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Ultra-wide band reflective metamaterial wave plates for terahertz waves

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Abstract – Terahertz (THz) electromagnetic waves have important applications in science and technology but available functional devices suffer from the issues of bulky size, low efficiency and narrow bandwidth. Here, based on Jones matrix and Poincaré sphere analyses, we present a set of general criterions to help design high-efficiency ultra-wide band THz wave plates using ultra-thin reflective metamaterials. Two half-wavelength and one quarter-wavelength THz wave plates are designed and fabricated based on the general criterions, and their excellent polarization manipulation capabilities are demonstrated experimentally. In particular, the realized devices, with thicknesses ∼λ/7, exhibit polarization-conversion efficiencies higher than 80% in ultra-wide working bandwidths (relative bandwidth >80% at about ∼0.7 THz).

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Introduction. – The terahertz (THz) frequency regime has recently attracted much attention, due to many exciting applications of electromagnetic (EM) waves [1], such as imaging [2], sensing [3], communications [4] and non-invasive testing [5]. To realize these fascinating applications, high-performance THz wave manipulation devices are desired, other than the frequently mentioned THz sources/d Detectors [6–8]. Among these functional devices, polarization manipulators (e.g., polarizers and wave plates) are of great importance. However, devices fabricated with conventional materials (e.g., dichroic crystals [9] and birefringent materials [10]) are typically too bulky in size and less efficient, being inconvenient for practical applications.

Recently, considerable efforts were devoted to making THz functional devices with metamaterials (MTMs) —artificial materials composed of subwavelength microstructures with tailored EM properties [11–13]. In particular, many MTM-based polarization manipulation devices have been designed/fabricated in both transmission and reflection geometries, such as polarizers [14–17] and wave plates [18–28], some of which were inspired from their counterparts in other frequency domains [29–31]. Having so many polarization manipulation devices with different structures/symmetries, it is interesting to ask the following questions: Are there any general criterions to guide designing such devices? Can we have other designing possibilities to achieve the same polarization manipulation effects?

In this article, based on Jones matrix and Poincaré sphere analyses, we establish a general strategy to design THz reflective wave plates with the desired polarization manipulation functionalities. In particular, we show that there are infinite ways to design THz MTM-based wave plates, while previously realized devices belong to a particular type of solution in our general theory. To illustrate the applicability of our approach, we design and fabricate two half-wavelength wave plates (HWP) and a quarter-wavelength wave plate (QWP), and then experimentally characterize their excellent polarization...
manipulation performances. We show that all these devices exhibit very high polarization manipulation efficiencies (>80%) over ultra-wide frequency bands (relative bandwidth >80%). Our theory can not only offer immediate assistance to designing wave plates in the THz regime, but also shed light on designing polarization-related functional devices in higher-frequency domains.

**Results and discussions.**

Criteria for designing wave plates in reflection geometry. We start from analyzing the EM properties of a generic ultra-thin MTM (e.g., metasurface) placed on the yz-plane, with EM characteristics described two Jones matrices:

\[
R = \begin{bmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{bmatrix} \quad \text{and} \quad T = \begin{bmatrix} t_{xx} & t_{xy} \\ t_{yx} & t_{yy} \end{bmatrix},
\]

expressed in the linear polarization (LP) bases. In this paper, we only consider the polarization manipulation in the reflection mode, and our derivation can be easily extended to the transmission mode. Consider the case in which the metasurface is shined by a normally incident EM wave with \(E_{\text{inc}} = E_{x\text{inc}} |x\rangle + E_{y\text{inc}} |y\rangle\) (see fig. 1(a)), the \(E\) field in the reflected EM wave can be expressed as \(E_{\text{ref}} = E_{x\text{ref}} |x\rangle + E_{y\text{ref}} |y\rangle\) with

\[
\begin{pmatrix} E_{x\text{ref}} \\ E_{y\text{ref}} \end{pmatrix} = R \cdot \begin{pmatrix} E_{x\text{inc}} \\ E_{y\text{inc}} \end{pmatrix}.
\]

Changing the LP bases to circular-polarization (CP) bases with the following transformation |±\rangle = (|x\rangle ± i|y\rangle)/\sqrt{2}, we get

\[
\begin{pmatrix} E_{+\text{ref}} \\ E_{-\text{ref}} \end{pmatrix} = \tilde{R} \cdot \begin{pmatrix} E_{+\text{inc}} \\ E_{-\text{inc}} \end{pmatrix},
\]

where \(\tilde{R} = SRS^{-1}\) and \(\begin{pmatrix} E_{+} \\ E_{-} \end{pmatrix} = S \cdot \begin{pmatrix} E_{x} \\ E_{y} \end{pmatrix}\) with the transformation matrix being

\[
S = \sqrt{\frac{1}{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}.
\]

