High-performance meta-devices based on multilayer meta-atoms: interplay between the number of layers and phase coverage

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1. Introduction

Metasurfaces refer to the ultra-thin metamaterials constructed by subwavelength planar microstructures (e.g., “meta-atoms”), which exhibit tailored electromagnetic (EM) properties. They have drawn intensive attention recently [1–4]. Through carefully adjusting the structures of the “meta-atoms”, one can realize metasurfaces to exhibit desired distributions of phase and amplitude for reflected/transmitted waves. They can be used to efficiently reshape the wavefronts of incident EM waves based on Huygens-Fermat’s principle. Many fascinating wave-manipulation effects have been realized based on metasurfaces, such as anomalous reflection and refraction [5,6], vortex beam generation [5,7], surface wave coupling [8], and giant and flexible spin-Hall effect [9,10]. Such extraordinary wave-manipulation capabilities inspire researchers to realize many functional metasurface-based optical devices such as flat lenses [11–14], ultrathin wave plates [15,16], and beam deflectors [17–19].

Among these applications, thickness and performance are two critical issues that must be considered simultaneously during the design of a meta-device. Ideally, ultra-thin meta-devices with high performances (e.g., high efficiency and broad bandwidth) are needed; but in reality thickness and performance affect each other due to the subtle interplay between them. To build a high-performance meta-device, a series of meta-atoms exhibiting nearly 100% transmission/reflection amplitudes with phases covering the full 360° range are required. For reflection-type metasurfaces, metal-insulator-metal (MIM) meta-atoms can fulfill these requirements, which hence have been widely used to build functional meta-devices with ultrathin thicknesses and high performances [8,12,20–24]. However, for transmission geometry, ultra-thin transparent meta-atoms with full-range phase coverage are quite difficult to obtain [25]. In order to make a meta-atom optically transparent, it must simultaneously exhibit electric and magnetic responses so that its impedance can match that of air [4]. In 2013, Pfeiffer and Grbic [17] realized the first transmissive metasurface (usually called Huygens’ surfaces) based on a series of composite meta-atoms combining carefully designed electric resonators and magnetic split-ring-resonators. Based on a similar concept, researchers also proposed all-dielectric metasurfaces that were consisted of dielectric meta-atoms possessing spectrally overlapping electric and magnetic resonances with equal strengths [26]. Although such an idea was proven useful in designing high-efficiency transmissive meta-devices [18,27,28], the adopted meta-atoms usually involve complex three-dimensional (3D)
structures, which are optically thick and non-flat, and thus unfavorable for practical applications. To address this issue, multilayer meta-atoms were widely used to build transmissive metasurfaces at frequencies ranging from microwave to infrared \[18,19,29\]. In these meta-devices, the adopted meta-atoms all exhibit broadband transparent windows covering large phase-variation ranges (inset of Fig. 1), which enable researchers to select appropriate meta-atoms with desired transmission phases. Such meta-atoms are flat and easy to fabricate, and are widely used to realize functional meta-devices for different purposes \[30–35\].

However, despite this impressive progress, several fundamental issues remain unsolved. For example, having seen so many meta-devices with different layers, naturally, scientists are curious to know the minimum layer number and thickness required to build high-performance transmissive meta-devices. Several prior works \[2,19,36,37\] pointed out the phase coverage for meta-atoms with different layers. Unfortunately, unknowing the inherent physics underlying the intriguing interplay between the number of layers and performance, people usually have to employ brutal-force simulations to search for appropriate meta-atoms. It is not only time consuming, but also lacks physical guidance.

The main purpose of this work is to answer this fundamental question and to provide useful guidance for the practical design of transmissive metasurfaces. To achieve this goal, first coupled-mode-theory (CMT) analyses are performed to reveal the subtle interplays between thickness and phase-variation in multilayer meta-atoms of distinct types. It is found that meta-atoms with a particular type can provide transmission phases covering only a particular range (Fig. 1b–d). Therefore, combinations of meta-atoms with distinct types are highly suggested when building particular range (Fig. 1b–d). Therefore, combinations of meta-atoms with distinct types can provide transmission phases covering only a particular range (Fig. 1b–d). Therefore, combinations of meta-atoms with distinct types can provide transmission phases covering only a particular range (Fig. 1b–d).

