Prepared by Jiang Xiao

第九章：波动

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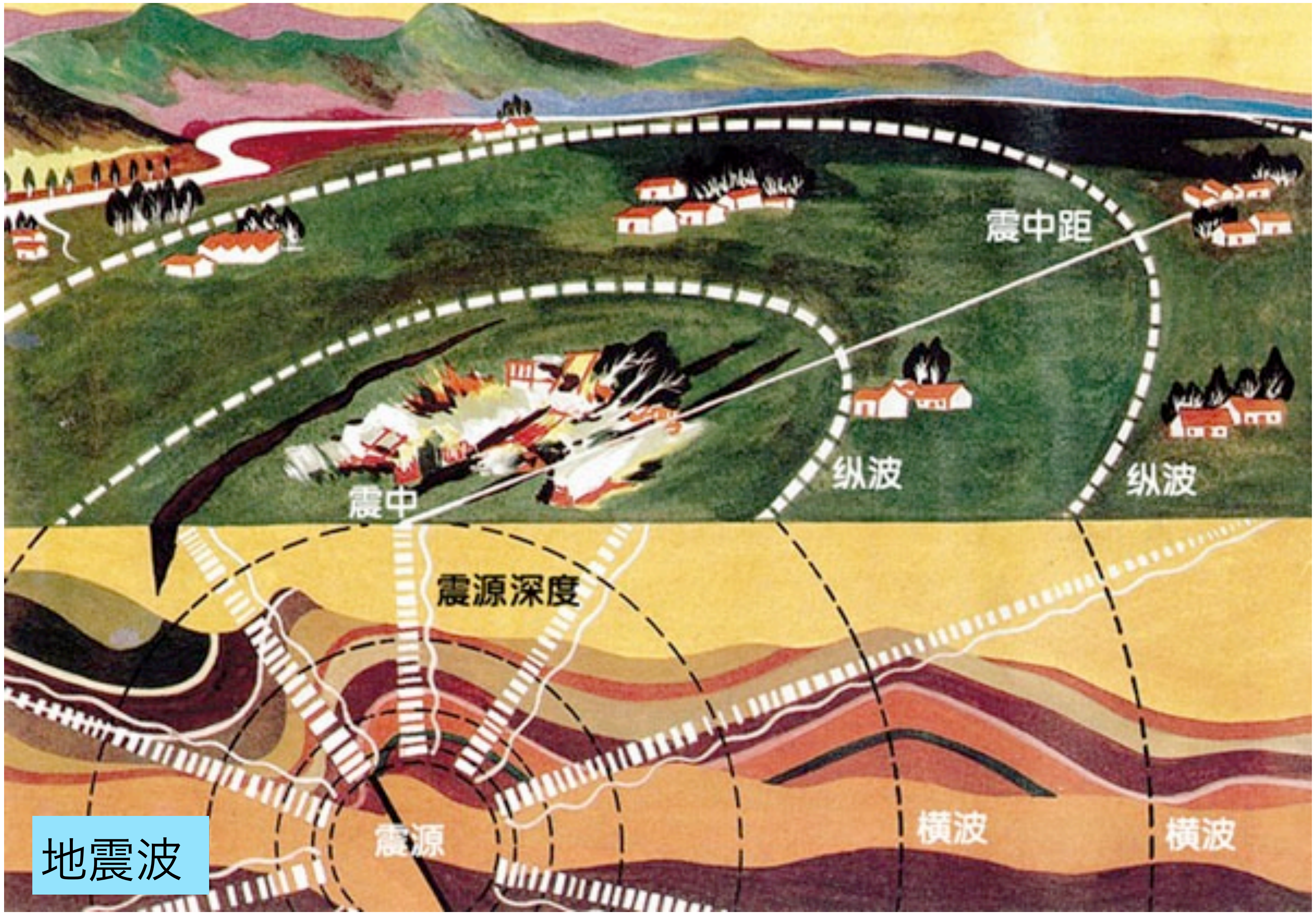
波动