We use Poincaré’s sphere to visualize the polarization operations. As we know, any polarization state described by a vector \(\begin{pmatrix} E_{+} \\ E_{-} \end{pmatrix}\) can find a corresponding point \((\theta, \phi)\) on Poincaré’s sphere, with \(\theta\) and \(\phi\) determined by \(E_{+} = \cos(\theta/2)\) and \(E_{-} = \sin(\theta/2)e^{i\phi}\), respectively (see fig. 1(b)). For example, the north/south pole on Poincaré’s sphere represents a pure left/right CP state, while the point (90°, φ) represents a LP state with \(φ/2\) denoting the angle between the \(E\) vector and the \(x\)-axis. To facilitate our later discussions, we assume that \(|R| = 1\), which is indeed the case for a totally reflective system without absorption. In the general case with \(|R| \neq 1\), we can always add a normalization constant to make \(|R| = 1\). Such a normalization constant does not affect the polarization manipulation functionality discussed in this paper. From eq. (1), we understand that the role of \(\tilde{R}\) is just to move the polarization state from its initial position to the final one on Poincaré’s sphere.

We now derive the concrete form of \(\tilde{R}\) to realize a certain polarization manipulation. In principle, any movement on Poincaré’s sphere can be described by a rotation by an angle \(ξ\) with respect to a certain axis \(\hat{n}\) = \((\theta, \phi)\), which is equivalent to rotating the coordinate system by an angle \(-ξ\) along the axis \(\hat{n}\). In the present \(SU(2)\) representation [32], rotating the (effective) coordinate system by an angle \(\phi\) along the \(i\)-th axis is described by the transformation matrix \(g_i(\phi) = \exp(i\phi \cdot \sigma_i/2)\) acting on the bases, with \(\sigma_i\) \((i = x, y, z)\) being the \(i\)-th Pauli matrix. Therefore, rotating the coordinate system by an angle \(ξ\) along the general \(\hat{n}\)-axis is obtained by a series of successive operations, resulting in the following transformation matrix:

\[
\tilde{g}_n(ξ) = \tilde{g}_z(\phi) \cdot \tilde{g}_y(\theta) \cdot \tilde{g}_x(ξ) \cdot \tilde{g}_y(\theta) \cdot \tilde{g}_z(\phi) = \exp(iξ\hat{n} \cdot \hat{σ}/2),
\]

acting again on the corresponding bases in the \(SU(2)\) representation [32]. The inverse of \(\tilde{g}_n(ξ)\) finally gives the desired transformation matrix acting on the expansion components,

\[
\tilde{R}_{n,ξ} = \exp(-iξ\hat{n} \cdot \hat{σ}/2) = \begin{pmatrix} \cos(ξ/2) - i \sin(ξ/2) \cos θ & -i \sin θ \sin(ξ/2) \cdot \exp(-iφ) \\ -i \sin θ \sin(ξ/2) \cdot \exp(iφ) & \cos(ξ/2) + i \cdot \sin(ξ/2) \cos θ \end{pmatrix}.
\]

Therefore, the desired form of \(\tilde{R}\) can be easily retrieved with the help of eq. (2), if one can determine the three angles \((θ, φ, ξ)\), after the initial/final polarization states are specified. In the following, we take two examples...
(i.e., HWP and QWP) to illustrate how to construct the desired $\mathbf{R}$.

Consider first the HWP. Suppose that in general the initial state is an LP state located at a point $(90^\circ, \phi_0)$ on Poincaré’s sphere, then the final LP state must locate at $(90^\circ, \phi_0 + 180^\circ)$ (see fig. 1(c)). A simple analysis shows that any $\xi = 180^\circ$ rotation along a vector $\mathbf{n} = (\theta, \phi_0 \pm 90^\circ)$ (with $\theta$ being an arbitrary angle) can do the job, and therefore, the reflection Jones matrix (in CP bases) must take the form