2. CMT analyses on meta-atoms with different numbers of resonators

We start from analyzing the generic EM responses of two distinct types of meta-atoms, widely used in building transmissive meta-devices \[25,32,34,35,38–41\]. In the first type, the meta-atoms contain different layers of planar resonators (metallic crosses). Each is embedded inside a subwavelength metallic mesh, which is optically opaque below around working frequencies (Fig. 1b, c). In the second type, the only difference is that there is no metallic mesh surrounding the resonators. Here, the metallic mesh processes a cutoff frequency (much higher than the working frequency of the device), below which EM waves are strongly reflected. Thus, the metallic mesh serves as an optically opaque background for the whole system. Firstly, a 2-layer meta-atom of the first type is studied, which is described by a 2-port 2-mode model consisting of two coupled resonators (i.e., the loaded resonating structures) embedded in an opaque background (i.e., the mesh), as shown in Fig. 1b. Due to the presence of the opaque background, these two resonators can only be excited by external light coming from one particular port; also they can only radiate to that port. Meanwhile, these two resonators can couple with each other through near-field coupling with a strength of $\kappa$. Therefore, according to the tight binding method (TBM) established for photonic systems \[42\], the time-evolution of the two modes’ amplitudes (denoted as $a_1$ and $a_2$) can be expressed as

$$\frac{1}{2\pi} \frac{d}{dt} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = i \mathbf{H} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} f_0 & \kappa \\ \kappa & f_0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$

where $f_0$ denotes the resonant frequency of the mode supported by a single resonator. Here, assume the neighboring resonators are weakly coupled (i.e., $|\kappa| \ll f_0$) to satisfy the assumption of TBM. In Eq. (1), we have purposely “turned off” the mode’s radiations to the far field at the moment, in order to see clearly the dynamical evolutions of the “modes” \[43,44\]. Diagonalizing the Hamiltonian matrix with a matrix $M = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$, we obtain

![Fig. 1. Illustration of the transparent meta-atoms. (a) Schematics of transmissive meta-atoms for realizing polarization-control and beam deflecting meta-devices; Smith diagrams and phase coverage achieved by coupled-mode models with (b) two and (c) three resonators embedded in opaque background, and a coupled-mode model with (d) two resonators in a transparent background.](image-url)
\[ M \cdot \left( \begin{array}{c} f_0 \\ \kappa \\ f_0 \end{array} \right) = \left( \begin{array}{c} f_s \\ 0 \\ f_s \end{array} \right), \]

where the frequencies and wave-functions of two eigen collective modes are

\[
\begin{align*}
\psi_s &= \psi_{s1} + \psi_{s2}, \\
\psi_{s1} &= \frac{1}{\sqrt{2}} (\psi_1 + \psi_2), \\
\psi_{s2} &= \frac{1}{\sqrt{2}} (\psi_1 - \psi_2),
\end{align*}
\]

with \( \psi_1 \) and \( \psi_2 \) being the wave-functions of two original (uncoupled) resonators respectively. Here, “s” and “a” denote the symmetric and anti-symmetric nature of the two eigen collective modes correspondingly. Turning on the radiation channels of the modes, it is found through the CMT analyses [45] that the (complex) transmission coefficient \( t \) of the whole system (without considering material losses) can be expressed as

\[
t = \frac{\Gamma_s}{-\hbar \Omega_f - \Gamma_s} - \frac{\Gamma_s}{-\hbar \Omega_f - \Gamma_s},
\]

where \( \Gamma_s = \frac{1}{2} \sum (|k_i| \psi_{s1} + |k_i| \psi_{s2})^2 \) and \( \Gamma_s = \frac{1}{2} \sum (|k_i| \psi_{s1} - |k_i| \psi_{s2})^2 \) denote the radiation damping rates of two collective modes towards two ports \( (i = 1 \text{ and } 2) \), respectively. The parameters \( f_s, f_s, \Gamma_0 \) (or \( f_0, \kappa, \Gamma_0 \), equivalently) can in principle be calculated by direct integrations between wave-functions and external scattering waves. Here, for simplicity, we treat them as fitting parameters, which are retrieved by fitting the CMT curves with their simulation counterparts. The opaque background blocks the radiations of the resonator to one of the available ports; thus \( \langle k_s | \psi_s \rangle = \langle k_1 | \psi_2 \rangle = 0 \). Further considering the mirror symmetry possessed by the system, it is found that the radiation damping rates of two collective modes are identical (\( \Gamma_s = \Gamma_s = \Gamma_0 \)).

Solid lines in Fig. 2a, b are the CMT-computed spectra with parameters \( \kappa/f_0 = -0.2 \), \( \Gamma_0/f_0 = \Gamma_0 = 0.15 \) (both the near field coupling and the radiation damping rate are normalized by \( f_0 \)). To further reveal the underlying physics, Fig. 2c depicts the transmission spectra calculated by two independent-oscillator models, i.e., \( \tau_s = \Gamma_0/(\hbar (f - f_1) + \Gamma_0) \) and \( \tau_a = -\Gamma_0/(\hbar (f - f_3) + \Gamma_0) \), as red and blue lines, respectively. Compared to the CMT-calculated transmission spectrum, though \( \tau_s \) and \( \tau_a \) can approximately describe the transmission spectrum at the immediate vicinities of these two peaks, significant deviations exist in frequencies away from these two peaks. This is quite understandable since the final transmission spectrum is dictated by \( t = \tau_s + \tau_a \) rather than \( \tau_s \) or \( \tau_a \) alone.