机械波：波在媒介（空气、水、地壳）中传播,

在波传播过程中媒介中的物质作某种振动。

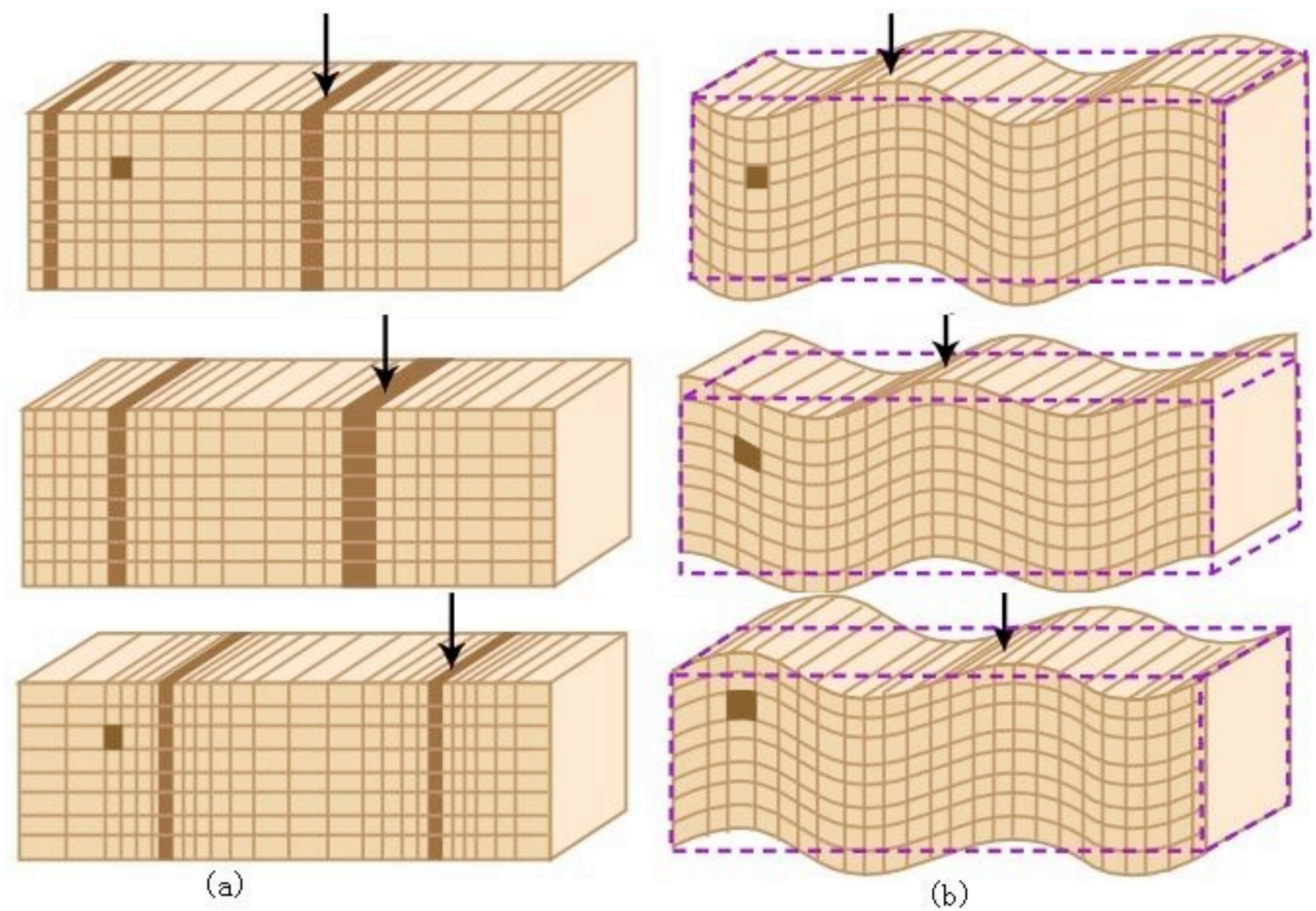
电磁波：可以在真空中传播。



地震的P波和S波

P波

S波



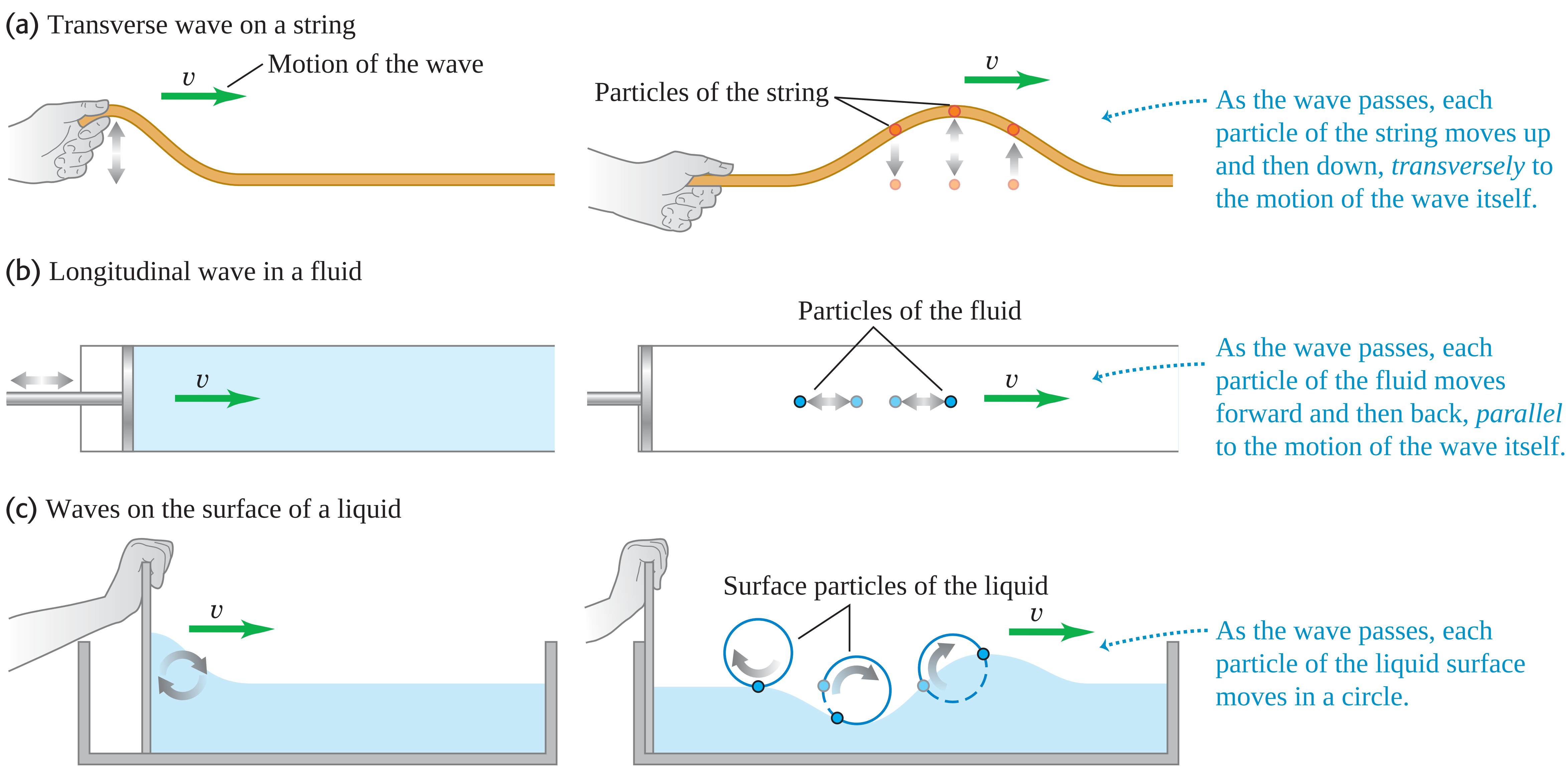
P波传播比S波快

横波和纵波

横波

纵波

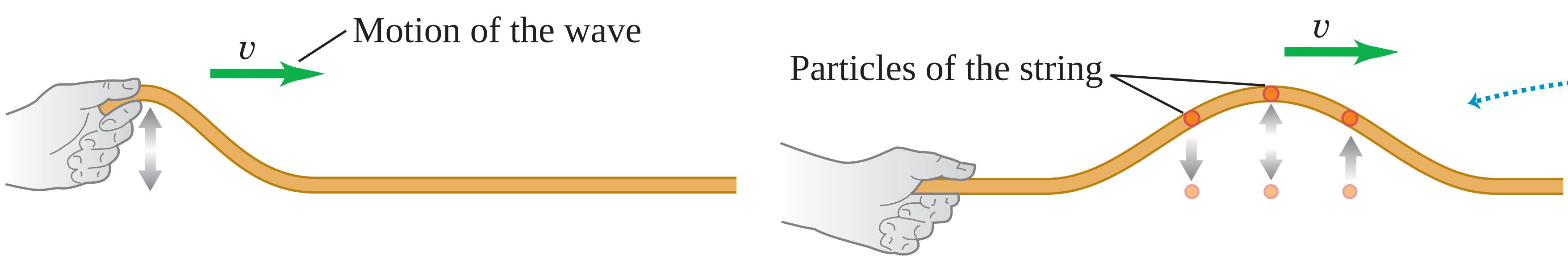
横波
+
纵波



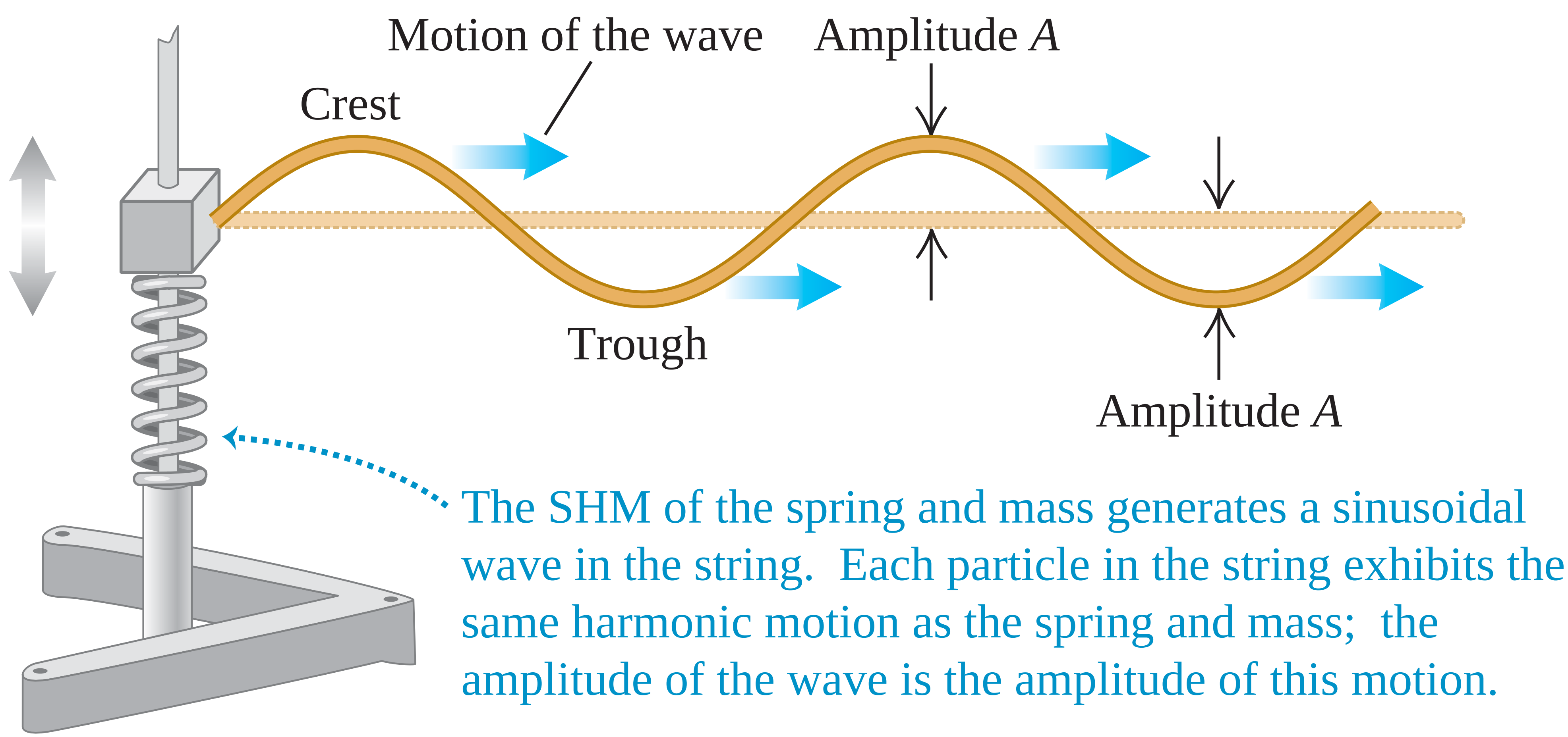
波传播能量，不传播物质。

http://v.youku.com/v_show/id_XMzg1NTQ1NTQ0.html

定态波和脉冲波



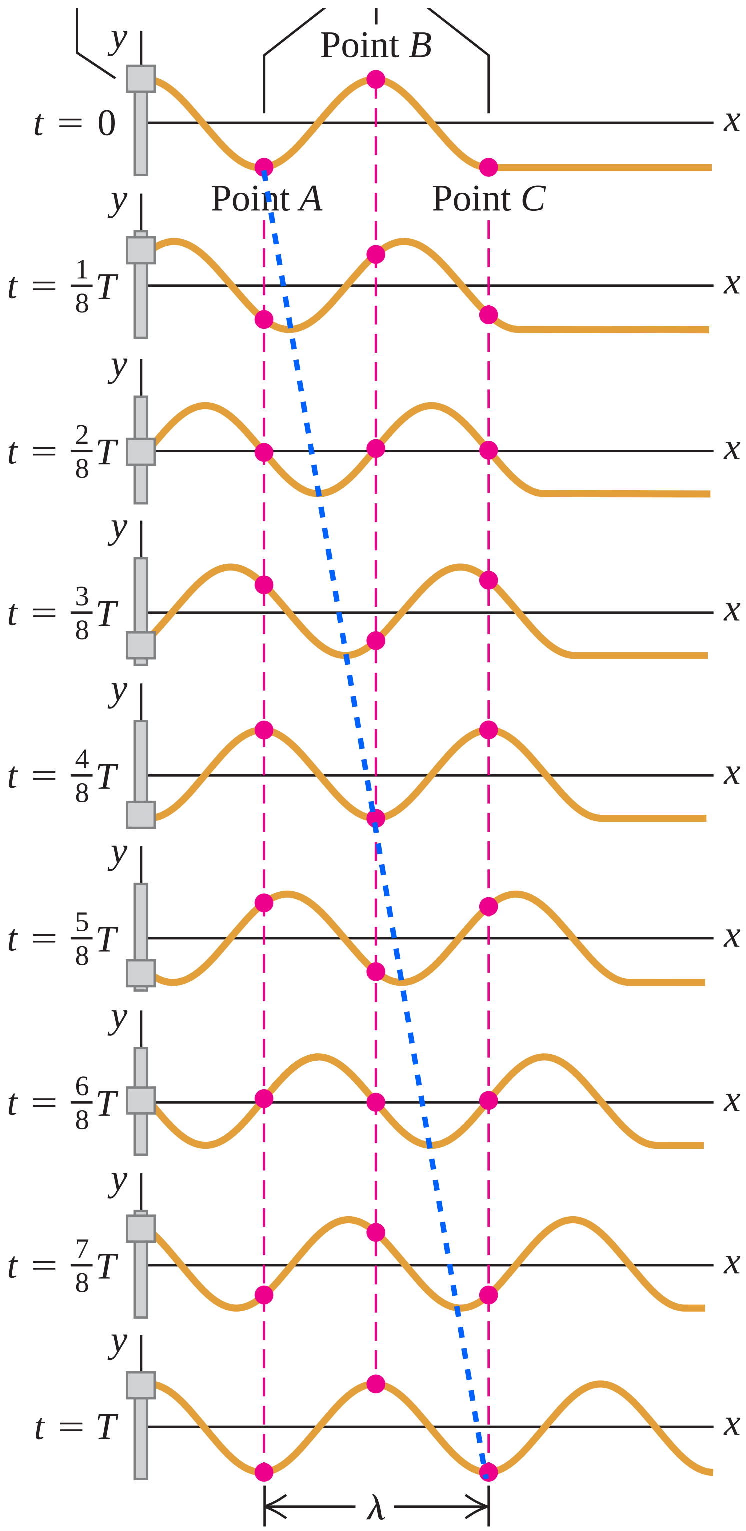
脉冲波



定态波

周期波

定态波的数学描述



$$\omega = 2\pi/T \qquad k = 2\pi/\lambda$$

频率　周期　波数　波长

$$u(x,t) = A\cos(\omega t + \phi_0(x))$$

$$u(x,t) = A\cos(kx + \psi_0(t))$$

$$u(x,t) = A\cos(\omega t - kx + \phi_0)$$

$$= A\cos\left[\omega\left(t - \frac{x}{v}\right) + \phi_0\right]$$

波速： $v = \frac{\lambda}{T} = \frac{\omega}{k} \quad \Rightarrow \quad \omega = vk$

相速度

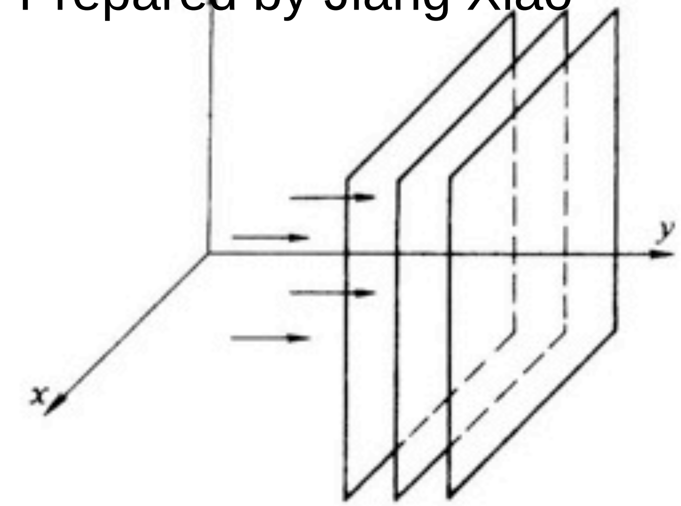
等相位面： $\phi = \omega\left(t - \frac{x}{v}\right) + \phi_0$

平面波、柱面波、球面波

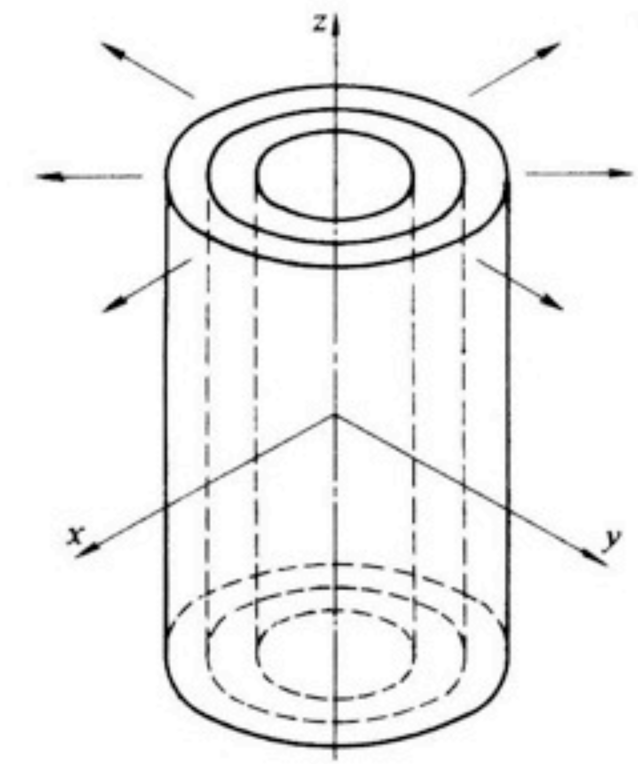
平面波：
$$u(r,t) = a\cos(\omega t - kx + \phi_0)$$
$$\phi = \omega t - kx + \phi_0$$

柱面波：
$$u(r,t) = \frac{a}{\sqrt{\rho}}\cos(\omega t - k\rho + \phi_0)$$
$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \omega t - k\rho + \phi_0$$

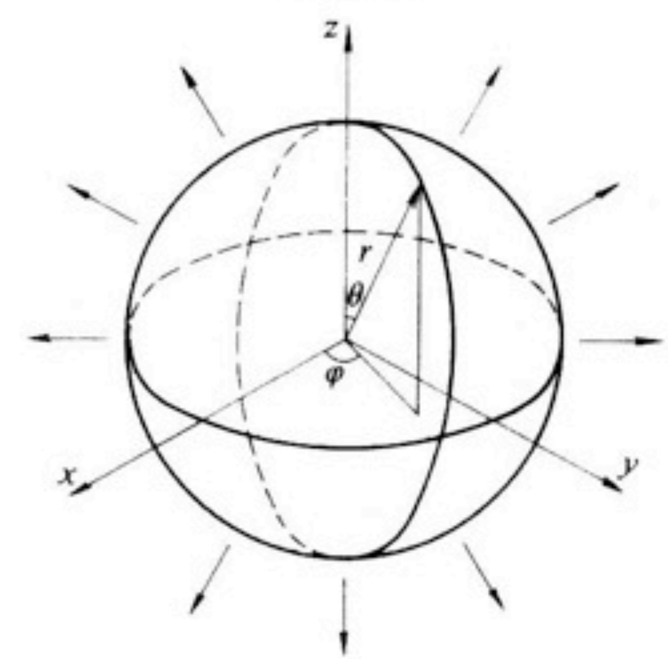
球面波：
$$u(r,t) = \frac{a}{r}\cos(\omega t - kr + \phi_0)$$
$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\phi = \omega t - kr + \phi_0$$



平面波



柱面波



球面波

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波动方程

$$\dot{u}(x,t) = \frac{d}{dt}u(x,t)$$

$$u'(x,t) = \frac{d}{dx}u(x,t)$$

$$u(x,t) = A\cos(\omega t - kx + \phi_0)$$

$$\dot{u}(x,t) = v(x,t) = -\omega A\sin(\omega t - kx + \phi_0)$$

$$\ddot{u}(x,t) = a(x,t) = -\omega^2 A\cos(\omega t - kx + \phi_0) = -\omega^2 u(x,t)$$

$$u''(x,t) = -k^2 u(x,t)$$

$$\frac{\ddot{u}(x,t)}{u''(x,t)} = \frac{\omega^2}{k^2} = v^2 \quad \Rightarrow \quad u''(x,t) - \frac{1}{v^2}\ddot{u}(x,t) = 0$$

波动方程： $$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)u(x,t) = 0$$

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波动方程

波动方程： $$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)u(x,t) = 0$$

通解： $$u_1(x,t) = F\left(t - \frac{x}{v}\right) \quad \text{and} \quad u_2(x,t) = G\left(t + \frac{x}{v}\right)$$

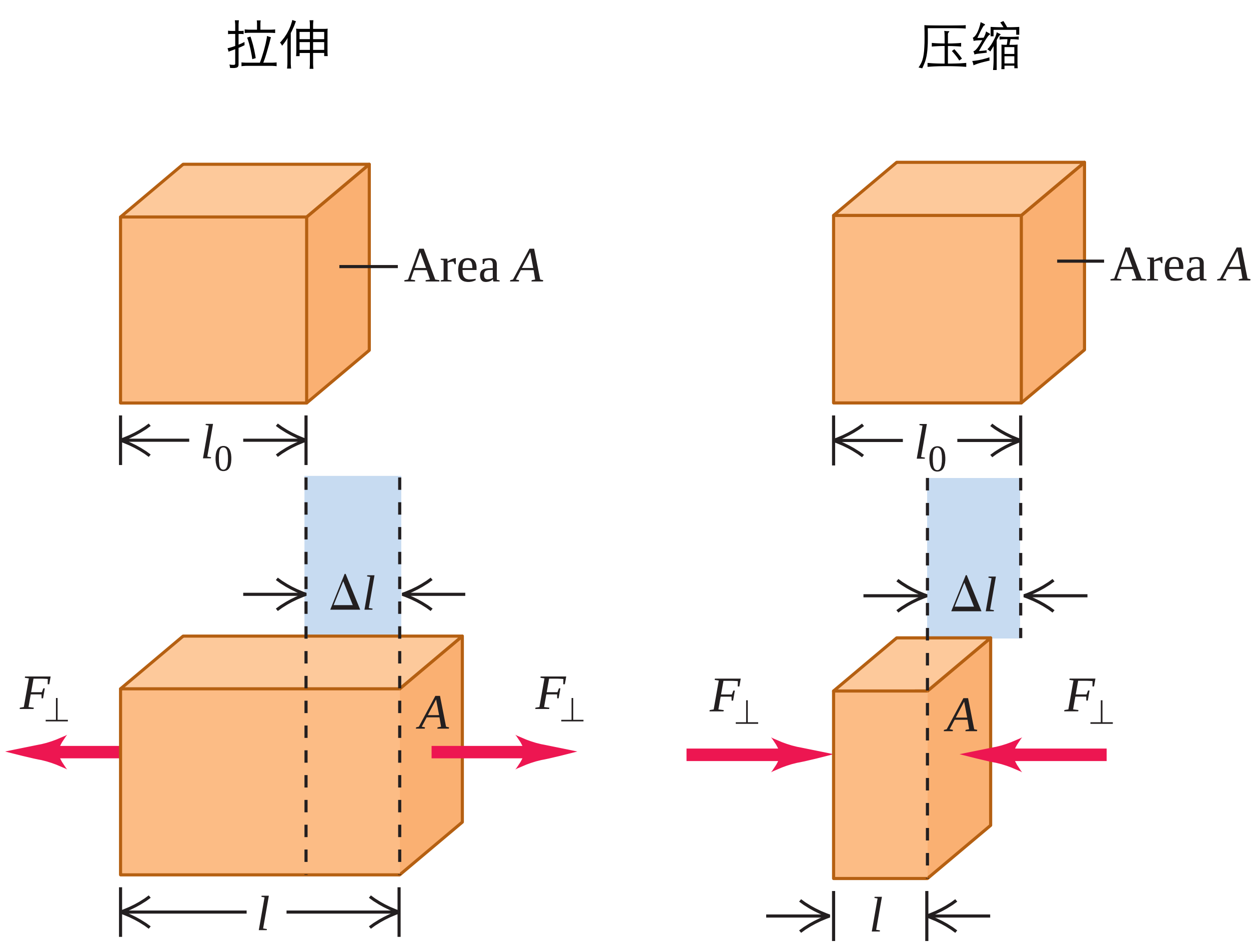
一般解： $$u(x,t) = c_1 F\left(t - \frac{x}{v}\right) + c_2 G\left(t + \frac{x}{v}\right)$$

三维： $$\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(\mathbf{r},t) = 0 \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

通解： $$\psi_\pm(\mathbf{r},t) = \psi\left(t \pm \frac{\mathbf{v}\cdot\mathbf{r}}{v^2}\right)$$

一般解： $$\psi(\mathbf{r},t) = c_+\psi\left(t + \frac{\mathbf{v}\cdot\mathbf{r}}{v^2}\right) + c_-\psi\left(t - \frac{\mathbf{v}\cdot\mathbf{r}}{v^2}\right)$$

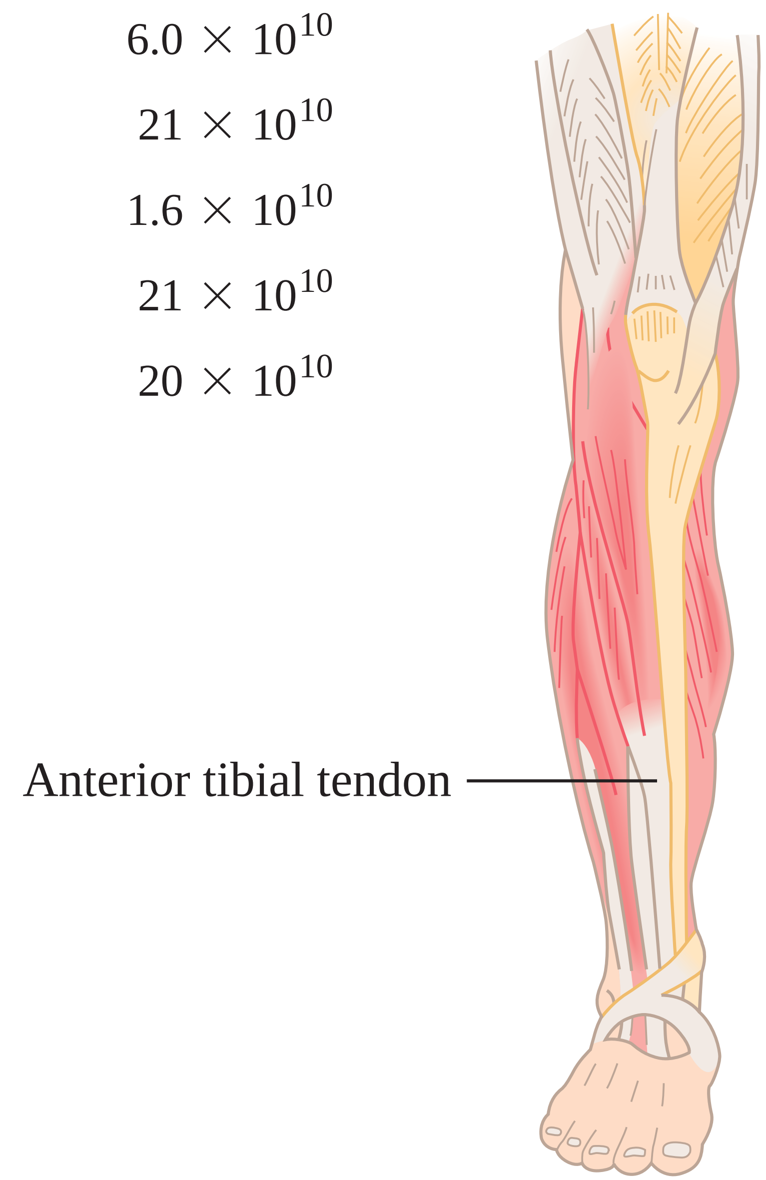
杨氏模量



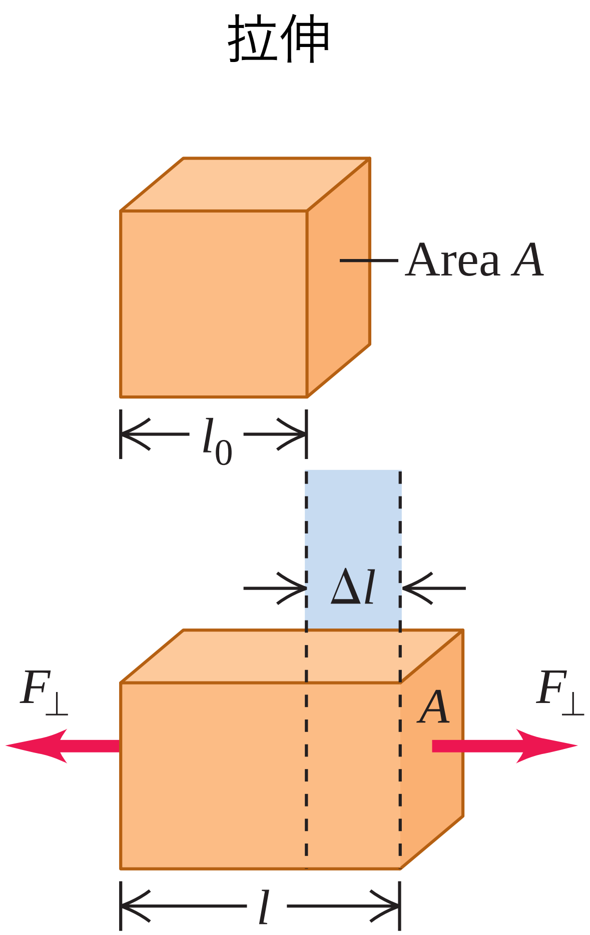
Material	Young's Modulus, Y (Pa)
Aluminum	7.0×10^{10}
Brass	9.0×10^{10}
Copper	11×10^{10}
Crown glass	6.0×10^{10}
Iron	21×10^{10}
Lead	1.6×10^{10}
Nickel	21×10^{10}
Steel	20×10^{10}

$$\frac{F}{A} \propto \frac{\Delta l}{l} \quad \Rightarrow \quad \frac{F}{A} = E\,\frac{\Delta l}{l}$$