$$
\mathbf{R}_{\text{HWP}} = a \begin{pmatrix}
-\cos \theta + i \sin \theta \cdot \exp(-i\phi_0) & i \cos \theta \\
\pm \sin \theta \cdot \exp(i\phi_0) & -\cos \theta \cdot \exp(i\phi_0)
\end{pmatrix}, \quad (3)
$$

which is transformed to

$$
\mathbf{R}_{\text{HWP}} = 
\pm i a \begin{pmatrix}
-\sin \theta \sin \phi_0 & -\sin \theta \cos \phi_0 \pm i \cos \theta \\
-\sin \theta \cos \phi_0 & -\sin \theta \sin \phi_0
\end{pmatrix}, \quad (4)
$$

in LP bases ($xyz$ coordinate system). Here $a$ is an arbitrary complex number with unit amplitude. Detailed derivations of eqs. (3) and (4) can be found in the Supplementary Material SupportInformation-PCR-EPL_1.docx (SM). It is interesting to note that we can have infinite possibilities to design our HWP (see eqs. (3) and (4)), since both $\theta$ and $\phi_0$ can take arbitrary values.

We next consider the QWP. Suppose the initial LP state is the point $(90^\circ, \phi_0)$ while the final CP state is the north/south pole on Poincaré’s sphere (see fig. 1(d)). Based on a straightforward analysis, we find that the three angles $(\theta, \phi, \xi)$ to describe such a rotation are determined by three coupled equations,

$$
\begin{cases}
\cos(\phi - \phi_0) = \pm \cot \theta, \\
\cos \xi = -\cot^2 \theta, \\
\sin \xi = \frac{1}{2} \sin 2(\phi - \phi_0).
\end{cases} \quad (5)
$$

Here the sign $"+/-"$ denotes the case in which the final state is the left CP or the right CP state, respectively (see the SM for derivations of eq. (5)). Solving eq. (5) to obtain the appropriate angles and to put them into eq. (2), we thus obtain the reflection Jones matrix for QWP. Again, eq. (5) indicates that there are infinite ways to construct a QWP.

However, not all Jones’ matrices predicted by eqs. (3)–(5) can be realized by practical MTM systems. In totally reflective metasurfaces, time-reversal symmetry and energy conservation restriction require that $r_{xy} = r_{yx}$ [33]. Take these constraints into consideration, we find that Jones’ matrices for HWP and QWP, achievable in totally reflective systems, take the following forms (see the SM for derivations of eqs. (6) and (7)):

$$
\mathbf{R}_{\text{HWP}} = a \begin{pmatrix}
\sin \phi_0 & -\cos \phi_0 \\
-\cos \phi_0 & \sin \phi_0
\end{pmatrix}, \quad (6)
$$

and

$$
\mathbf{R}_{\text{QWP}}^+ = a \begin{pmatrix}
\pm 1 - i \cdot \sin \phi_0 & i \cdot \cos \phi_0 \\
\pm 1 + i \cdot \sin \phi_0 & i \cdot \cos \phi_0
\end{pmatrix}. \quad (7)
$$

We emphasize that eqs. (6) and (7) are only for totally reflective metasurfaces. In general, the original forms of the reflection Jones matrices can be possibly realized in more complex MTM systems without such constraints. In the following, we will discuss how to design reflective metasurfaces to achieve these Jones matrices (e.g., eqs. (6), (7)).

Ultra-wideband half-wavelength wave plates. As mentioned in the previous section, there are infinite solutions of Jones’ matrices that can be used to help us design HWPs. Here, we choose two specific solutions of eq. (6) to design, fabricate, and characterize two HWPs in the THz regime.

Case 1: Consider first $\mathbf{R} = a \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, corresponding to a solution of eq. (6) with $\phi_0 = 90^\circ$. Such a device can work for incident LP wave with the $E$ vector lying at an angle $45^\circ$ with respect to the $x$-axis. To realize such a Jones matrix, we choose the tri-layer meta-atom in a metal/insulator/metal (MIM) configuration as shown in fig. 1(a), which contains a metallic cross and a continuous metal layer separated by a dielectric spacer. Since our meta-atom exhibits mirror symmetries with respect to the $xz$ and $yz$ planes, the off-diagonal terms of its Jones matrix must be zero. Meanwhile, the bottom metallic layer ensures that our meta-atom is a totally reflective system and, thus, $|r_{xx}| \approx |r_{yy}| \approx 1$ if the losses are not significant (e.g., in the off-resonance frequency regime or the involved resonances exhibit low $Q$ factors). However, the reflection phases for two polarizations (i.e., $\Phi_{xx}, \Phi_{yy}$) can be very different and strongly depend on the geometric parameters of the structure. Thus, the criterion of designing an HWP can be further simplified to $\Phi_{xx} - \Phi_{yy} = 180^\circ$.