Fig. 2. Phase coverage for a 2-mode 2-port model in an opaque background. (a) CMT-computed spectra of transmission amplitude (solid line) phase (dashed line), and (b) Smith curve obtained for a 2-mode 2-port model in the opaque background with parameters \( \kappa/f_0 = -0.2 \), \( \Gamma_0/f_0 = \Gamma_0 = 0.15 \). (c) The transmission amplitude (solid line), phase (dashed line), and (d) Smith curves of the anti-symmetric mode \( \tau_s \) (blue) and symmetric \( \tau_a \) mode. Phase diagrams of (e) phase coverage \( \Delta \phi \) and (f) transparency bandwidth \( \Delta BW \) versus \( \Gamma_0/f_0 \) and \( \kappa/f_0 \) calculated with CMT for the 2-mode-2-port model. Inset of (e) depicts the typical one-peak transmission spectrum within the weak-coupling region, and inset of (f) shows how transparency bandwidth \( \Delta BW \) is defined and calculated.
Hence, the interference between the two modes are not negligible. In fact, the validities of two independent oscillators can only be justified when the considered two modes exhibit extremely large quality (Q) factors, which are highly undesired in metamaterial designs requiring flat transparency windows.

Such mode-interference effect can help us understand the phase variation inside the transparency window. Smith curves of $t_1$ and $t_2$ are separately depicted as blue and red lines in Fig. 2d, which represent two independent circles covering half space of the whole polar region without mutual interferences. Obviously, ignoring the mode interference effect, ideally, the transmission phase must vary from 0 to 180°, which covers a range of 180° as the frequency changes from $f_1$ to $f_2$. However, in reality, the Smith curve of $t$ (Fig. 2b) deviates significantly from that shown in Fig. 2d. In particular, now the two peak frequencies ($f_1$ and $f_2$) do not locate exactly at the real axis (Fig. 2b); also the Smith curve does not touch the origin at a frequency in between the two peaks, both caused by the mutual interference between the two modes. As a result, the phase variation range inside the transparency window is only 82.8° (see the shaded region in Fig. 2a), less than 180° as predicted by Fig. 2d without considering the mode-interference effect.

Based on the above analyses, how to control the phase variation range in such 2-mode systems is discussed. Obviously, the mode-interference effects have close relations with the Q factors of two collective modes, which are in turn, dictated by the coupling strength $\kappa$ and the radiation damping $\Gamma_0$ of the original mode. To quantify these arguments, CMT is used to compute the phase coverage $\Delta \phi$ enabled by such two-mode systems as varying two model parameters $\Gamma_0$ and $\kappa$. Fig. 2e depicts the value $\Delta \phi$ (color map) as a function of $\kappa$ and $\Gamma_0$. It is interesting to note that the line $\kappa = \Gamma_0$ separates the whole space into two sub-regions: the right-lower region with $0 < \Delta \phi < 180°$ and the left-upper region with $\Delta \phi = 0$. The physics is described in the following. Whereas the two resonators are coupled tightly yielding two well-defined transmission peaks in the strong-coupling ($\kappa > \Gamma_0$) region, no well-defined "modes" can be formed in the weak coupling ($\kappa < \Gamma_0$) region (see corresponding insets in Fig. 2e), where the rate of the radiation decay is higher than that of the inter-mode exchange. It is quite expectable that $\Delta \phi$ covers a wide range in the strong-coupling region, but drops to zero in the weak-coupling region where no perfect transmission peak can be clearly identified.

In addition to the phase variation range $\Delta \phi$, it is also essential to consider the amplitude fluctuations within the transparency band, for the purpose of practical meta-atoms designs. To quantitatively measure such amplitude fluctuations, we define $\Delta A_{\text{BW}} = (|BW_1 + BW_2|)/2\kappa$ as the transparency bandwidth (see inset of Fig. 2f), which corresponds to the frequency region with high transmittance ($T > 0.85$). Fig. 2f depicts how $\Delta A_{\text{BW}}$ varies against $\kappa$ and $\Gamma_0$, which reveals a clear trade-off between $\kappa$ and $\Gamma_0$, as $\Delta A_{\text{BW}}$ is maximized (approaching to 180°) at the limit of $\kappa < \Gamma_0$ (Fig. 2e), the transparency bandwidth $\Delta A_{\text{BW}}$ approaches to zero in that limit. Instead, a large bandwidth is found in the region around $\kappa \sim 1.4\Gamma_0$, defined as "intermediate region" (shaded region in Fig. 2e, f), where the phase variation is only 90°, about half of the maximum phase coverage. Such intriguing trade-off between phase coverage and transparency bandwidth makes the 2-mode systems unsuitable for designing high-efficiency meta-devices with the purposes of wavefront-control, since these devices require full-range (360°) phase coverage coinciding with high transmission amplitudes. Instead, balancing the contradictory requirements on $\Delta \phi$ and $\Delta A_{\text{BW}}$, it is found that such systems in the intermediate region are particularly suitable for designing high-performance polarization-control devices that do not need large phase coverage, such as quarter wave-plates.