杨氏弹性模量



杨氏模量

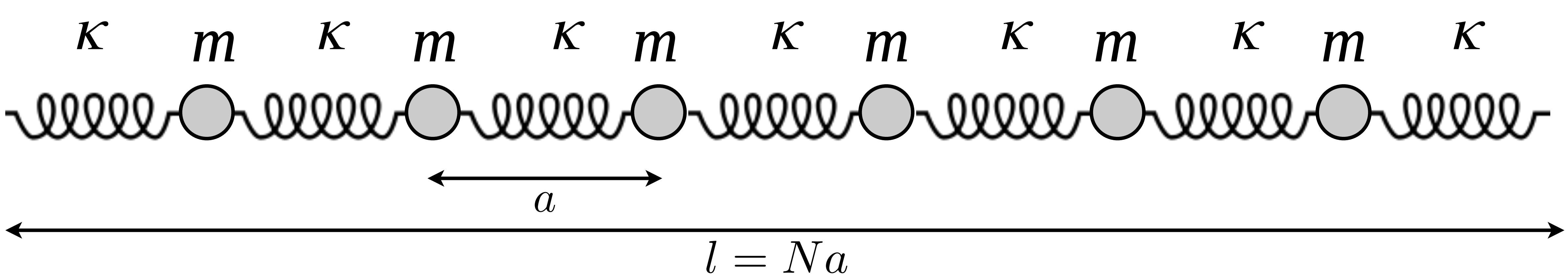


$$\frac{F}{A} \propto \frac{\Delta l}{l} \quad \Rightarrow \quad \frac{F}{A} = E\,\frac{\Delta l}{l}$$

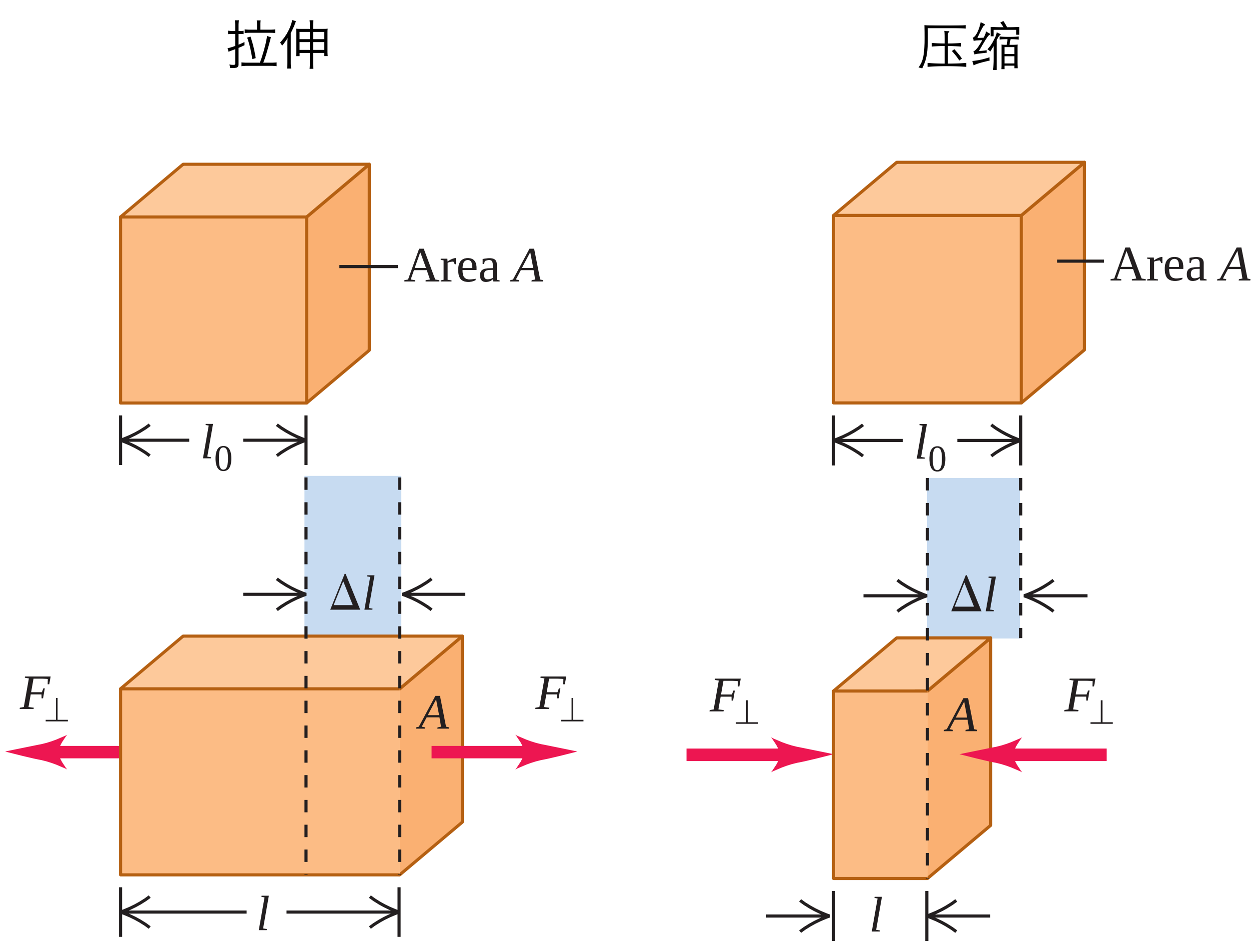
杨氏弹性模量

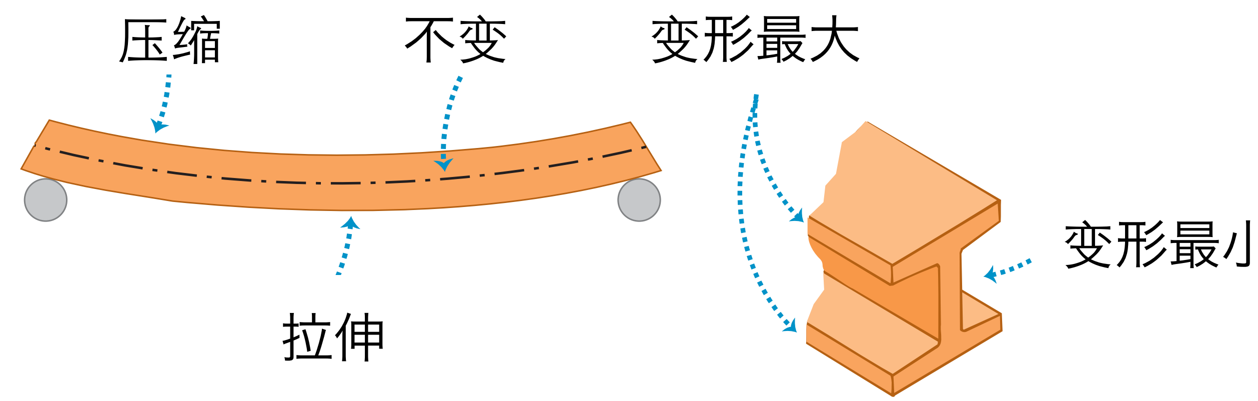
$$M = \rho A l = N m \quad \Rightarrow \quad m = \rho A a$$

$$AE\frac{\Delta l}{l} = \kappa\frac{\Delta l}{N} \quad \Rightarrow \quad \kappa = \frac{AE}{a}$$



杨氏模量

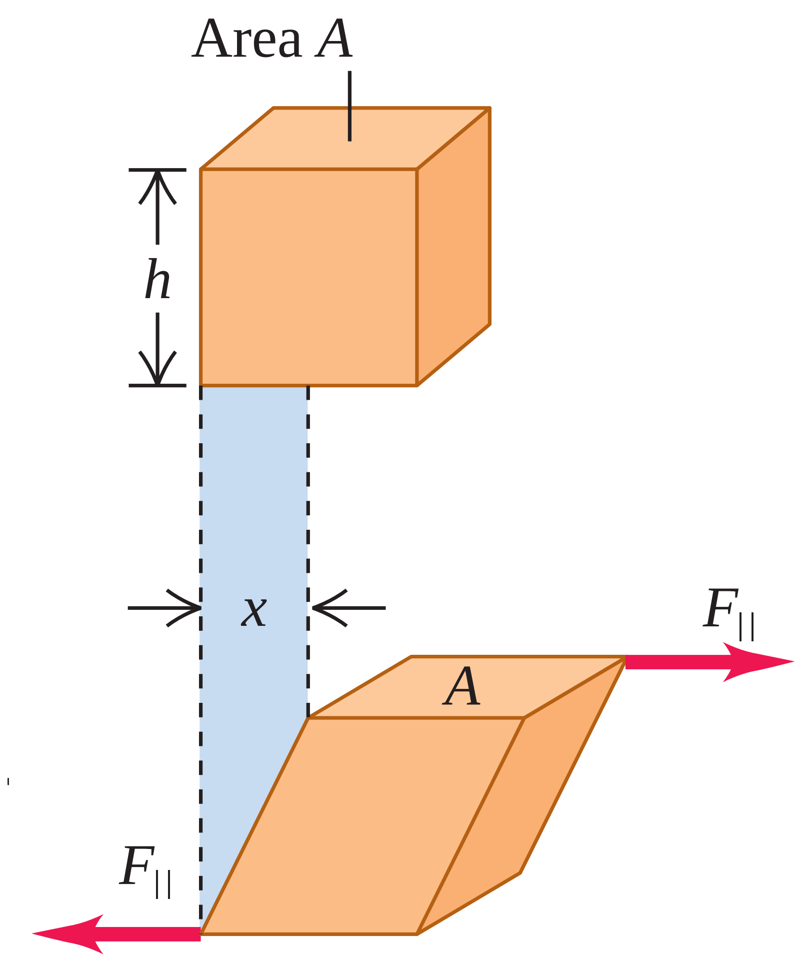


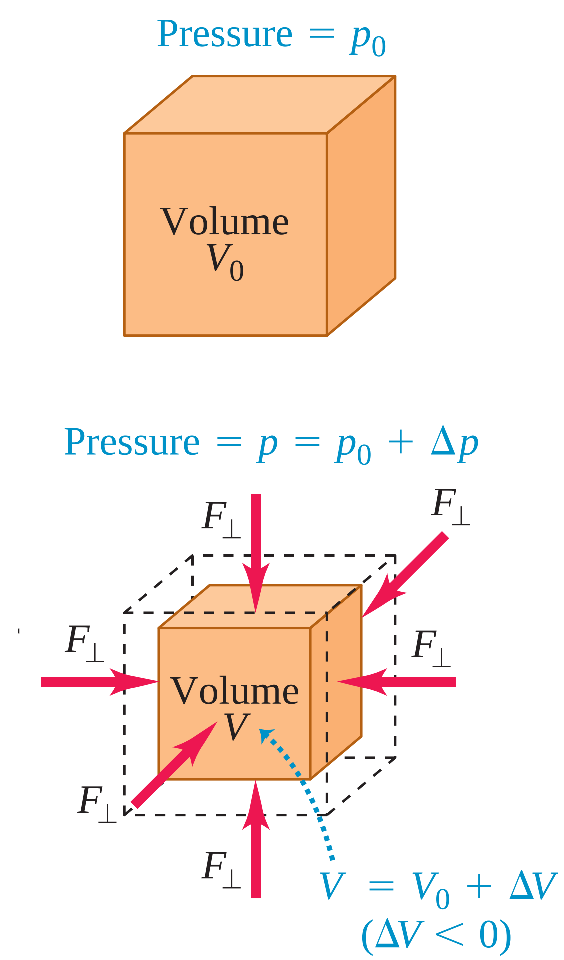


$$\frac{F}{A} \propto \frac{\Delta l}{l} \quad \Rightarrow \quad \frac{F}{A} = E\,\frac{\Delta l}{l}$$

杨氏弹性模量

切变模量和体变模量





$$\frac{F}{A} \propto \frac{x}{h} \quad \Rightarrow \quad \frac{F}{A} = G\,\frac{x}{h}$$

切变模量

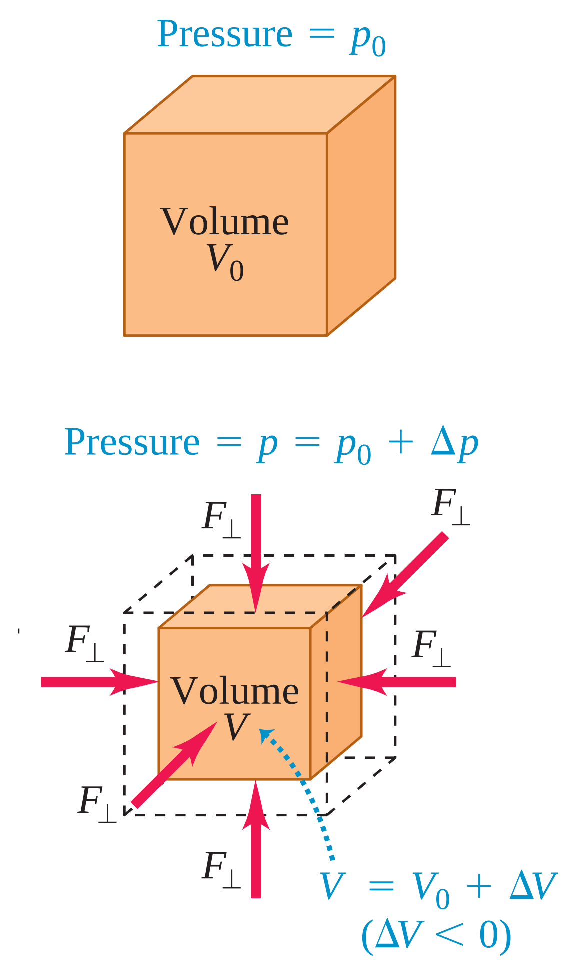
$$\Delta p = -K\frac{\Delta V}{V} = K\frac{\Delta \rho}{\rho}$$

体变模量

Prepared by Jiang Xiao

切变模量和体变模量

Anglerfish
生活在海底约1000米（100个大气压）

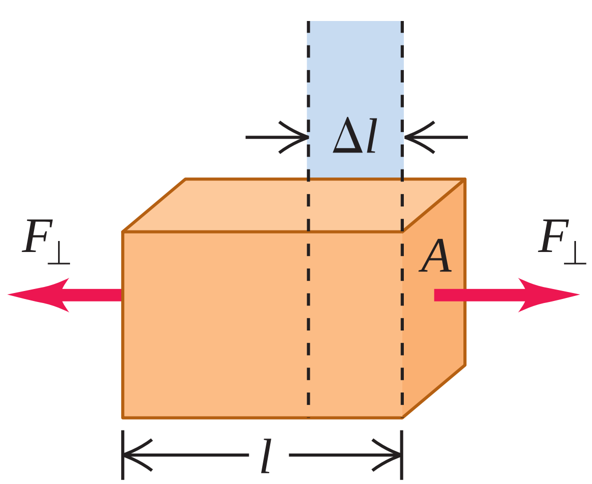


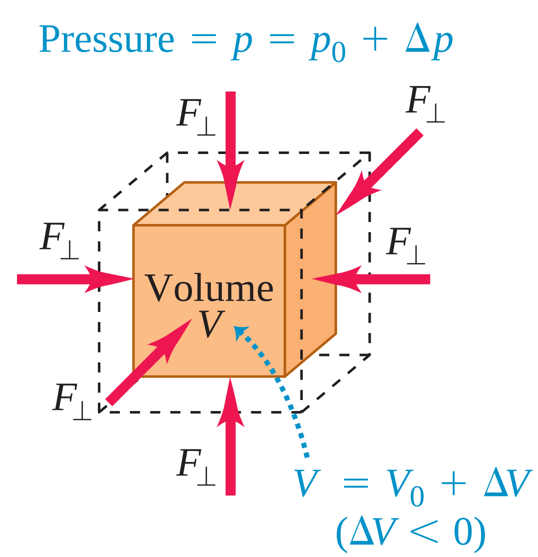
$$\Delta p = -K\frac{\Delta V}{V} = K\frac{\Delta\rho}{\rho}$$

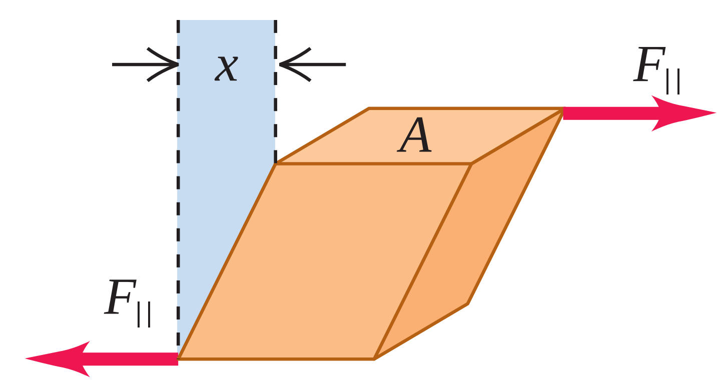
体变模量

杨氏模量、切变模量、体变模量

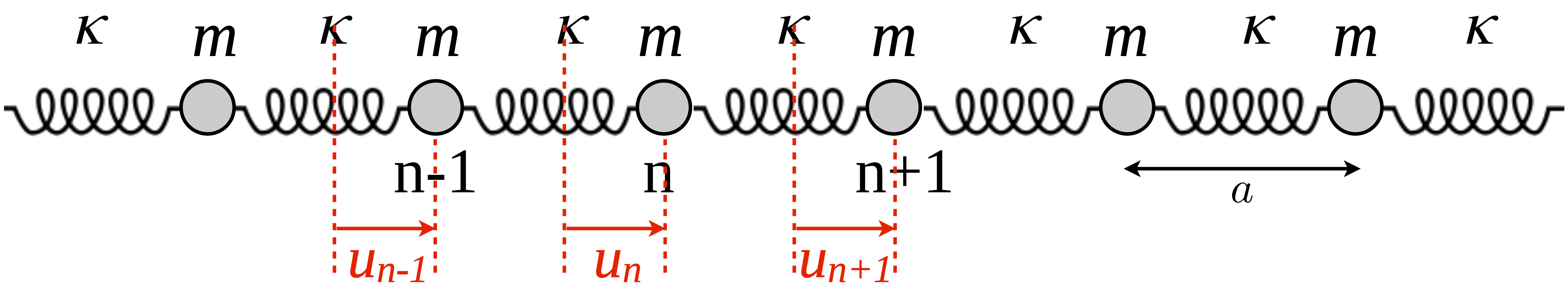
Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Crown glass	6.0×10^{10}	5.0×10^{10}	2.5×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}