Although in principle one can rely on brute-force simulations to design such a device, understanding the general properties of the MIM structure offers great help. According to the complete phase diagram of MIM structures as presented in ref. [34], we understand that we must choose the MIM system working in the underdamped regime so that it can exhibit a $360^\circ$ reflection phase coverage as frequency varies and a low absorption. To achieve this end, we choose a relatively thick dielectric spacer ($d = 60 \mu m$, which is about $\lambda/7$) in our design to ensure that the meta-atom works in the underdamped regime exhibiting a low resonance $Q$ factor. Moreover, to further enlarge the working bandwidth, we purposely design the structure to exhibit two magnetic resonances (i.e., the fundamental and the first high-order mode) for the polarization $\hat{E} \parallel \hat{x}$ so that the condition $\Phi_{xx} - \Phi_{yy} = 180^\circ$ can be met at two different frequencies. Since both resonance modes are of low $Q$ factors, cascading them appropriately can significantly enlarge the working bandwidth.

Based on the above consideration, we obtained the final design for the meta-atom aided by FDTD simulations$^1$.

\footnote{Concerto 7.0, Vector Fields Limited, England, 2008. In FDTD simulations, gold was treated as lossy metal with conductivity $1.0e6 S/m$, while the dielectric constant of the dielectric spacer was assumed as $\varepsilon = 3.1 + 0.04i$.}
In this paper, we employed two different methods to characterize the  \(\hat{\mathbf{E}}\parallel \mathbf{\hat{y}}\) polarization. As shown in Fig. 2(e), the measured cross-polarization response is much larger than the co-polarization one \(|r_{uu}|\) within the working band, already implying a large PCR. We can quantitatively compute the PCR spectrum using the formula PCR = \(|r_{xx} - r_{yy}|^2/2(|r_{xx}|^2 + |r_{yy}|^2)\). As shown in Fig. 2(e), the PCR spectrum thus obtained (blue squares) is in good agreement with that obtained with indirect measurement (red circles), both showing that the PCR values are close to 1 within an ultra-wide frequency band. FDTD simulation results are shown in Fig. 2(b)–(e) as solid lines, which are in excellent agreement with their corresponding experimental results. Slight discrepancies between experiments and simulations can be attributed to the sample imperfections.

Finally, we employed FDTD simulations to analyze the incidence angle dependence of the devices polarization manipulation functionality. As shown in Fig. 2(f), while the devices working bandwidth gets shrunk at large incidence angles, the obtained PCR can persist at > 80% as the incident angle is less than 55\(^\circ\). Even for very large incident angles, our device can still exhibit a narrow band inside where its PCR approaches 100%.

Case 2: We choose another solution of eq. (6) to design another HWP. Setting \(\phi_0 = 0^\circ\) in eq. (6), we have the Jones matrix \(\mathbf{R} = a(0 1)\). Instead of trivially rotating the sample presented in case 1 by an angle of 45\(^\circ\), here we propose a different approach to realize such a Jones matrix. Noting that energy conversion dictates that \(|r_{xx}|^2 + |r_{yx}|^2 = 1\) and \(|r_{uu}|^2 + |r_{uy}|^2 = 1\) in such reflective systems (if we neglect losses), to realize the desired form of the Jones matrix, we need to adjust the geometry to strengthen the off-diagonal responses (\(i.e., r_{xy}, r_{yx}\)) and consequently weaken the diagonal terms. Obviously, symmetric meta-atoms can never do the job since they naturally do not support off-diagonal terms. Therefore, we and then fabricated the device using the standard photolithography method. Figure 2(a) shows the optical image of the fabricated device. We then experimentally characterized the functionality of our device (with a dimension of 1 cm \(\times\) 1 cm) using a terahertz time domain spectroscopy (THz-TDS) system (see the SM for the details of the sample fabrication and experimental setup).