After the intrinsic limitations of the 2-mode systems with an opaque background in providing wide-range phase coverage are understood, the possible strategies to solve the issue will be discussed. Obviously, adding more modes to the system is one possible solution. Still assuming that the background is opaque, CMT is used to analyze a 3-mode model (see Fig. 1c), obtained by adding another identical layer. First TBM [42] is used to obtain all collective modes. The Hamiltonian matrix can be written as

$$\frac{d}{dt} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = i \hat{H} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = i \begin{pmatrix} 0 & \kappa \\ \kappa & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}.$$  \hspace{1cm} (5)

Diagonaling this Hamiltonian matrix, we obtain three eigen collective modes as

$$f_1 = f_0 + \sqrt{2} \kappa, \quad |\Psi_1\rangle = \left( |\phi_1\rangle + \sqrt{2} |\phi_2\rangle + |\phi_3\rangle \right)/2,$$

$$f_2 = f_0, \quad |\Psi_2\rangle = \left( |\phi_1\rangle - |\phi_2\rangle \right)/\sqrt{2},$$

$$f_3 = f_0 - \sqrt{2} \kappa, \quad |\Psi_3\rangle = \left( |\phi_1\rangle - \sqrt{2} |\phi_2\rangle + |\phi_3\rangle \right)/2.$$  \hspace{1cm} (6)

Now turn-on the radiation channels and perform the CMT analyses [45] following the same processes as in the 2-mode system, then we get the transmission coefficient as

$$t = \frac{\Gamma_0}{i(f-f_0)-\Gamma_0} - \frac{\Gamma_0(f-f_0)}{i(f-f_0) - \sqrt{2}\kappa} - \frac{\Gamma_0(f-f_0)}{i(f-f_0) + \sqrt{2}\kappa} - \frac{\Gamma_0(f-f_0)}{i(f-f_0) + \sqrt{2}\kappa}.$$  \hspace{1cm} (7)

Fig. 3a and b depict the CMT-computed transmission spectra of a 3-mode system with $\kappa/f_0 = -0.2, \Gamma_0/f_0 = 0.04$. Three perfect transmission peaks are found in the transmission spectrum, which correspond to three collective modes with the resonance frequency $f_1, f_2, f_3$, respectively. Neglecting far-field interactions (valid at the limit of $\Gamma_0 \ll \kappa$), the transmission amplitude can be expressed as $t \sim t_1 + t_2 + t_3$, where

$$t_1 = \frac{\Gamma_0}{i(f-f_0) + \sqrt{2}\kappa} + \Gamma_0,$$

$$t_2 = \frac{\Gamma_0}{i(f-f_0) + \sqrt{2}\kappa} + \Gamma_0,$$

$$t_3 = \frac{\Gamma_0}{i(f-f_0) + \sqrt{2}\kappa} + \Gamma_0.$$  \hspace{1cm} (8)

to represent the transmission coefficients of three independent oscillators. Considering the symmetries of these modes, one may easily find that at three (bare) resonant frequencies, $t_1, t_2$ and $t_3$ exhibit phases 0, 180°, 360°, respectively (see Fig. 3c). Therefore, ideally, if the mutual interferences between the three modes are completely neglected, the phase of $t$ can cover up to 360°. However, in practice, mode-interference inevitably affects the phase coverage of the entire system, with the same physics as in the 2-mode system. As shown in Fig. 3d, mode-interference shifts the first and third peaks anticlockwise and clockwise, respectively; but the second peak is unaffected due to the symmetry protection, resulting in a smaller phase-variation range centered at 180°, as shown in Fig. 3b.
the transmission peaks. Balancing these two effects, it is found that
the largest phase coverage that a 3-mode system (with an opaque
background) can open is roughly 240° inside the intermediate
region (i.e., with transmittance higher than 0.85).