一维弹性波



弹簧链的等效密度和杨氏模量

$$m = \rho A a \quad \text{and} \quad \kappa = \frac{AE}{a} \quad \Rightarrow \quad \rho = \frac{m}{Aa} \quad \text{and} \quad E = \frac{\kappa a}{A}$$

$$m\ddot{u}_n(t) = \kappa(u_{n+1} - u_n) - \kappa(u_n - u_{n-1}) \qquad \omega_0 = \sqrt{\kappa/m}$$

$$\frac{\partial^2}{\partial x^2}u(x,t) - \frac{1}{(\omega_0 a)^2}\frac{\partial^2}{\partial t^2}u(x,t) = 0$$

波动方程：$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)u(x,t) = 0$

波速

$$v = \omega_0 a = a\sqrt{\frac{AE/a}{\rho A a}} = \sqrt{\frac{E}{\rho}}$$

固体棒中的声速

纵波： $v_p = \sqrt{\frac{E}{\rho}}$ 杨氏模量

横波： $v_s = \sqrt{\frac{G}{\rho}}$ 切变模量

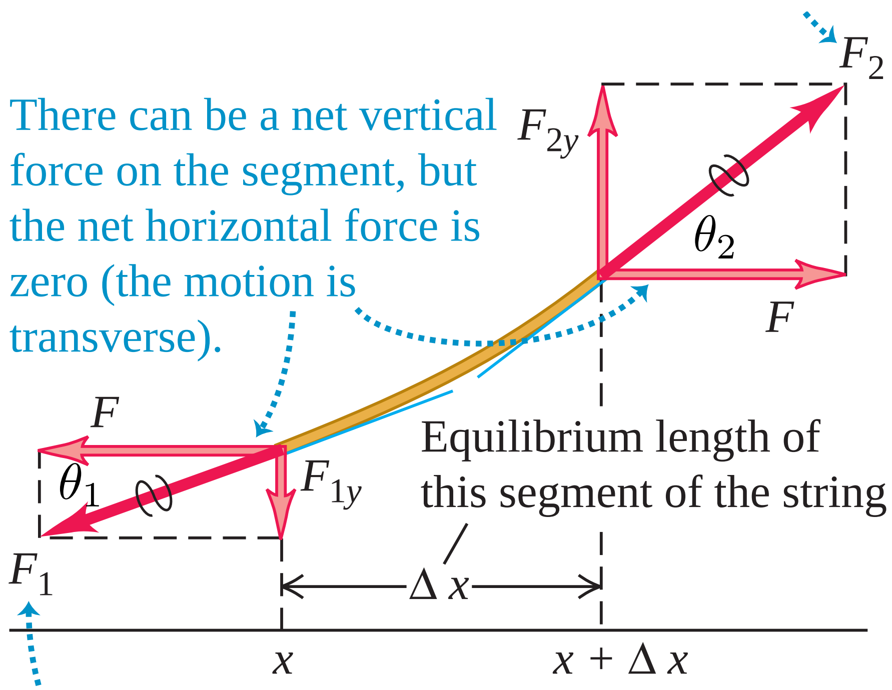
Material	**Young's Modulus, *Y* (Pa)**	**Bulk Modulus, *B* (Pa)**	**Shear Modulus, *S* (Pa)**
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Crown glass	6.0×10^{10}	5.0×10^{10}	2.5×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}

弦横波

$$F_1 = F_2 = T$$

$$T\sin\theta_1 \approx T\left(\frac{\partial u}{\partial x}\right)_1$$

$$T\sin\theta_2 \approx T\left(\frac{\partial u}{\partial x}\right)_2 = T\left(\frac{\partial u}{\partial x}\right)_1 + T\left(\frac{\partial^2 u}{\partial x^2}\right)_2 \Delta x$$



$$\Delta F = T\sin\theta_2 - T\sin\theta_1$$

$$= T\frac{\partial^2 u}{\partial x^2}\Delta x = \eta\Delta x\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial^2 u}{\partial x^2} - \frac{\eta}{T}\frac{\partial^2 u}{\partial t^2} = 0 \qquad v = \sqrt{\frac{T}{\eta}} \qquad \lambda = vf$$

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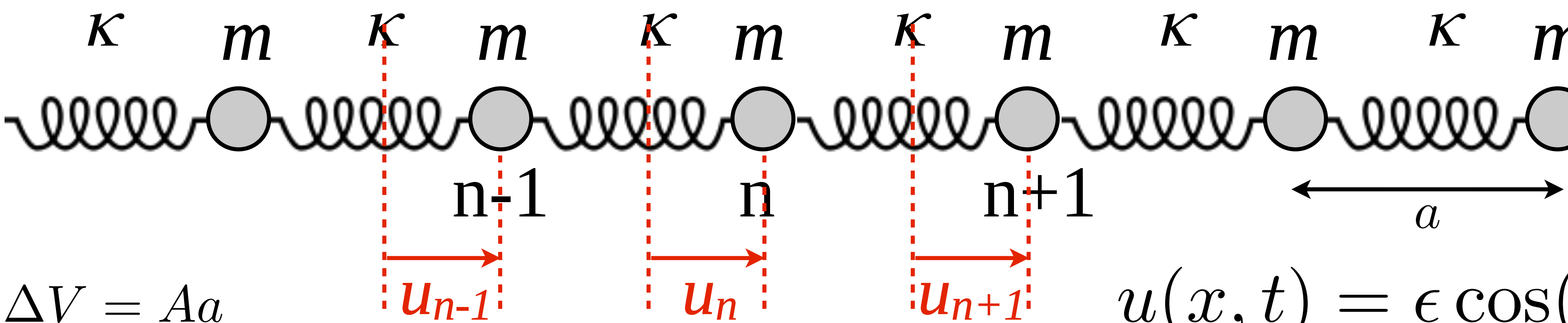
波的能量



$$\Delta V = Aa \qquad u(x,t) = \epsilon\cos(\omega t - kx)$$

$$m = \rho A a \quad \text{and} \quad \kappa = \frac{AE}{a} \quad \Rightarrow \quad \rho = \frac{m}{Aa} \quad \text{and} \quad E = \frac{\kappa a}{A}$$

动能：$\Delta E_k = \frac{1}{2} m v_n^2 = \frac{1}{2} m \left(\frac{\partial u_n}{\partial t}\right)^2 = \frac{1}{2} m \left(\frac{\partial u(x_n,t)}{\partial t}\right)^2 = \frac{1}{2}\Delta V \rho \omega^2 \epsilon^2 \sin^2(\omega t - kx)$

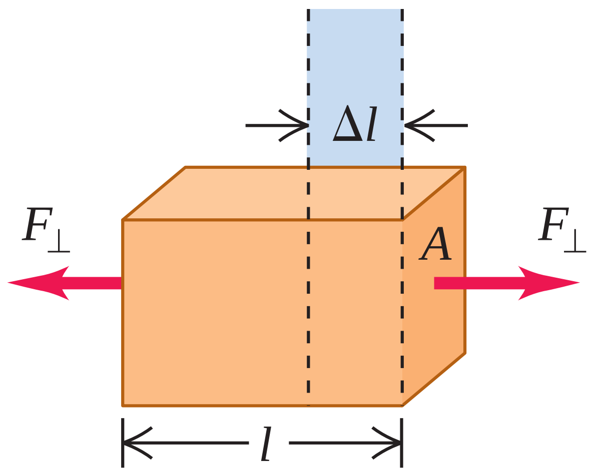
势能：$\Delta E_p = \frac{1}{2}\kappa(u_{n+1} - u_n)^2 = \frac{1}{2}\kappa\left[a\left(\frac{\partial u}{\partial x}\right)_{x_n}\right]^2 = \frac{1}{2}\kappa a^2 \left(\frac{\partial u(x_n,t)}{\partial x}\right)^2 = \frac{1}{2}\Delta V E k^2 \epsilon^2 \sin^2(\omega t - kx)$

$$\Delta E = \Delta E_k + \Delta E_p = \frac{1}{2}\Delta V(\rho\omega^2 + Ek^2)\epsilon^2 \sin^2(\omega t - kx) = \Delta V \rho \omega^2 \epsilon^2 \sin^2(\omega t - kx)$$

能量密度：$w(x,t) = \rho\omega^2\epsilon^2\sin^2(\omega t - kx) \quad \Rightarrow \quad \bar{w} = \frac{1}{T}\int_0^T w(x,t)dt = \frac{1}{2}\rho\omega^2\epsilon^2$

能流密度：$I = \bar{w} v = \frac{1}{2}\rho\omega^2\epsilon^2 v$

气体中的声速



$$E_a = \gamma p_0$$

绝热指数

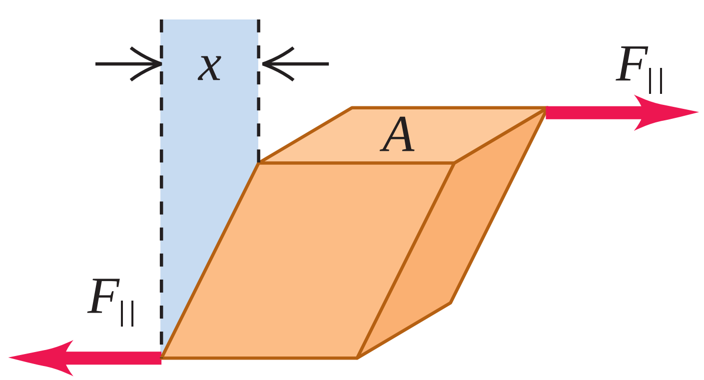
$$pV^\gamma = \text{const.}$$

$$\Delta p = -\gamma p \frac{\Delta V}{V}$$

$$v = \sqrt{\frac{\gamma p_0}{\rho_0}} \approx 331.2 \text{ m/s}$$

$$\gamma = 1.4, \quad p_0 = 10^5 \text{ N/m}^2, \quad \text{and} \quad \rho_0 = 1.3 \text{ kg}/m^3$$

$$\lambda = \frac{v}{f} = \frac{331.2 \text{ m/s}}{300 \text{ Hz}} = 1.1 \text{ m}$$



$$G_a = 0$$

气体中的压力波

$$u(x,t) = A\cos(\omega t - kx)$$

绝热指数

$$pV^{\gamma} = \text{const.}$$

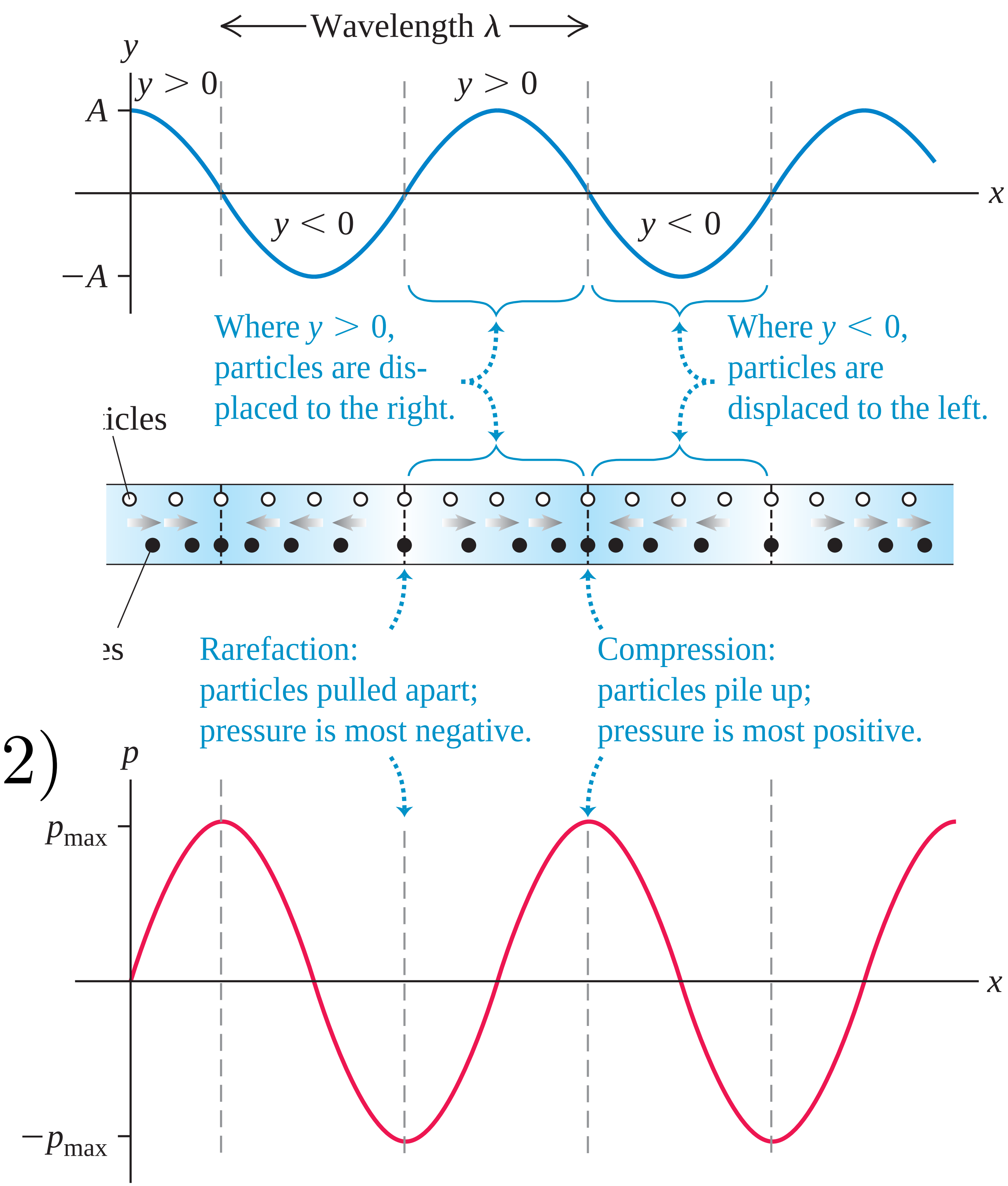
$$E_a = \gamma p_0$$

$$\Delta p = -\gamma p \frac{\Delta V}{V}$$

$$\Delta p(x,t) = -E_a \frac{\frac{\partial u}{\partial x}dx}{dx} = -E_a \frac{\partial u}{\partial x}$$

$$p(x,t) = A_p \cos(\omega t - kx + \pi/2)$$

$$A_p = E_a k A = \gamma p_0 k A$$



气体中的压力波

$$u(x,t) = A\cos(\omega t - kx)$$

能流密度：$I = \frac{1}{2}\rho_0\omega^2 A^2 v$

与频率有关

$$L = 20\text{dB}\ \log\frac{p}{p_b} = 10\text{dB}\ \log\frac{I}{I_b}$$

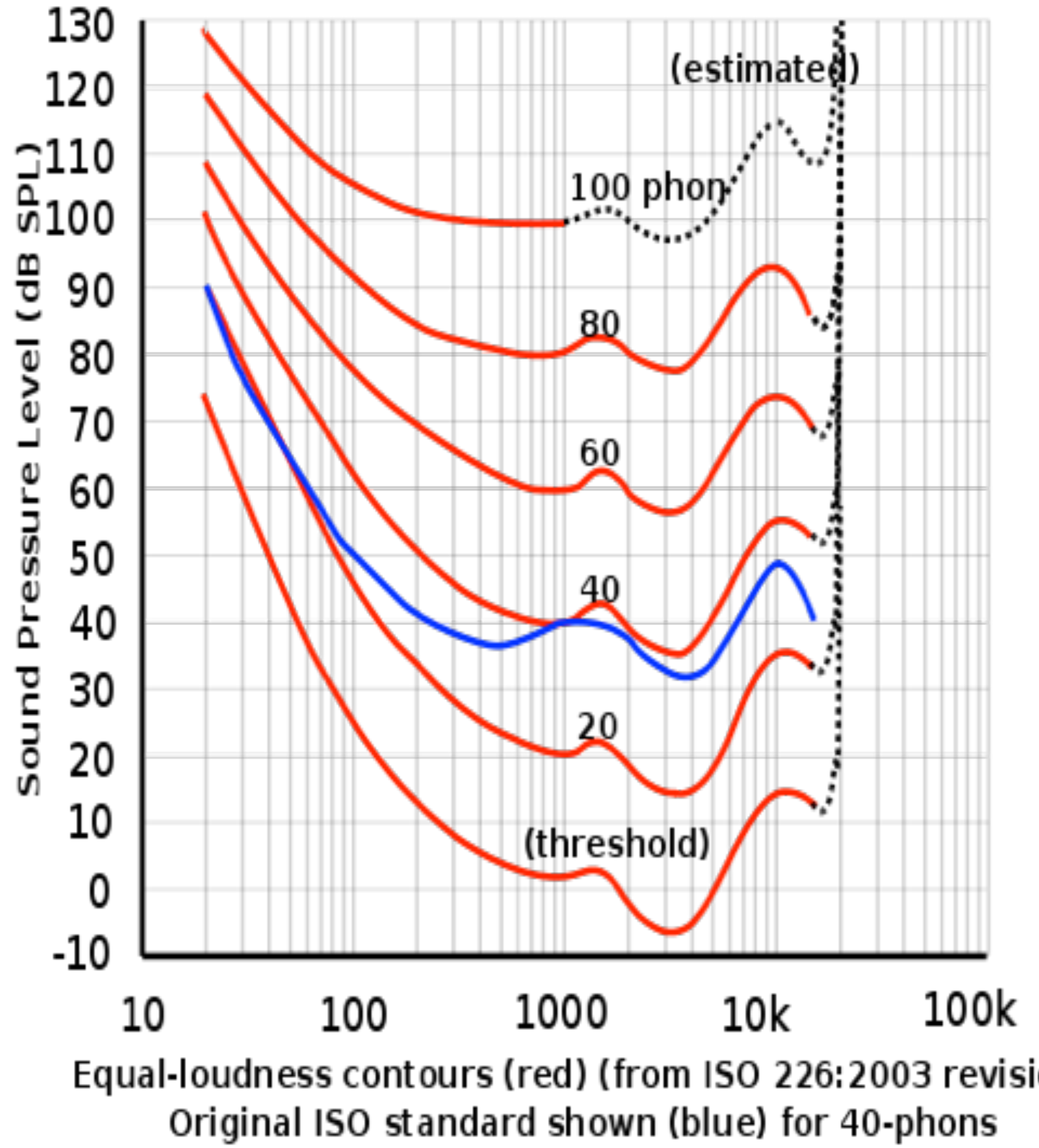
$p_b = 20\mu\text{Pa}$ and $I_b = 10^{-12}\text{W/m}^2$

