for these modes. Calculations show that the results are $54.7^\circ$ for the (110) mode and $66.3^\circ$ for the (111) mode.13 We then calculated $\Delta \phi_n(f)$ under these incidence angles and used the corrected $\Delta \phi_n(f)$ results [as shown in Fig. 3(a)] to recalculate the function $\delta_{\text{inc}}$ and depicted the corrected $\delta_{\text{inc}} \sim f$ curves in Fig. 3(b). The resonance frequencies obtained following these procedures are 8.0, 9.08, and 9.63 GHz for these modes, which are in excellent agreements with both simulations and experiments. In particular, we find that the (110) mode becomes lower than the (111), after considering the incidence angle corrections. Such incidence angle dependence is a unique feature of the present metamaterial cavity, distinct from a conventional cavity.

We also measured the radiation patterns of the antenna placed inside the cavity, for the mode at $f = 8.0$ GHz (Fig. 4). The H-plane pattern is symmetrical with respect to the $45^\circ$ axis, consistent with the symmetry properties of the (011) mode. However, the E-plane pattern is not entirely symmetrical, due to the intrinsic asymmetry of the monopole antenna itself. FDTD simulation results are shown in the same figures as solid lines. Good agreement between theory and experiment is noted.

In principle, there is no restriction on the cavity size if we can freely tune $\Delta \phi_n(f)$ as desired. The latter can be achieved via tuning the magnetic resonance frequencies of the metamaterial reflectors. To illustrate this point, we fabricated two new metamaterial reflectors in which $\varepsilon_r$ of the inner PCB layer changes to 10.2 and applied them to construct a $14 \times 14 \times 14$ mm$^3$ cavity [picture shown in Fig. 1(c)] and redo the same measurements and simulations. These metamaterials possess even lower magnetic resonance frequencies, so that the eigenmodes inside the cavity are in a much smaller subwavelength size. Shown in Fig. 5 are the S11 spectra of a 4-mm-long antenna inside such a cavity, obtained by measurements (open circles) and FDTD simulations (solid line). Although the quantitative agreement between theory and experiment is not perfect, the positions of the resonance modes indicated in two spectra accurately agree with each other. In particular, we note that the lowest mode of the present cavity is at $f = 5.5$ GHz, which is roughly 1/3 of that for a conventional cavity with the same size ($\sim 15.15$ GHz). Meanwhile, the side length of the cavity is only (roughly) 1/4 of the working wavelength $\sim 54.5$ mm.

In short, we demonstrated a mechanism to construct all-dimensional subwavelength cavities based on metamaterials. The present idea may be useful in applications such as a subwavelength high-$Q$ filter15 and subwavelength cavity antenna.

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10Here we define the TE (TM) mode to have nonzero $H_z(E_y)$ components.
11Here we adopt the convention that $e^{i\phi_{\text{inc}}}$ denotes a wave propagating along the $-\hat{y}$ direction.
12If the cavity has only one magnetic surface and five perfect metallic surfaces, boundary conditions of the two pairs of the perfectly metallic surfaces in parallel require that the size of the cavity along one axis has to be larger than a half of the cutoff wavelength, so no less than two magnetic surfaces are required to make the cavity much smaller than the cutoff wavelength in all dimensions.
13Simulations were performed using the package CONCERTO 4.0, developed by Vector Fields Limited, England, 2004.
15The quality factor of the present cavity can be significantly improved to the order of 10$^3$ if we replace the metallic mesh layers by metal plates (adopt copper to construct the metallic layer and ignore the loss of the dielectric layer).