After the key EM properties of the first type of meta-atoms (e.g.,
Fig. 1b, c) are learned, the second type of multilayer meta-atoms is
studied, which can be described by CMT in a transparent back-
ground, as shown in Fig. 1d. Such meta-atoms have also been much
widely used in building transmissive meta-devices[46,47]. Due to
the differences in background properties, such model exhibits
completely different EM properties (particularly the phase cover-
age properties) from its opaque-background counterpart, and
therefore it may provide appropriate phases that cannot be cov-
ered by the opaque-background models. Specifically, the transmis-
sion coefficient of the 2-mode CMT (in a transparent background)
is given by (see Supplementary Information Section 1 for mathe-
matical derivation)
\[
t = 1 + \frac{4\Gamma_0 \sin^2(\eta)}{i[f - f_0 + \kappa] - 4\Gamma_0 \sin^2(\eta)} + \frac{4\Gamma_0 \cos^2(\eta)}{i[f - f_0 - \kappa] - 4\Gamma_0 \cos^2(\eta)},
\]
where \(\eta\) represents the phase difference of two resonators’
radiations toward forward and backward directions, closely related
to the distance between two resonators. Solid lines in Fig. 4a, b are
the CMT-computed spectra with parameters \(\kappa/f_0 = -0.2, \Gamma_0/f_0 = 0.04\).
Unlike the case with an opaque back-
ground, here it is found two dips in the transmission spectrum,
caused by the contribution from the transparent background.
Specifically, Eq. (9) shows that the final transmission contains
three parts, which can be rewritten as \(t = 1 + t_1 + t_2\), where
\(t_1 = \frac{4\Gamma_0 \cos^2(\eta)}{i[f - f_0 - \kappa] - 4\Gamma_0 \cos^2(\eta)}\) and \(t_2 = \frac{4\Gamma_0 \sin^2(\eta)}{i[f - f_0 + \kappa] - 4\Gamma_0 \sin^2(\eta)}\) represent the scatterings from the two
modes. Both \(t_1\) and \(t_2\) are equal to \(-1\) (possessing a 180° phase) at
their resonance frequencies (Fig. 4c). The presence of the back-
ground transmission (i.e., the first term in Eq. (9)) completely
changes the EM responses of the whole system. The phase coverage
is mainly contributed by two transmission bands at low and high
frequencies (Fig. 4a, b), which is quite different from its opaque-
background counterpart (Fig. 2a, b). Such difference can be better
understood from the evolutions of Smith curves shown in Fig. 4d,
where the interference between \(t_1\) and \(t_2\) first slightly merge two
individual circles while adding the background transmission, and

![Fig. 3. Phase coverage for a 3-mode 2-port model in an opaque background. (a) CMT-computed spectra of transmission amplitude (solid line), phase (dashed line) and (b) Smith curve obtained for a 3-mode 2-port model in the opaque background with parameters \(\kappa/f_0 = -0.2, \Gamma_0/f_0 = 0.04\). (c) The transmission amplitude (solid line), phase (dashed line), and (d) Smith curves of three collective modes \(t_1\) (red), \(t_2\) (blue) and \(t_3\) (black) calculated by Eq. (8). Phase diagrams of (e) the phase coverage \(\Delta \varphi\) and (f) transparency bandwidth versus \(\Gamma_0/f_0\) and \(\kappa/f_0\) calculated with CMT for 2-port 3-mode model. Inset of (e) depicts the typical single-peak transmission spectrum within the weak coupling region.](image-url)
then shifts the whole curve from the left region to the right one. It is not surprising that the phase coverage is strongly dependent on the inter-mode coupling \( j = C_0 \) (Fig. 4e), sharing the same physics as that discussed in the opaque-background model (Fig. 2). Moreover, there is a new degree of freedom in such a system, i.e., the phase difference \( g \) between two original resonators. It can be exploited to significantly modify the Smith curve, resulting in different phase coverage (Fig. 4f). Through adjustment of \( j \) and \( g \), the response of the system can be effectively changed, thus yielding phases covering different ranges. In particular, when the condition meets \( \eta = \pi/4, \kappa = 0 \), the transmission coefficient has a magnitude of 1 with the phase covering from 0 to 2\( \pi \). Many applications have been implemented around the optical band on this basis [26,48]. However, such a condition \( \eta = \pi/4 \) and \( \kappa = 0 \) usually requires wavelength-scale 3D structures, which is not easy to be realized by metasurfaces with subwavelength thickness.

The above CMT analyses help us establish the following guidelines to build transmissive meta-devices with distinct functionalities. (1) Subwavelength multilayer meta-atoms with 2 or 3 layers with opaque background, no matter in an opaque background or a transparent one, can only provide phases covering particular ranges. Therefore, using such meta-atoms alone is not suitable for constructing wavefront-control meta-devices, but is acceptable in many polarization-control applications. (2) It is possible to adopt meta-atoms of the same type to build wavefront-control meta-devices, but the number of layers must be large enough, which is unfavorable for many application scenarios. (3) A more advisable approach is combining meta-atoms of different types, with phase coverage ranges compensating each other, so as to build wavefront-control devices requiring full-range phase coverage.

3. Practical meta-atom designs and application demonstrations

In this section, numerical simulations and microwave experiments are carried out to demonstrate three different meta-devices constructed by different types of multilayer meta-atoms, following the general design guidelines discussed in last section.

3.1. Quarter wave-plate

First how to use a 2-layer meta-atom to design a polarization-control meta-device is illustrated. The meta-atom is schematically depicted in Fig. 5a, consisting of a cross-shape resonator inserted in subwavelength-size metallic mesh, where each metallic layer is fabricated on a 0.5-mm-thick dielectric substrate (\( e = 2.95 + 0.005i \)) and the inter-layer distance is denoted by \( h \). Obviously, such a meta-atom can be described by the 2-port 2-mode model (Fig. 2b) studied in last section [49]. We first search
for meta-atoms with balanced $j$ and $C_0$ that can be used for our purpose. Fig. 5b depicts the FDTD-simulated spectra of a series of such structures with identical bar lengths ($l_{1} = l_{2} = 6$ mm) but different inter-layer distance $h$. Simulation results show that the EM properties of two transmission peaks, associated with symmetric and anti-symmetric modes of the model, can be dramatically controlled by the parameter $h$. All simulated spectra are well described by the 2-mode CMT model established in last section, with one particular example ($h = 5$ mm, corresponding to the dashed line in Fig. 5b) explicitly studied in Fig. 5e. Thus all model parameters