三米远处蚊子飞行

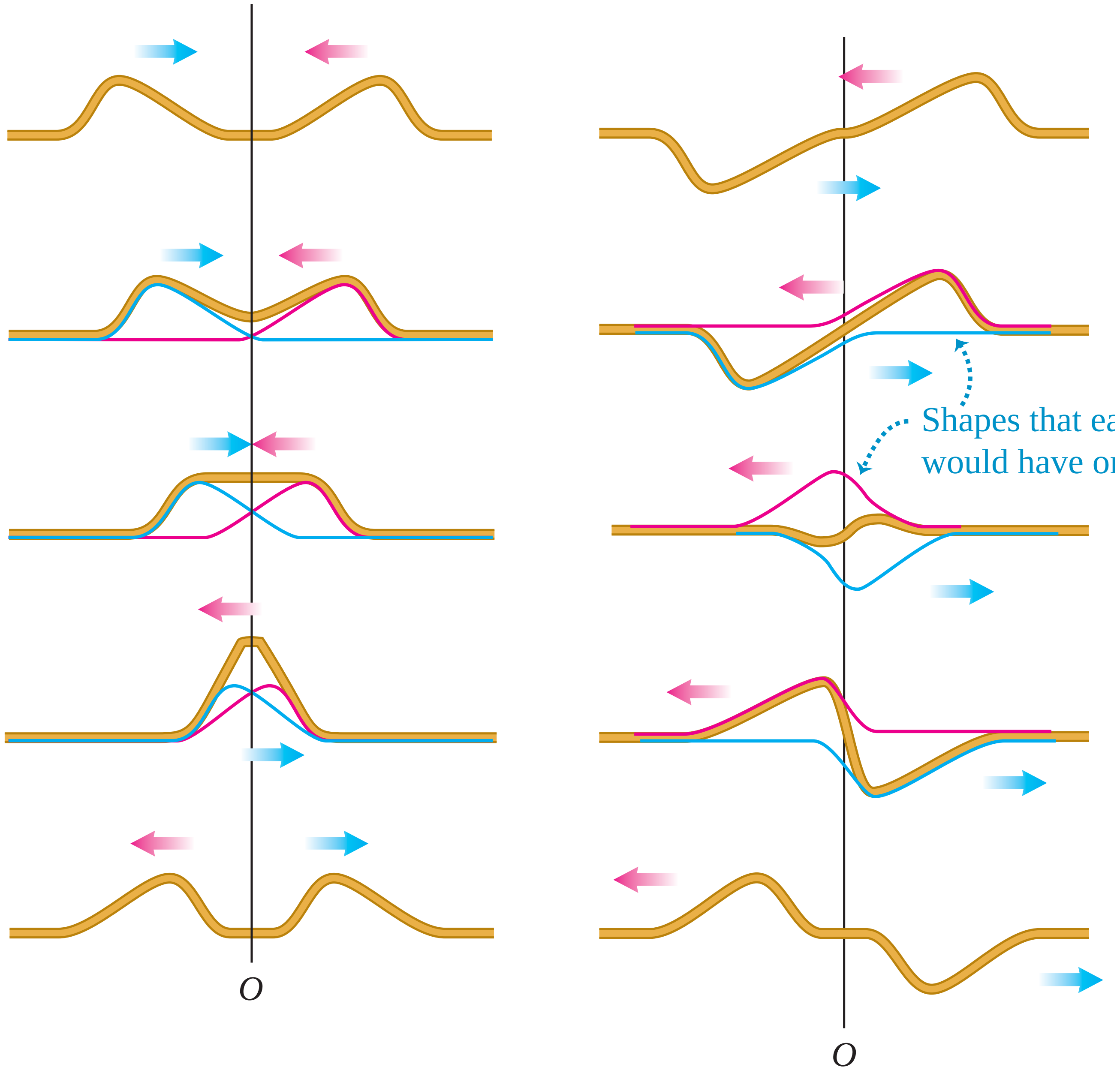
$$p(x,t) = A_p\cos(\omega t - kx + \pi/2)$$

$$A_p = E_a kA = \gamma p_0 kA$$

能流密度：$I = \frac{1}{2}\frac{A_p^2}{\rho_0 v}$



波的叠加



线性叠加：

$$u(x,t) = u_1(x,t) + u_2(x,t)$$

波的叠加

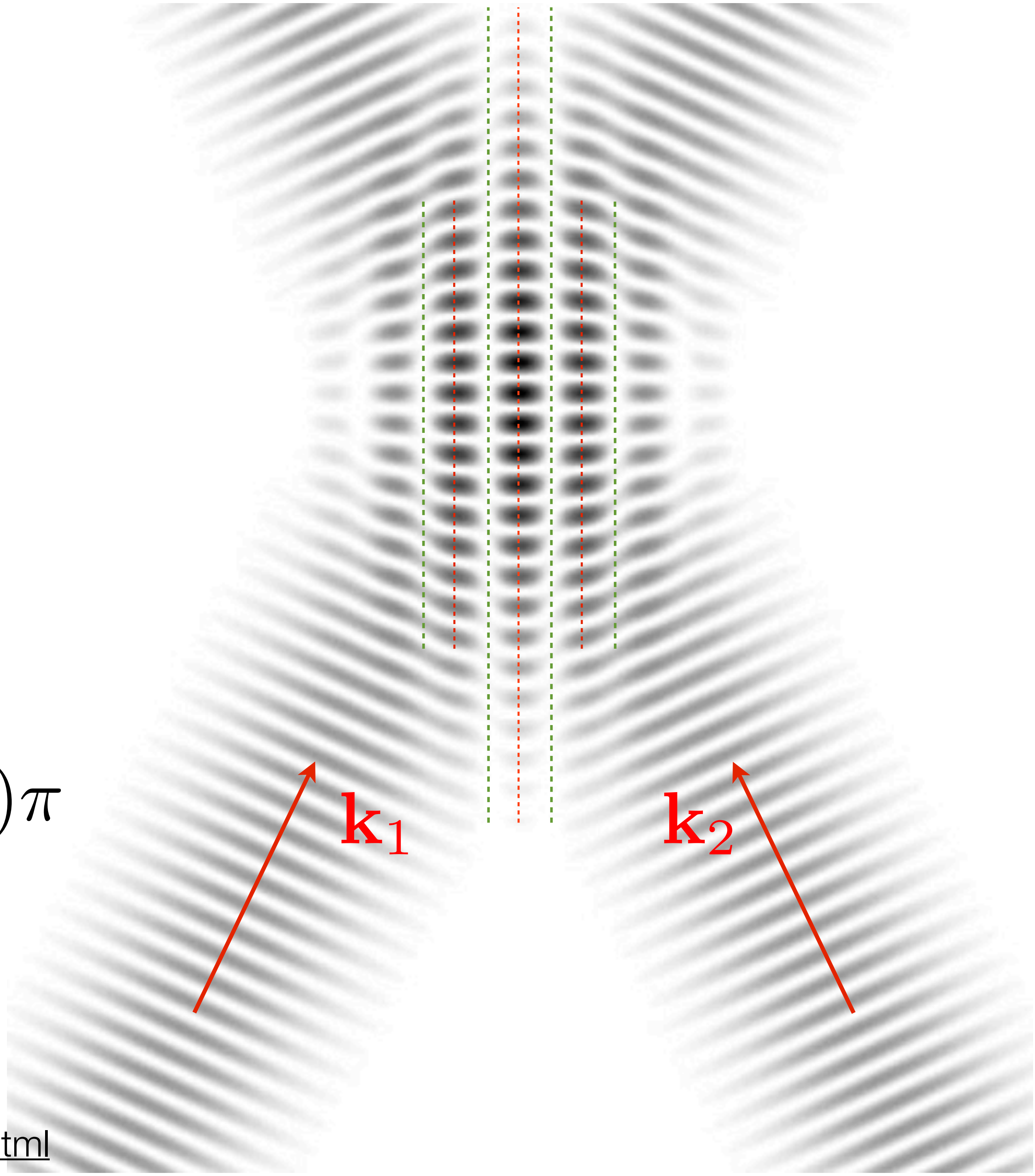
$$u_1(\mathbf{r},t) = A_1\cos(\omega t + \mathbf{k}_1\cdot\mathbf{r})$$
$$u_2(\mathbf{r},t) = A_2\cos(\omega t + \mathbf{k}_2\cdot\mathbf{r})$$

$$|\mathbf{k}_1| = |\mathbf{k}_2| = \omega/v$$

线性叠加：
$$u(\mathbf{r},t) = u_1(\mathbf{r},t) + u_2(\mathbf{r},t)$$

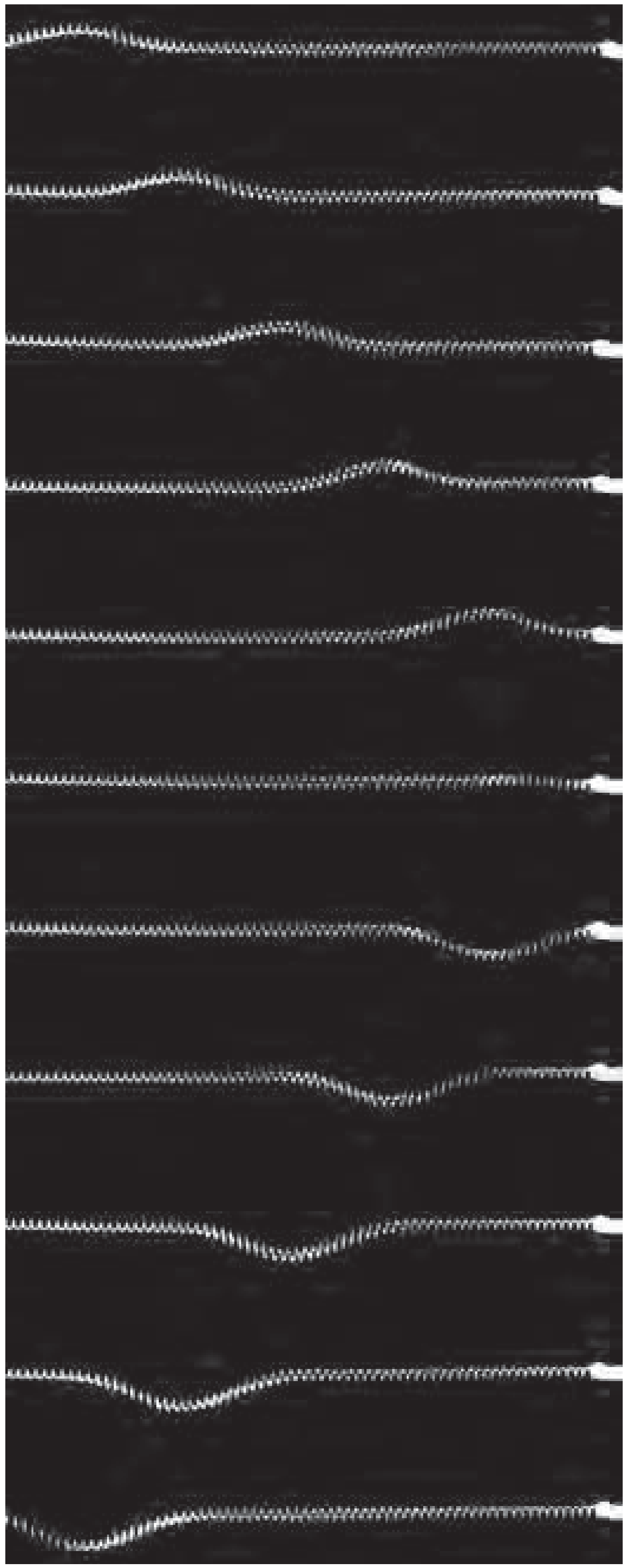
相长叠加：$(\mathbf{k}_1 - \mathbf{k}_2)\cdot\mathbf{r} = 2n\pi$

相消叠加：$(\mathbf{k}_1 - \mathbf{k}_2)\cdot\mathbf{r} = (2n+1)\pi$



波的干涉：http://v.youku.com/v_show/id_XMzg2ODA5MjY4.html

波的反射 - 固定端点



波动方程：$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)u(x,t) = 0$

通解：$u_1(x,t) = F\left(t - \frac{x}{v}\right)$ and $u_2(x,t) = G\left(t + \frac{x}{v}\right)$

一般解：$u(x,t) = c_1 F\left(t - \frac{x}{v}\right) + c_2 G\left(t + \frac{x}{v}\right)$

边界条件：端点**固定**

$x_0 = 0$

$$u(x_0,t) = 0 \quad \Rightarrow \quad G\left(t + \frac{x_0}{v}\right) = -\frac{c_1}{c_2}F\left(t - \frac{x_0}{v}\right)$$

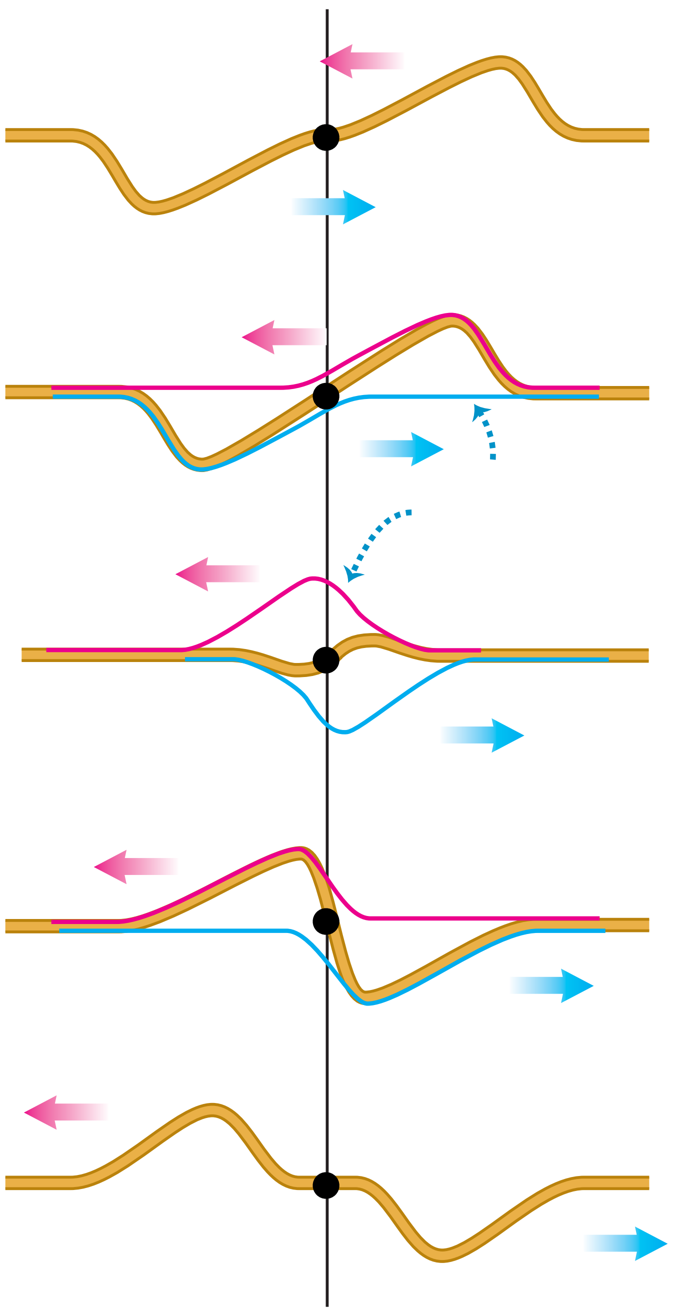
$$\Rightarrow \quad G\left(t + \frac{x}{v}\right) = -\frac{c_1}{c_2}F\left(t + \frac{x}{v} - \frac{2x_0}{v}\right) = -\frac{c_1}{c_2}F\left(t + \frac{x}{v}\right)$$

$$u(x,t) = F\left(t - \frac{x}{v}\right) - F\left(t + \frac{x}{v}\right)$$

波的反射（固定端点）：http://v.youku.com/v_show/id_XMzg5NTI1MTQw.html

Prepared by Jiang Xiao

波的反射 - 固定端点



波动方程： $\left(\dfrac{\partial^2}{\partial x^2} - \dfrac{1}{v^2}\dfrac{\partial^2}{\partial t^2}\right) u(x,t) = 0$

通解： $u_1(x,t) = F\left(t - \dfrac{x}{v}\right)$ and $u_2(x,t) = G\left(t + \dfrac{x}{v}\right)$