We employed two different methods to characterize the polarization manipulation functionalities of the device. In the first approach, we directly measured each component of the Jones matrix for our device and then calculated the polarization conversion ratio (PCR) based on the measured data. Figures 2(b) and (c) depict the measured spectra of the reflection amplitudes (\(|r_{xx}|\) and \(|r_{yy}|\)) and phases (\(\Phi_{xx}\) and \(\Phi_{yy}\)) of the device for input THz waves polarized along two principle axes. The other two Jones matrix components (\(i.e., r_{uy}, r_{yx}\)) are zero due to the symmetry requirement. Two magnetic resonances can be clearly identified at 0.37 THz and 1.05 THz for the \(\hat{\mathbf{E}}\parallel \mathbf{\hat{x}}\) polarization (evidenced by the reflection dips and the fact that \(\Phi_{xx} = 0^\circ\)), between which a magnetic resonance for the \(\mathbf{R}\) polarization is identified at 0.74 THz. In accordance with our expectations, all resonances involved are of the desired low \(Q\) factors, yielding weak absorptions and mild frequency-dependent phase variations. As a result, the condition \(\Phi_{xx} - \Phi_{yy} = 180^\circ\) can be approximately satisfied within a broad working band (0.37–1.05 THz) in which the reflection amplitudes are close to 1 (>85%), both being highly desired. To check the polarization manipulation functionality, we used the formula PCR = \(|r_{xx} - r_{yy}|^2/2(|r_{xx}|^2 + |r_{yy}|^2)\) to calculate the PCR of our device, under illuminations of an input THz wave polarized along an in-plane axis lying at an angle with respect to the \(\mathbf{\hat{x}}\)-axis. Figure 2(e) shows that the PCR realized by our HWP exceeds 80% within 0.37–1.05 THz, exhibiting a relative bandwidth \(\Delta f/f_0 = 96%\) with \(f_0 = 0.71\) THz being the central working frequency.

Such a conclusion was reinforced by our direct measurements. In this second characterization approach, we rotated the sample along the \(\mathbf{\hat{z}}\)-axis by an angle of 45\(^\circ\), and then measured the co-polarization and cross-polarization reflection coefficients of our sample, respectively. As shown in fig. 2(d), the measured cross-polarization response \(|r_{uv}|\) is much larger than the co-polarization one \(|r_{uu}|\) within the working band, already implying a large PCR. We can quantitatively compute the PCR spectrum using the formula PCR = \(|r_{uv}|^2/(|r_{uu}|^2 + |r_{uv}|^2)\). As shown in Fig. 2(e), the PCR spectrum thus obtained (blue squares) is in good agreement with that obtained with indirect measurement (red circles), both showing that the PCR values are close to 1 within an ultra-wide frequency band. FDTD simulation results are shown in Fig. 2(b)–(e) as solid lines, which are in excellent agreement with their corresponding experimental results. Slight discrepancies between experiments and simulations can be attributed to the sample imperfections.
choose the asymmetrical tri-layer meta-atom structures as shown in fig. 3(a). For such structures, an incident $E$ field along the $\hat{x}$-direction can excite the resonance mode with non-zero average currents $\langle j_y \rangle$, yielding strong off-diagonal responses $r_{yx}$. Via engineering the structural asymmetry (say, by varying the length $a_1, a_2, b$, etc.), we can control the off-diagonal responses $r_{yx}$ of the structure at a pre-determined frequency, and finally arrive at an optimized meta-atom structure. We fabricated the device (see fig. 3(a)) based on the optimized design and experimentally characterized its polarization manipulation functionality. Noting that this device works for $\phi_0 = 0^\circ$, we do not need to rotate the sample but rather directly measure all four reflection coefficients with the incident polarization along the $\hat{x}$- or $\hat{y}$-axis, respectively. Figure 3(b) depicts the measured spectra of four Jones matrix elements, from which one can clearly see that the cross-polarization response dominates the co-polarization ones within an ultra-broad band (0.41–0.96 THz). Figure 3(c) shows the PCR spectrum calculated with these Jones matrix coefficients using the formula $\text{PCR} = |r_{yx}|^2/(|r_{yx}|^2 + |r_{xx}|^2)$. Obviously, this device is an excellent HWP (with PCR larger than 80%) within the noted working band. Again, FDTD simulation results are in excellent agreement with their corresponding experimental data. We also employed FDTD simulations to investigate the incidence-angle dependence of the devices functionality (fig. 3(d)). Similarly to case 1, the PCR of this device remains at a high value for incidence angles up to $55^\circ$, although the working bandwidth is narrowed at large incident angles.