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**Fig. 5.** Simulation results of the two-layer structure. (a) Schematics of the two-layer structure. (b) Transmittance spectra versus thickness $h$ computed by FDTD simulations, where the solid black lines denote the perfect transmission peak, and the symbols (error-bars) denote the resonating frequencies (radiation damping rates) calculated with CMT. (c) CMT parameters $\kappa$, $\Gamma_0$ as a function of thickness $h$ obtained by fitting FDTD-simulations. (d) $\kappa - \Gamma_0$ phase diagram, where the shaded area indicates the intermediate region and the red (blue) star symbols denote the designed meta-atoms for $y$-polarization ($x$-polarization). (e) Transmission amplitude (red) and phase (blue) spectra, and (f) Smith curves of two-layer system, where dashed lines denote simulated curves and solid lines denote CMT curves with $f_0 = 7.76$, $\kappa = 0.81$, $\Gamma_0 = 0.59$, all in units of GHz. (g) $T_y$ and (h) $T_x$ vary against bar length $l_2$, where the black lines represent the frequencies $f_{1,y}$ and $f_{2,y}$ for the 1st mode with $x$-polarization and the 2nd mode with $y$-polarization.
are successfully retrieved from the simulation results for realistic structures. Fig. 5c presents how the two model parameters \((|\kappa|/f_0, \Gamma_0/f_0)\) vary against \(h\). It is found \(|\kappa| < f_0\), indicating a weak coupling between resonators in neighboring layers. To see how well the CMT analyses work, Fig. 5b depicts the positions of transmission peaks (symbols) and associated bandwidths (error bars), obtained by the CMT model with parameters shown in Fig. 5c. Comparisons show that the CMT model has indeed captured all essential features of the FDTD ones. The two CMT parameters have distinct dependences on \(h\). Clearly, \(\kappa\) is a decreasing function of \(h\), since the wave-function overlapping decreases as \(h\) increases. On the contrary, \(\Gamma_0\) is an increasing function of \(h\), caused by the decreased capability of a metallic mesh to block EM waves as \(h\) increases.

The intrinsic connection between \(h\) and the CMT parameters (Fig. 5c) motivates us to establish the following strategy to search for appropriate meta-atoms. Clearly, adjusting \(h\) can effectively control the two CMT parameters (\(\kappa\) and \(\Gamma_0\)), which in turn, drives the model moving in the phase diagram (see red open circles shown in Fig. 5d), finally arriving at an appropriate position (denoted as the red star) in the optimized region. Fig. 5e and f depict the simulated and CMT-calculated transmission spectra of the optimized structure (See Supplementary Information Section II for more fitting results), both exhibiting an optical transparency window (OTW) during 7.1 – 8.3 GHz covering a phase variation range of around 90°.

After the basic structure with optimized \(h\) is found, how to design the quarter wave plate is discussed by further exploiting the lateral anisotropy of the structure. Obviously, it needs to adjust one of the length bars (say, \(l_2\)), while other geometric parameters (\(l_1\) and \(h\)) are kept unchanged, in order to open enough cross-polarization phase difference at a given frequency. Fig. 5g and h depict how the transmission spectrum changes with the adjustment of \(l_2\) for two different incident polarizations. The adjustment of \(l_2\) is found to drastically change the \(x\)-polarized transmission spectra (Fig. 5h) but have little impact on the \(y\)-polarized spectra (Fig. 5g). Therefore, as \(l_2\) takes a particular value (4.9 mm), the low-frequency peak in the \(x\)-polarized spectrum coincides well with the high-frequency peak in the \(y\)-polarized spectrum, yielding a 90° phase difference at this particular frequency yet with almost perfect transmittance for both polarizations, as illustrated in Fig. 5h. To better illustrate the physics, the CMT parameters are retrieved for the series of \(x\)-polarized spectra as shown in Fig. 5h (see Supplementary Information Section III for details), and then their positions are depicted as blue circles in the phase diagram (Fig. 5d). Interestingly, the adjustment of \(l_2\) only drives the system to move inside the optimized region. In fact, while adjusting \(l_2\) changes the resonant frequency \((f_0)\) of the original mode and in turn changes both \(\Gamma_0\) and \(\kappa\) associated with that mode, it does have significant impacts on the ratio of \(\Gamma_0/\kappa\). As a result, the system still stays in the intermediate region. Thus, the lateral anisotropy is fully applied to finally reach a design, with two polarizations both located inside the intermediate region yet exhibiting large enough phase difference.