一般解： $u(x,t) = c_1 F\left(t - \dfrac{x}{v}\right) + c_2 G\left(t + \dfrac{x}{v}\right)$

边界条件：端点**固定**

$x_0 = 0$

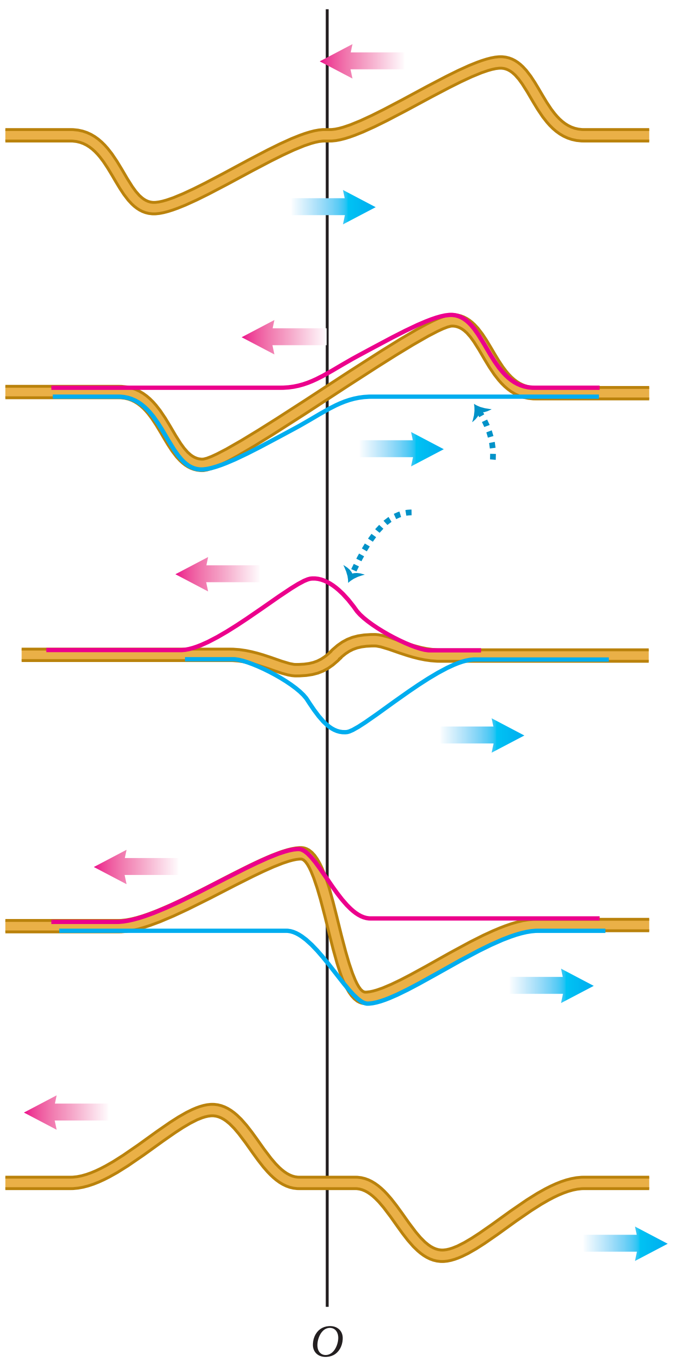
$$u(x_0,t) = 0 \quad \Rightarrow \quad G\left(t + \frac{x_0}{v}\right) = -\frac{c_1}{c_2} F\left(t - \frac{x_0}{v}\right)$$

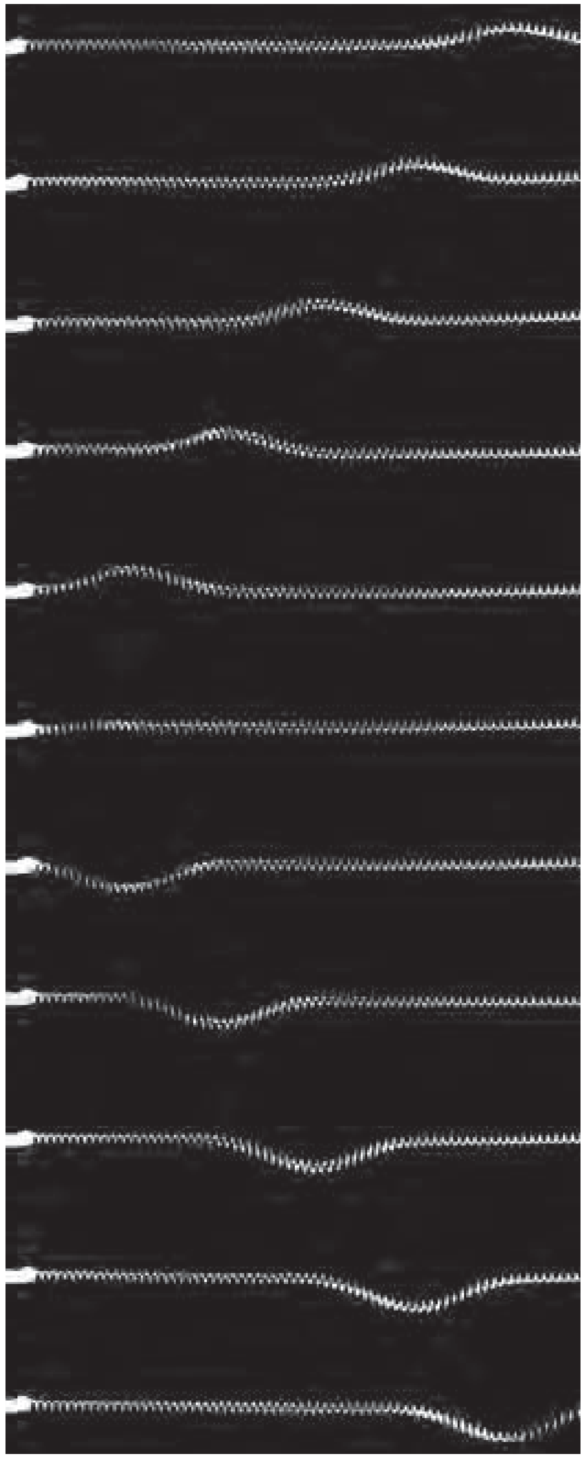
$$\Rightarrow \quad G\left(t + \frac{x}{v}\right) = -\frac{c_1}{c_2} F\left(t + \frac{x}{v} - \frac{2x_0}{v}\right) = -\frac{c_1}{c_2} F\left(t + \frac{x}{v}\right)$$

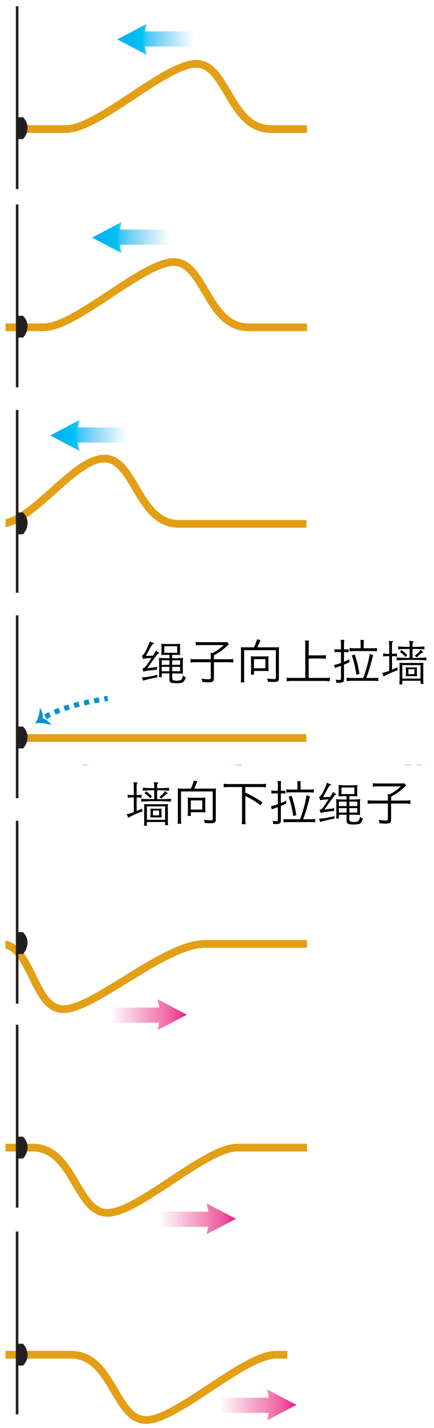
$$u(x,t) = F\left(t - \frac{x}{v}\right) - F\left(t + \frac{x}{v}\right)$$

Prepared by Jiang Xiao

波的反射 - 固定端点

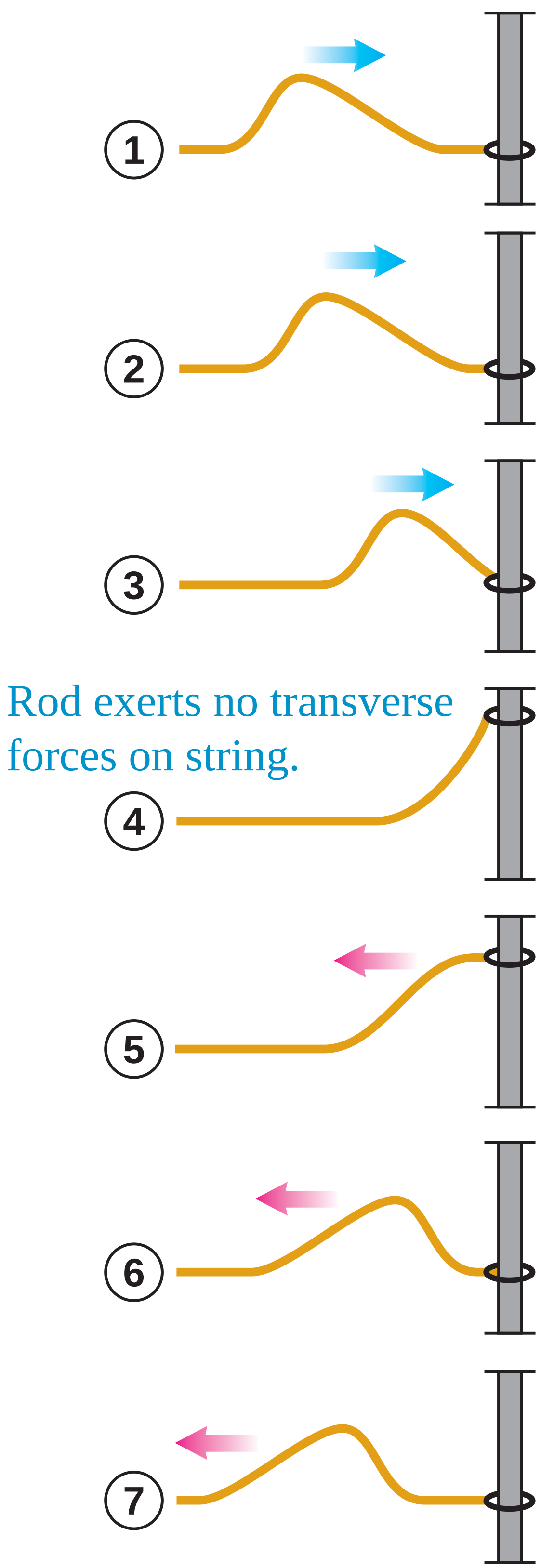






Prepared by Jiang Xiao

波的反射 - 固定自由



波动方程：$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right) u(x,t) = 0$

通解：$u_1(x,t) = F\left(t - \frac{x}{v}\right) \quad \text{and} \quad u_2(x,t) = G\left(t + \frac{x}{v}\right)$

一般解：$u(x,t) = c_1 F\left(t - \frac{x}{v}\right) + c_2 G\left(t + \frac{x}{v}\right)$

边界条件：端点**自由**

$$\left.\frac{\partial}{\partial x} u(x,t)\right|_{x_0} = 0 \quad \Rightarrow \quad G'\left(t + \frac{x_0}{v}\right) = \frac{c_1}{c_2} F'\left(t - \frac{x_0}{v}\right) \qquad x_0 = 0$$

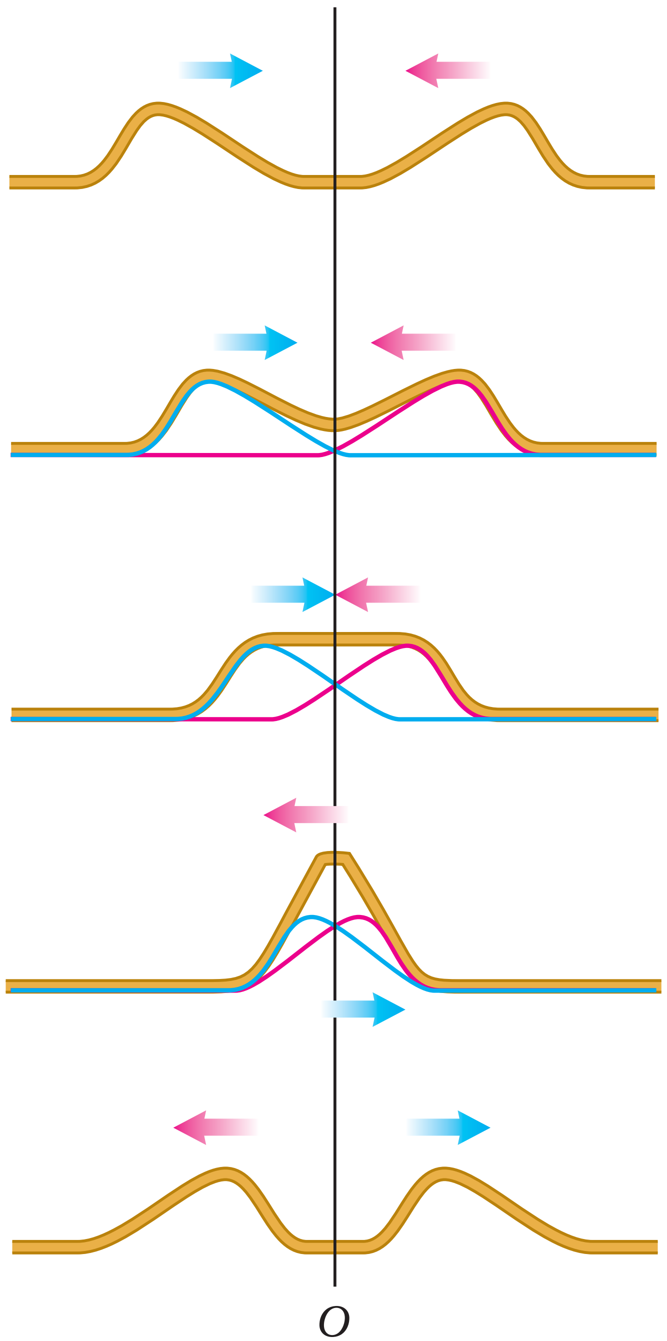
$$\Rightarrow \quad G\left(t + \frac{x}{v}\right) = \frac{c_1}{c_2} F\left(t + \frac{x - 2x_0}{v}\right) = \frac{c_1}{c_2} F\left(t + \frac{x}{v}\right)$$

$$u(x,t) = F\left(t - \frac{x}{v}\right) + F\left(t + \frac{x}{v}\right)$$

波的反射（端点自由）：http://v.youku.com/v_show/id_XMzg5NTI1OTA4.html

Prepared by Jiang Xiao

波的反射 - 固定自由



波动方程：$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right) u(x,t) = 0$

通解：$u_1(x,t) = F\left(t - \frac{x}{v}\right)$ and $u_2(x,t) = G\left(t + \frac{x}{v}\right)$

一般解：$u(x,t) = c_1 F\left(t - \frac{x}{v}\right) + c_2 G\left(t + \frac{x}{v}\right)$

$\boxed{\text{边界条件：端点自由}}$

$x_0 = 0$

$x_0 = 0$

$$\left.\frac{\partial}{\partial x} u(x,t)\right|_{x_0} = 0 \quad \Rightarrow \quad G'\left(t + \frac{x_0}{v}\right) = \frac{c_1}{c_2} F'\left(t - \frac{x_0}{v}\right)$$

$$\Rightarrow \quad G\left(t + \frac{x}{v}\right) = \frac{c_1}{c_2} F\left(t + \frac{x - 2x_0}{v}\right) = \frac{c_1}{c_2} F\left(t + \frac{x}{v}\right)$$

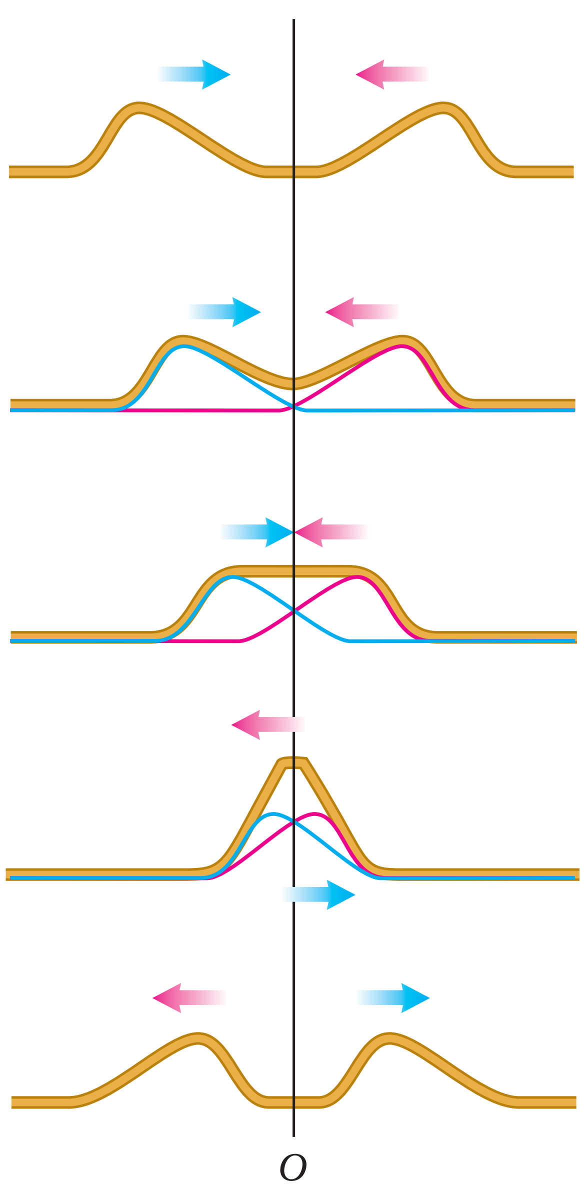
$$u(x,t) = F\left(t - \frac{x}{v}\right) + F\left(t + \frac{x}{v}\right)$$