**Ultra-wide band quarter-wavelength wave plate.** As a final example, we designed/fabricated a QWP based on the same approach. As already discussed, there are infinite possible forms of Jones’ matrix based on which a QWP can be designed. Here we only choose one specific solution $R = a^1 b^0 c^1 d^{-1}$, corresponding to the case of $\phi_0 = 90^\circ$ with the final state being a right CP. Obviously, the meta-atom that we design must exhibit mirror symmetries, with reflectance for two orthogonal polarizations close to 100% ($|r_{xx}|, |r_{yy}| \approx 1$) with phase difference satisfying $\Phi_{xx} - \Phi_{yy} = 90^\circ$. Such a requirement is quite similar to that of case 1, so that here we adopt a meta-atom similar to that shown in fig. 2(a), but with structural parameters optimized to fulfill the present requirement. Figure 4(a) shows the optical picture of our device, fabricated based on the optimized design.

We experimentally characterized the functionality of the fabricated terahertz QWP using the THz-TDS system, starting from measuring the Jones matrix components of the device. Figures 4(b) and (c) depict the measured spectra of the reflection amplitudes and phases for two orthogonal Jones matrix elements $(r_{xx}, r_{yy})$, which are in good agreement with the corresponding FDTD simulations. Similarly to the case studied in fig. 2, here the present device also supports two magnetic resonances for the $\vec{E} \parallel \hat{x}$ polarization and one magnetic resonance for the $\vec{E} \parallel \hat{y}$ polarization, evidenced by the resonance
dips in the reflectance spectra (fig. 4(a)). However, to fulfill the requirement for the present QWP, the positions and Q factors of three resonance modes are tuned such that the phase difference $\Phi_{xx} - \Phi_{yy}$ are around 90° degrees within an ultra-wide frequency band (0.4–1.05 THz). The measured spectra shown in figs. 4(a) and (b) already implied that our device behaves as a good QWP within this frequency regime, which can convert a LP THz wave to a CP one. We define a quantity $\text{PCR}_{\text{QWP}} = (|r_{tu}|^2 - |r_{ru}|^2)/(|r_{tu}|^2 + |r_{ru}|^2)$ to quantitatively characterize the polarization conversion efficiency of our device, where $l$ and $r$ denote the LCP and RCP states, respectively, and $\hat{u}$ denotes the direction of the input linear polarization lying at 45° with respect to the $\hat{x}$-axis (see inset of fig. 4(a)). We can rewrite the formula as $\text{PCR}_{\text{QWP}} = 2\text{Im}(r_{ru}^* r_{yy})/(|r_{xx}|^2 + |r_{yy}|^2)$, which can be computed based on the measured spectra of the Jones matrix elements presented in figs. 4(b), (c).

Figure 4(f) shows that the PCR thus calculated (open circles) are larger than 90% within an ultra-wide frequency band (0.4–1.05 THz), showing the good functionality of our fabricated device. It should be noted that the present device can not only covert a linear polarization (for incident THz wave polarized along the $\hat{u}$-axis, see fig. 4(a)) to a right circular polarization, but it can convert also a left circular polarization to that linear polarization.

We also performed independent experiments to verify such a polarization manipulation effect. As schematically shown in figs. 4(d) and (e), when the device is shined by an LP wave polarized along the $\hat{u}$-axis, the reflected wave should in principle be a RCP wave. However, due to the limitation of our TDS system, we can only characterize the linearly polarized waves. Noting that a CP wave can always be decoupled to a linear combination of two equal-strength LP waves polarized along two orthogonal axes, as expected. By further checking the phase difference of these two components (see fig. 4(e)), we find that the reflected wave is indeed nearly a RCP one, within the frequency band (0.4–1.05 THz). Again, FDTD results are depicted in fig. 4(b)–(f), which are in good agreement with the measured results. Similarly to HWP for incidence angles up to 55°, the PCR of this device remains at a high value, although the working bandwidth is narrowed at large incident angles (see the SM, fig. S3 for FDTD simulation results).

**Conclusion.** – To sum up, based on the Poincaré sphere analysis, we presented the general forms of the Jones matrices to help researchers design appropriate metamaterials to achieve high-efficiency polarization conversions with different functionalities. We designed/fabricated three typical THz wave plates and experimentally demonstrated that all these devices exhibit excellent polarization manipulation properties within ultra-wide frequency bands. In addition to being immediately applicable in THz science and technologies, our mechanism also shed light on designing/realizing high-efficiency wide-band polarization manipulators in higher-frequency domains.

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Ultra-wide band THz wave plate