The quarter-wave plate is fabricated according to the design, with metallic structures printed on a 0.5-mm-thick F4B-PCB \((\varepsilon_r = 2.95 + 0.005i)\), separated by 5 mm – thick spacers made by polymethylacrylimide (PMI, with \(\varepsilon_r = 1.02\)) (see sample photo in Fig. 6a). In the measurements, the sample is shined normally with microwaves emitted from a horn antenna placed 1 m away from the sample, and then another horn antenna placed 1 m away from the sample is used to collect the transmitted signals. Both source and receiver are connected to a vector-field analyzer (Agilent E8362c) so that both the amplitudes and phases of the transmitted signals can be obtained. Open symbols in Fig. 6b, c are the measured transmission spectra for the designed device under two polarizations (see Supplementary Information Section III for details), both exhibiting an optical transparency window (OTW) during 7.1 – 8.3 GHz covering a phase variation range of around 90°.

Fig. 6. Experimentally measured results for QWP. (a) Photograph of the experimental QWP sample, and the inset is a side view. (b) Experimental measured (symbols) and simulated (solid line) transmittance \(T_x\) (red) and \(T_y\) (blue) spectrum for two polarizations. (c) Experimental measured (symbols) and simulated (solid line) PCRWP spectrum.
polarizations, which are in perfect agreement with FDTD simulations. Define polarization-conversion ratio \( \text{PCR}_{\text{QWP}} = - \text{Im}(t_x t_y) \) as the absolute conversion efficiency from linear polarization to circular polarization. The fabricated sample exhibits the largest
PCR (95%) at 8.13 GHz, with a 0.5 GHz bandwidth, inside which PCR is greater than 85% (Fig. 6c).

3.2. Half wave-plate

As the second demonstration, a half wave plate is designed based on a similar strategy. Previous theoretical analyses show that the 3-mode system can yield 180° phase coverage; therefore the three-layer structure (Fig. 7a) is adopted to perform the design. The design strategy is essentially the same as that adopted in the design of the quarter wave-plate. Specifically, first, the appropriate spacer thickness \( h \) is found to drive the system to reach the optimized region shown in Fig. 3e, and then the lateral anisotropy is fully utilized to open enough phase difference for two cross polarizations.

Fig. 7b depicts how the FDTD-simulated transmission spectrum of the three-layer system varies against \( h \), indicating the evolutions of three perfect transmission peaks associated with three collective modes defined in Eq. (6). Again, these simulated spectra can be well described by the 3-mode CMT model, with the best-fitted model parameters \( f_0, \kappa \) plotted in Fig. 7c as functions of \( h \). The two CMT parameters, \( f_0 \) and \( \kappa \), exhibit similar dependence on \( h \) as that in the two-layer case, sharing the same underlying physics. Therefore, adjusting \( h \) can efficiently change \( f_0 \) and \( \kappa \), which in turn, drives the system to move in the phase diagram until reaching an optimized position (with \( h = 10 \text{ mm} \) ) represented by the red star in Fig. 7d. Indeed, the transmission spectrum of the optimized system (\( h = 10 \text{ mm} \) ) exhibits a flat OTW with a phase change of 180° (Fig. 7e), in good agreement with the CMT-computed spectra with \( f_0 = 7.36 \text{ GHz} \), \( \kappa = -0.59 \text{ GHz} \), \( f_0 = 0.65 \text{ GHz} \) (see Supplementary Information Section II for more fitting results). Fig. 7f depicts the Smith curves of such a structure obtained by FDTD simulations and CMT calculations, in consistent with the generic CMT analyses presented in last section (Fig. 3b).

The second-step design is conducted by adjusting the bar length of \( l_2 \), while other parameters are kept unchanged. Again, adjusting \( l_2 \) can shift the x-polarized resonances rather than the y-polarized ones. Therefore, such a \( l_2 \) is chosen to make the first peak in x-polarization coincide with the third peak in y-polarization. Interestingly, such an optimization process essentially makes the system (in term of x-polarization response) move inside the intermediate region, thereby reaching at a design exhibiting the largest possible cross-polarization phase difference, as shown by the trajectory of retrieved CMT parameters (see Supplementary Information Section III for detailed parameters) as \( l_2 \) is adjusted (Fig. 7d). At \( l_2 = 3.5 \text{ mm} \), a phase difference with 180° can be opened with an almost perfect transmittance for two polarizations, as illustrated in Fig. 7h.

Next, a sample is fabricated, with geometric parameters as \( l_1 = 6, l_2 = 3.5, P = 10, s = 4, w = 0.5, a = 9 \), all in units of mm (Fig. 8a is a photograph of the sample), and a microwave experiment is performed to characterize its transmission property in the same way as in last sub-section. Symbols in Fig. 8b are the measured transmittance spectra, which are in excellent agreement with FDTD simulations. Symbols and solid lines in Fig. 8c depict the spectra of PCR-HWP calculated from the experimental and simulation data, respectively, where \( \text{PCR}_{\text{HWP}} = \frac{|T_{xx} - T_{yy}|^2}{4} \) describes the polarization conversion efficiency. Perfect agreement between experimental and simulation results is noted. Fig. 8c shows that an HWP with \( \text{PCR}_{\text{HWP}} = 95% \) at 7.59 GHz is achieved, exhibiting a 0.5 GHz bandwidth with \( \text{PCR}_{\text{HWP}} > 85\% \).