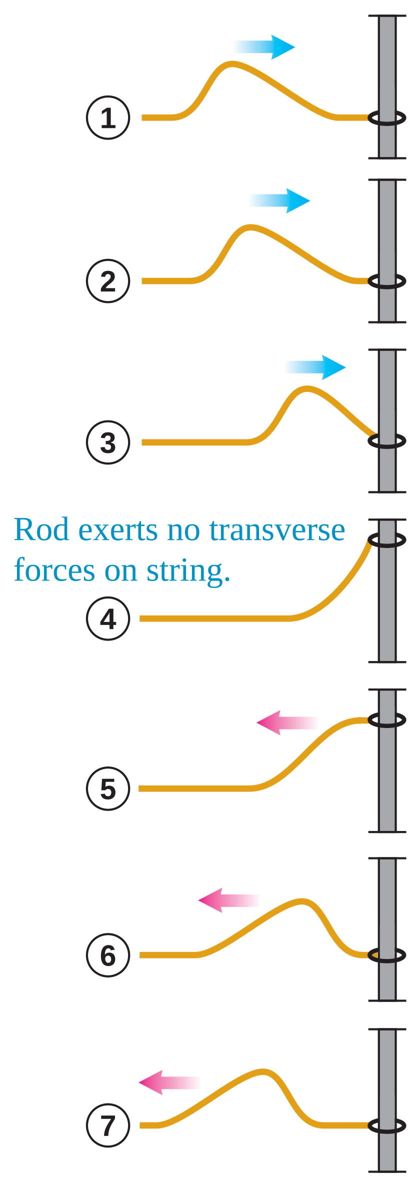
Prepared by Jiang Xiao

波的反射

边界条件：端点**自由**

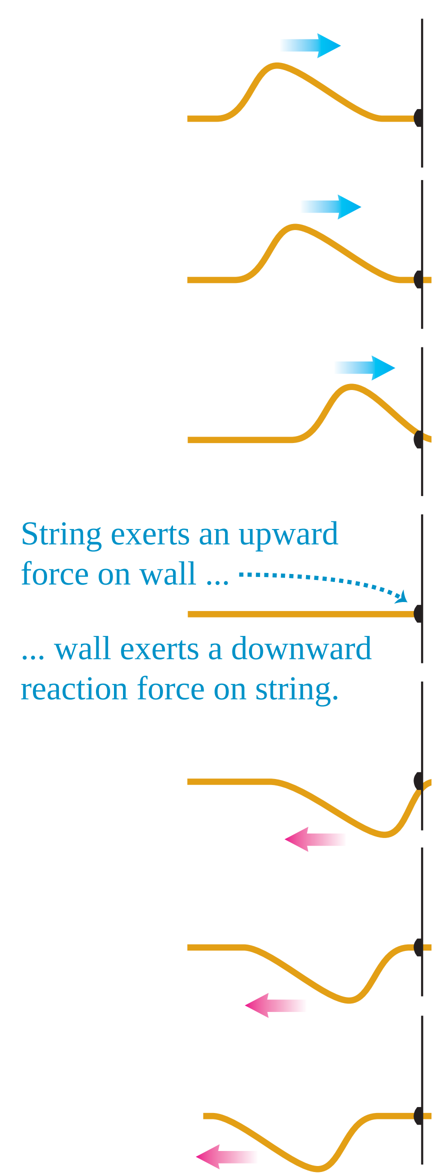
$$u(x,t) = F\left(t - \frac{x}{v}\right) + F\left(t + \frac{x}{v}\right)$$

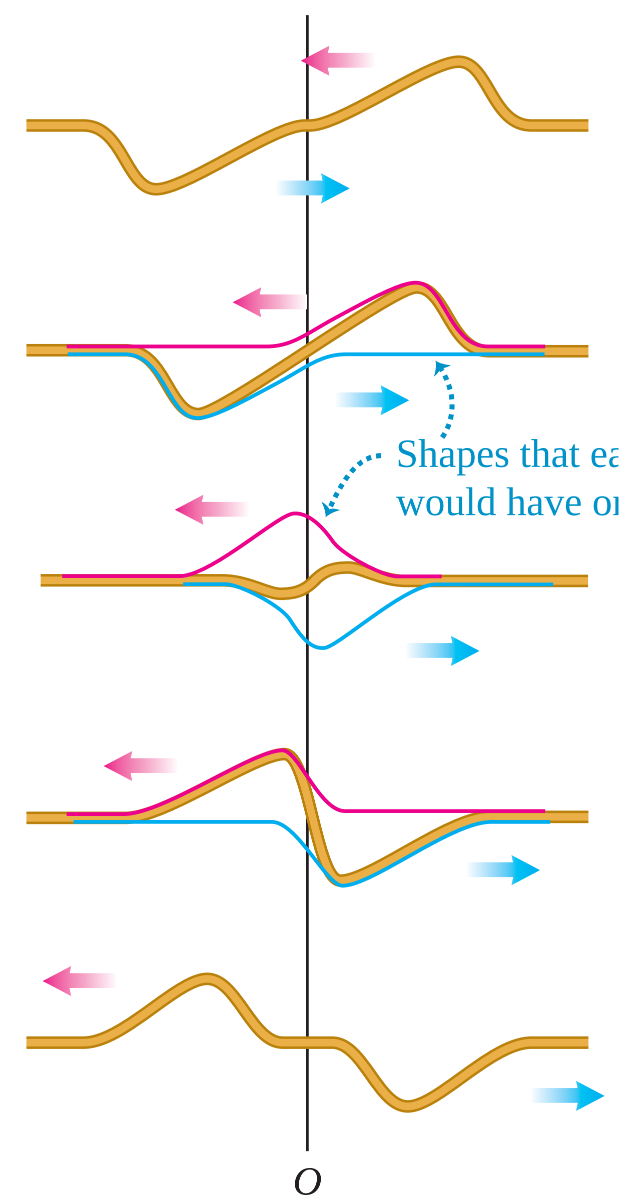




边界条件：端点**固定**

$$u(x,t) = F\left(t - \frac{x}{v}\right) - F\left(t + \frac{x}{v}\right)$$





驻波

$$u_1(x,t) = -A\cos(\omega t + kx)$$
$$u_2(x,t) = A\cos(\omega t - kx)$$

相消叠加：

$$2kx = 2n\pi \quad \Rightarrow \quad x = \frac{n\pi}{k} = \frac{n}{2}\lambda$$

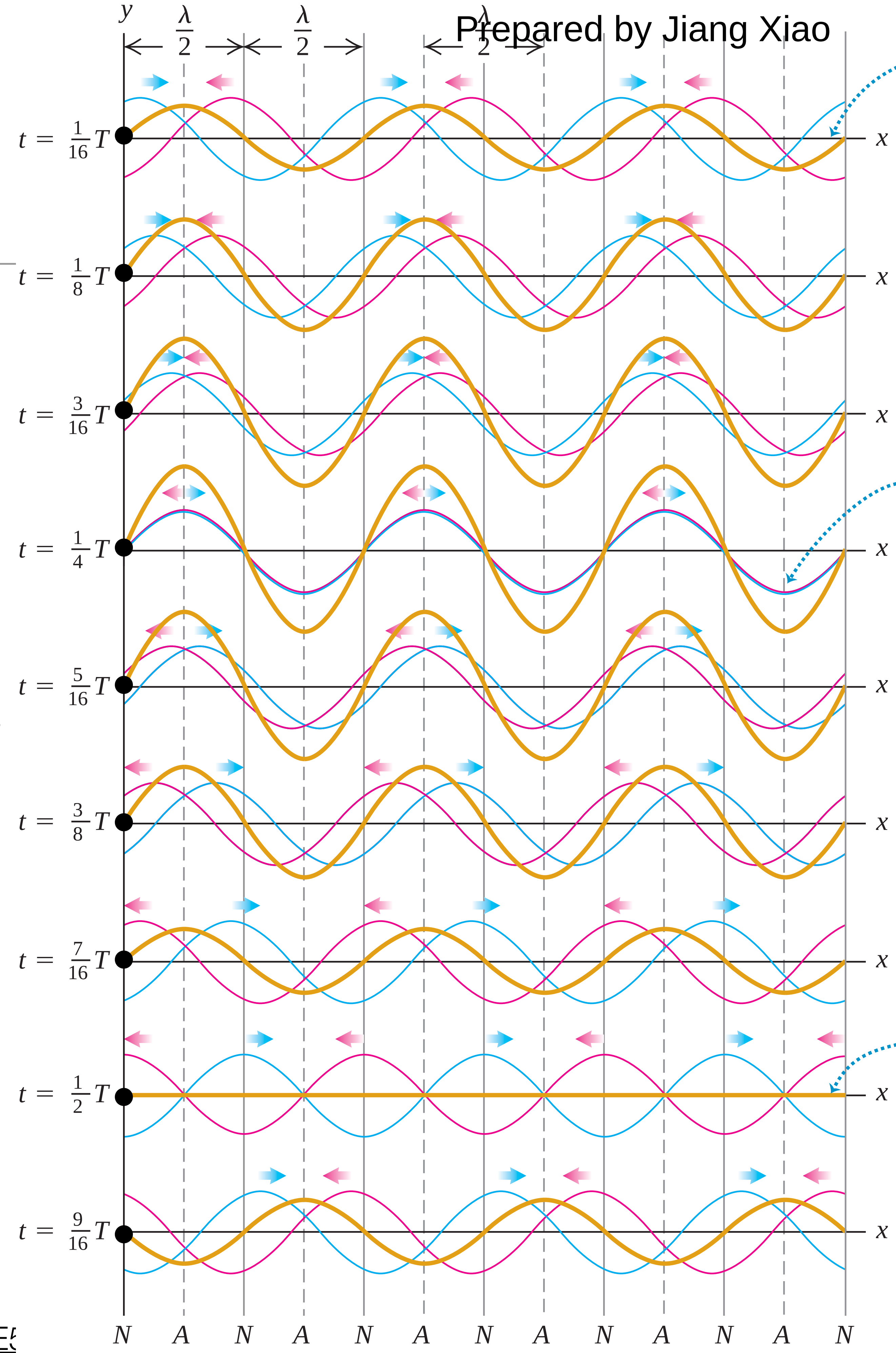
相长叠加：

$$2kx = (2n+1)\pi \quad \Rightarrow \quad x = \frac{(2n+1)\pi}{2k}$$

$$u(x,t) = u_1(x,t) + u_2(x,t)$$
$$= 2A\sin(kx)\sin(\omega t)$$
$$= 2A(x)\sin(\omega t)$$

端点固定：端点为波节　　Node (N): 波节

端点自由：端点为波腹　　Anti-node (A): 波腹

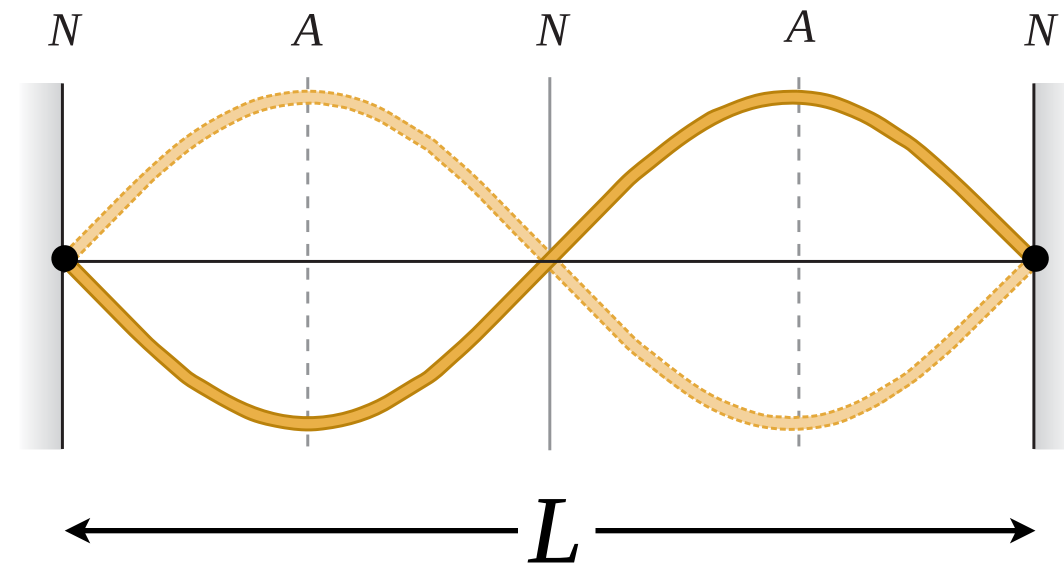


波的叠加、反射等：http://v.youku.com/v_show/id_XMzg5NjE5

驻波 - 两端固定

$$u_1(x,t) = -A\cos(\omega t + kx)$$
$$u_2(x,t) = A\cos(\omega t - kx)$$

Node (N): 波节, Anti-node (A): 波腹



相消叠加：

$$2kx = 2n\pi \quad \Rightarrow \quad x = \frac{n\pi}{k} = \frac{n}{2}\lambda$$

相长叠加：

$$2kx = (2n+1)\pi \quad \Rightarrow \quad x = \frac{(2n+1)\pi}{2k}$$

$$\begin{aligned} u(x,t) &= u_1(x,t) + u_2(x,t) \\ &= 2A\sin(kx)\sin(\omega t) \\ &= 2A(x)\sin(\omega t) \end{aligned}$$

$$L = \frac{n}{2}\lambda \quad \Rightarrow \quad \lambda_n = \frac{2L}{n}$$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$$

端点固定：端点为波节

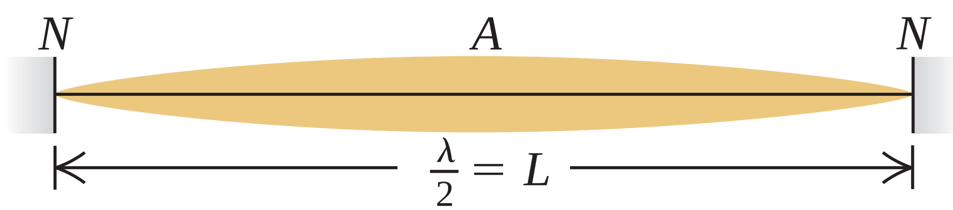
端点自由：端点为波腹

$$v = \sqrt{\frac{T}{\eta}} \quad \Rightarrow \quad f_n = \frac{n}{2L}\sqrt{\frac{T}{\eta}}$$

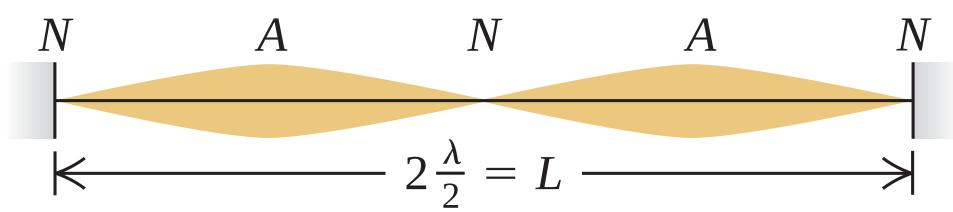
驻波：http://www.stmary.ws/highschool/physics/home/review/waves/standwave.swf

驻波 - 两端固定

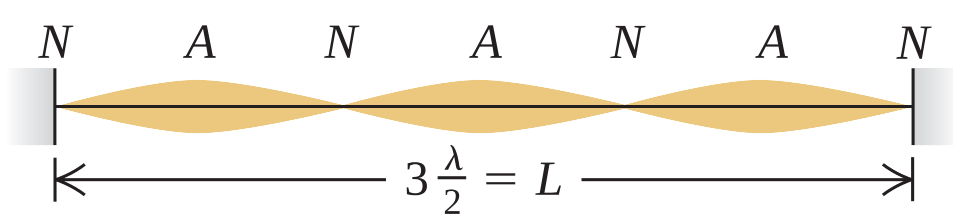
(a) $n = 1$: fundamental frequency, f_1



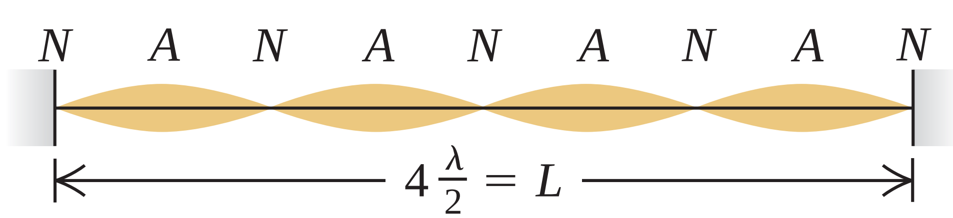
(b) $n = 2$: second harmonic, f_2 (first overtone)