![Fig. 8](https://example.com/fig8.png)

Fig. 8. Experimentally measured results for HWP. (a) Photograph of the experimental HWP sample, and inset is a side view. (b) Experimental measured (symbols) and simulated (solid line) transmittance \( T_x \) (red) and \( T_y \) (blue) spectrum for two polarizations. (c) Experimental measured (symbols) and simulated (solid line) PCR-HWP spectrum.
3.3. Beam deflector

Finally, how to design a beam deflector using multilayer meta-atoms with thin thickness is described for realizing high-efficiency anomalous refraction of EM waves. The construction of such a beam-deflection meta-device needs a series of meta-atoms, all allowing high transmissions of EM waves at the same frequency but exhibiting different transmission phases that are linearly varying in space covering the whole 360° region, as shown in Fig. 9b. Here, the working frequency of 8 GHz and the phase gradient of

![Diagram of beam deflector with phase distribution and transmission coefficient](image)

**Fig. 9.** Design of a beam deflector. (a) Schematic of a beam deflector meta-device. (b) Schematic of the phase distribution and transmission coefficient required by the beam deflector device. The open star symbols (solid star symbols) denote the transmission phase and amplitude of the opaque background three-layer model (the transparent background two-layer model). (c) Transmission amplitude (solid) and phase (dash) of the opaque background three-layer system as a function of scaling factor \( \chi \), where \( l_1 = l_0 - y_0, s = y_0, l_0 = 6 \text{ mm}, s_0 = 4 \text{ mm} \). (d) \( \kappa - \Gamma_0 \) phase diagram, where the red (blue) symbols denote CMT parameters with varying thickness \( h \) (scaling factor \( \chi \)). (e) Transmission amplitude (solid line) and phase (dashed line) of the transparent background two-layer system as a function of scaling factor \( \chi \). (f) Smith curves obtained by CMT (solid line) and FDTD (dashed line). (g) Far-field scattered intensity and (h) calculated electric field patterns at 8 GHz, where thin metallic sheets are inserted to separate two adjacent meta-atoms to reduce their mutual functional interferences.
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This is achieved based on meta-atoms with as few as possible layers is described here. Obviously, the meta-atoms with an identical type but more layers can be used to realize the desired phase coverage. However, the realized meta-device will be too thick. Alternatively, meta-atoms can be combined with different types, all possessing thin thicknesses but with phase coverage compensating each other. Since the phase coverage of the opaque-background 2-mode model (Fig. 1b) is too narrow (only compensating each other), the phase coverage of the thick. Alternatively, meta-atoms can be combined with different phase coverage. However, the realized meta-device will be too thick. Therefore, it is necessary to find a meta-atom of another type providing desired transmission phases. According to the results of Fig. 7b, $h = 10$ mm is selected. As shown in Fig. 9c, the structural details of the metallic crosses in each layer are adjusted; while $h = 10$ mm, $w = 0.5$ mm, $P = 10$ mm are fixed in the first-step optimization. Then, meta-atoms with transmission phases inside the range of 56°–30° and with high transmittance ($T > 0.85$) at 8 GHz (shaded region in Fig. 9c) can be obtained. Such calculations facilitate to select four meta-atoms (see Supplementary Information Section IV for their geometrical parameters) yielding transmission phases 63°, 135°, 207°, 279°, correspondingly, as shown in Fig. 9b. Again, the roles of such two-step optimizations can be clearly seen from the two paths shown in the phase diagram (Fig. 9d): adjusting $h$ effectively drives the system into the intermediate region (red symbols) while changing the lateral size of cross moves the system inside the intermediate region (blue symbols). However, a meta-atom with transmission phase 35° cannot be found based on such a searching strategy, due to the intrinsic limitation on phase coverage enabled by such a model (Fig. 9c).

Conflict of interest

The authors declare that they have no conflict of interest.

Acknowledgments

This work was supported by National Key Research and Development Program of China (2017YFA0303500), the National Natural Science Foundation of China (11704240, 11734007, and 11674068), Natural Science Foundation of Shanghai (17ZR1409500 and 18QA1401800), Shanghai Science and Technology Committee (16JC1403100), Shanghai East Scholar Plan, and Fudan University-CIOMP Joint Fund.

Author contributions

Shihi Xiao and Lei Zhou contributed to the conception of the research. Bowen Yang performed theoretical analyses, simulation and wrote the manuscript. Tong Liu helped theoretical analyses and wrote the manuscript. Huijie Guo performed the experimental measurement. All authors discussed the results and reviewed the manuscript.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.scib.2019.05.028.