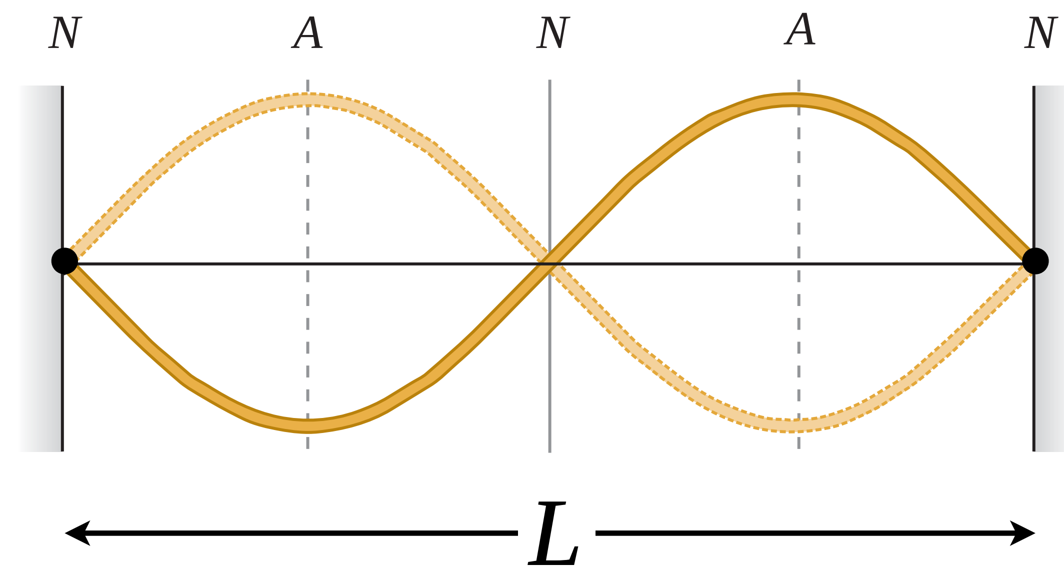
(c) $n = 3$: third harmonic, f_3 (second overtone)



(d) $n = 4$: fourth harmonic, f_4 (third overtone)



Node (N): 波节, Anti-node (A): 波腹



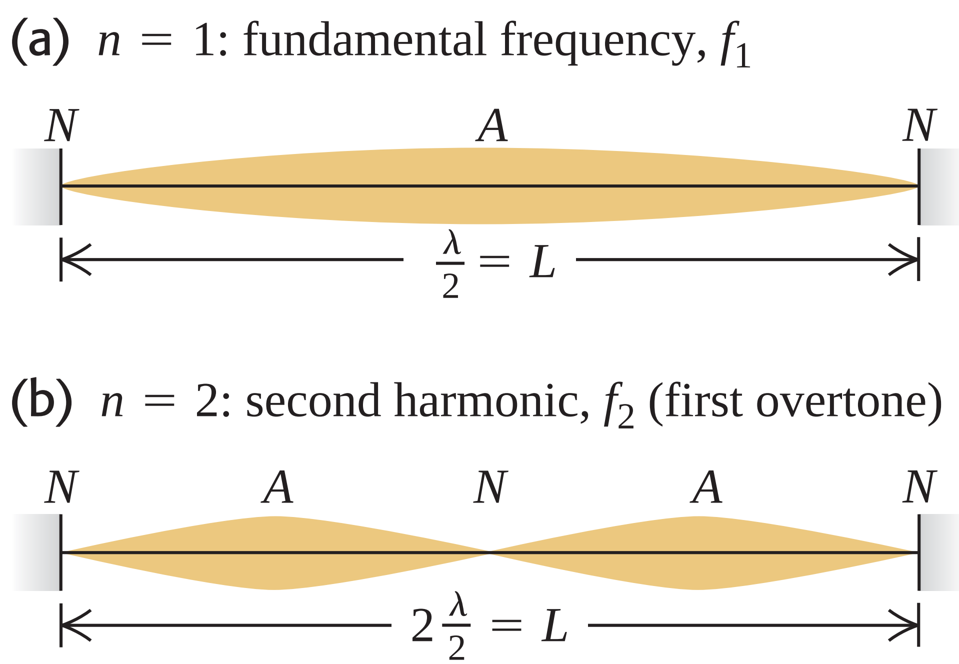
$$L = \frac{n}{2}\lambda \quad \Rightarrow \quad \lambda_n = \frac{2L}{n}$$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$$

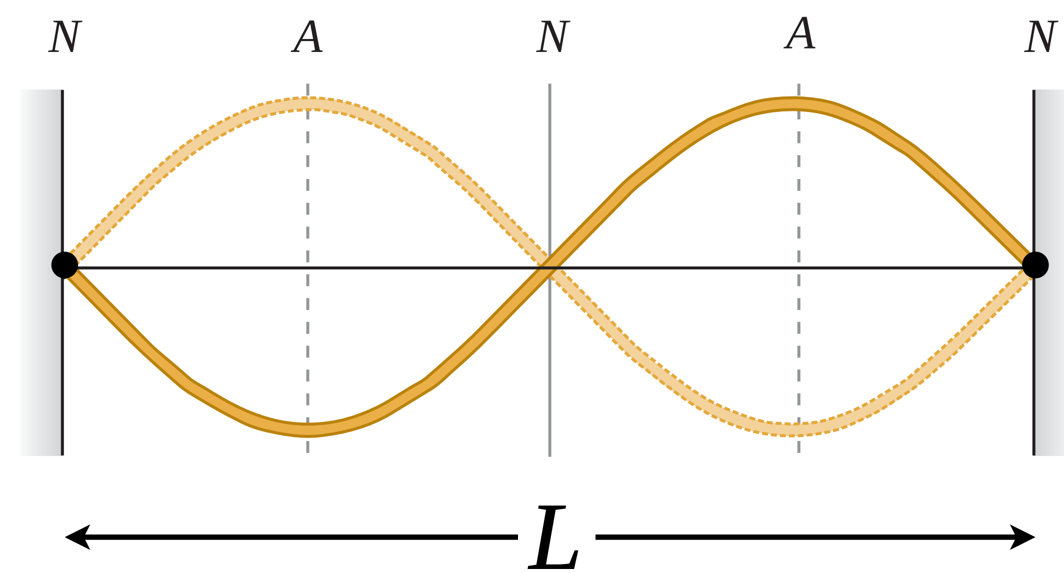
$$v = \sqrt{\frac{T}{\eta}} \quad \Rightarrow \quad f_n = \frac{n}{2L}\sqrt{\frac{T}{\eta}}$$

驻波：http://v.youku.com/v_show/id_XMzg5MDY2MTEy.html

驻波 - 两端固定

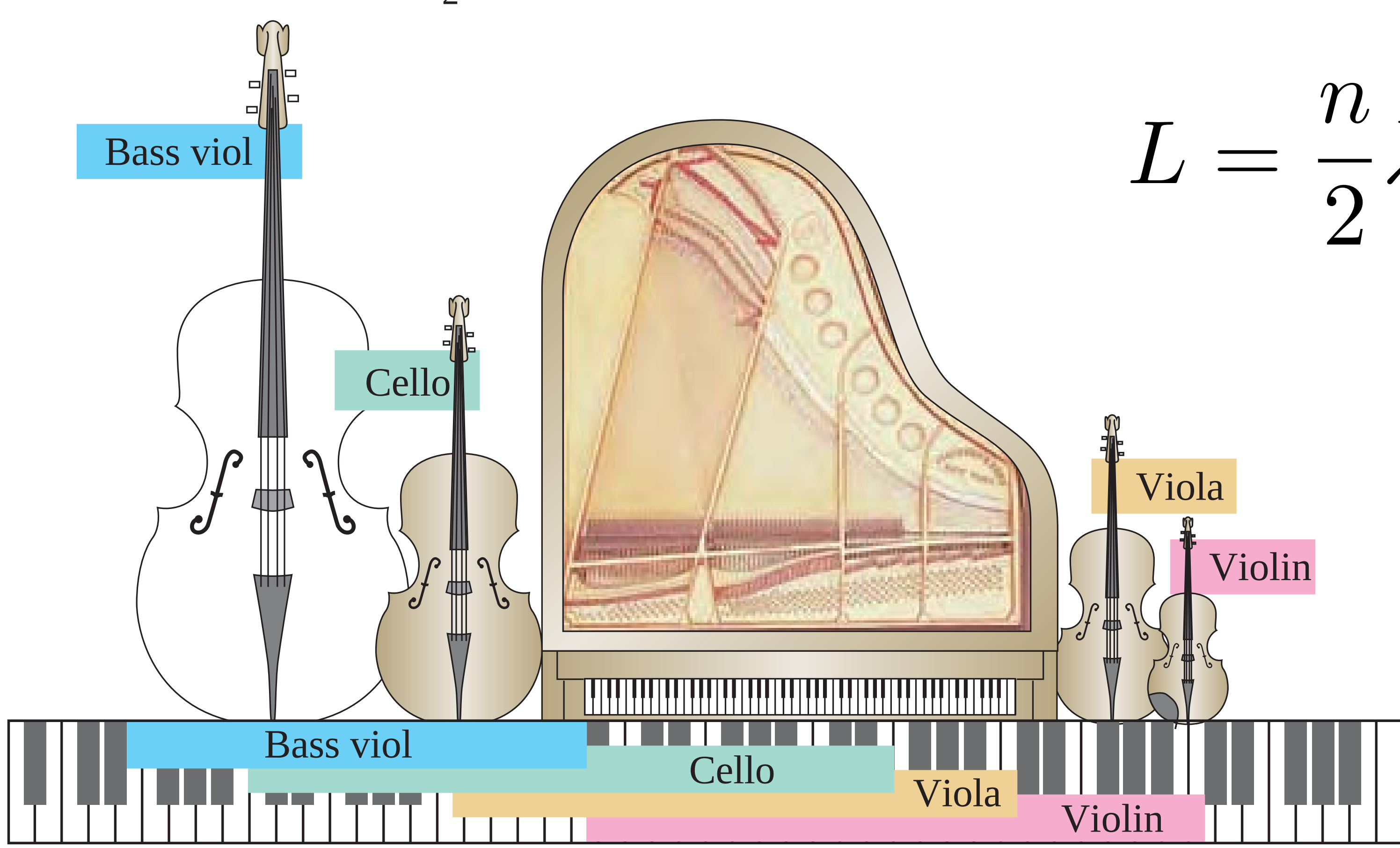


Node (N): 波节, Anti-node (A): 波腹



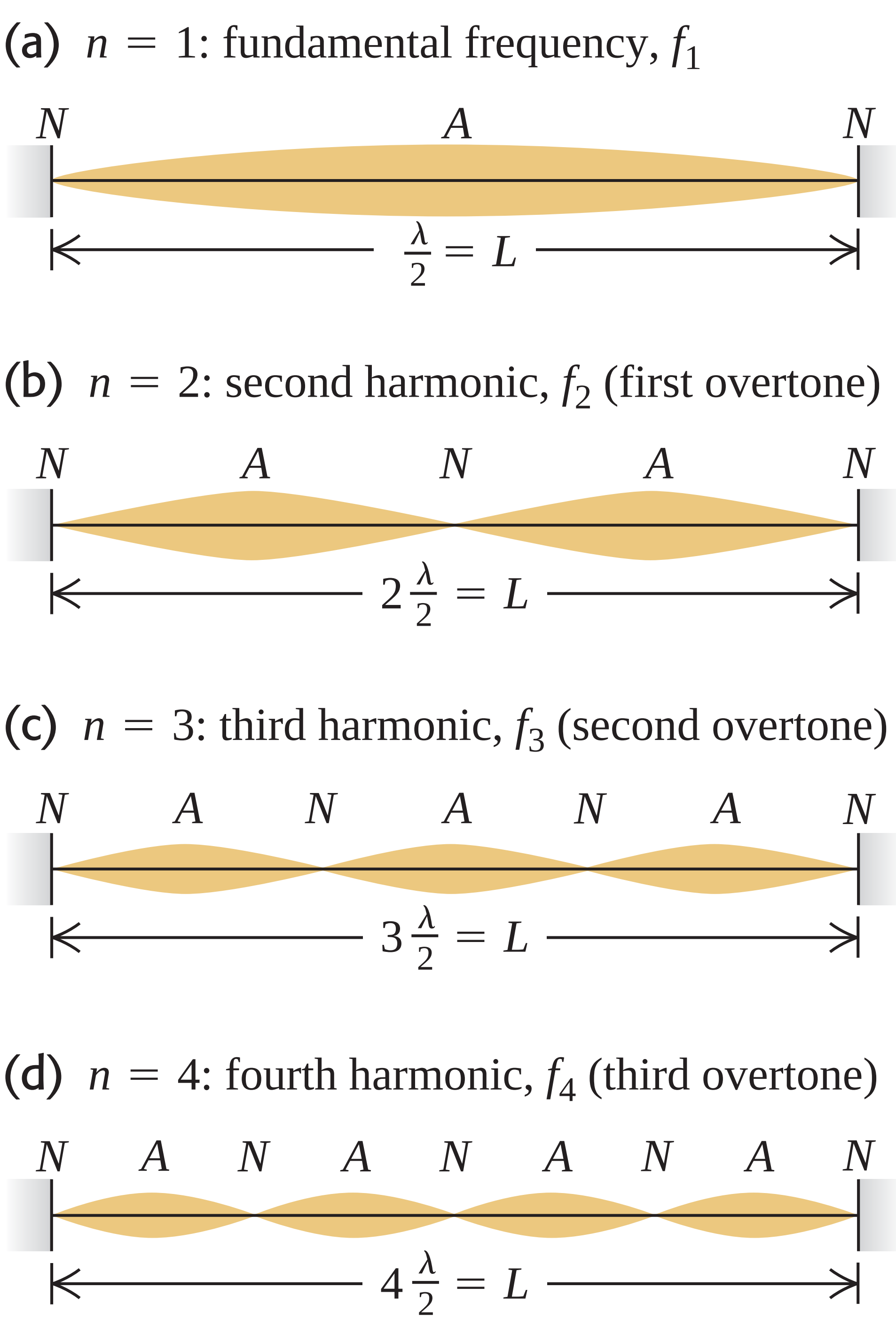
$$L = \frac{n}{2}\lambda \quad \Rightarrow \quad \lambda_n = \frac{2L}{n}$$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$$

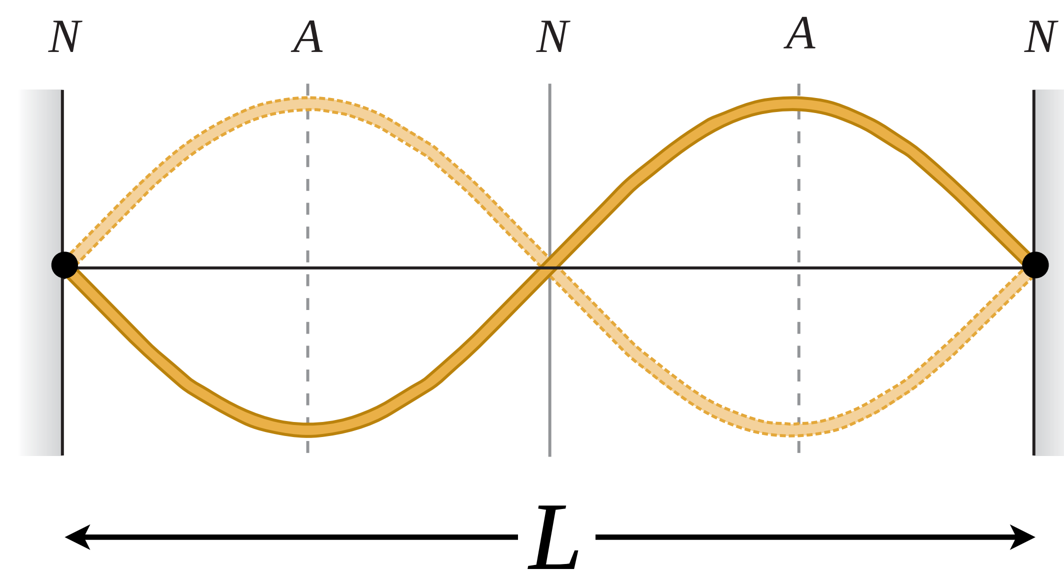


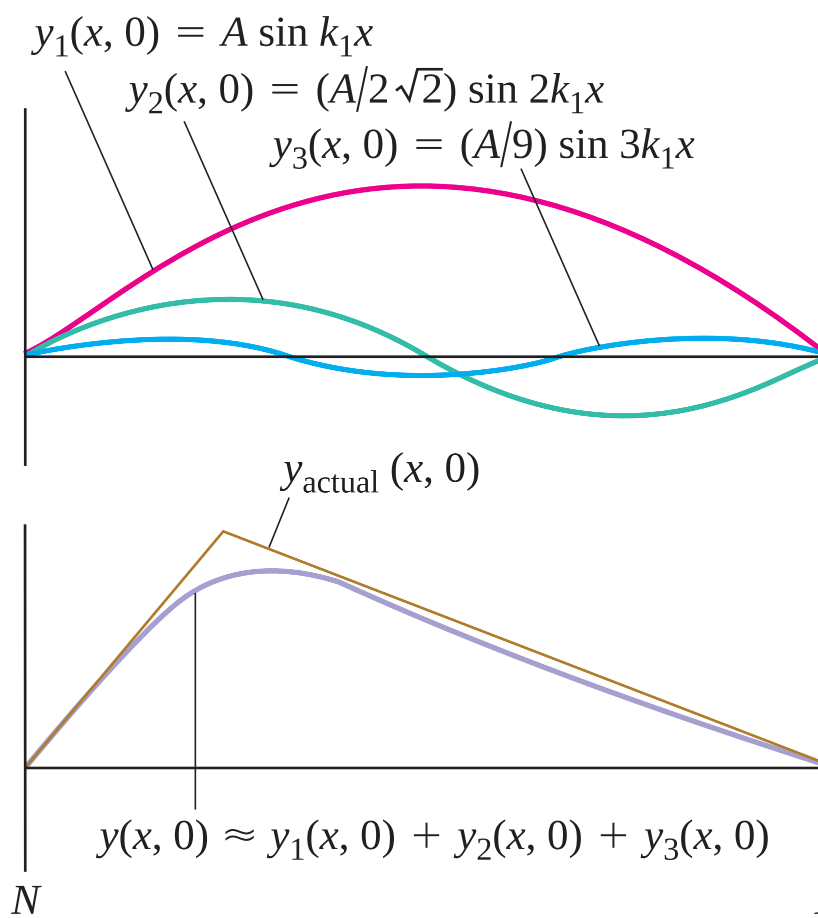
Prepared by Jiang Xiao

驻波 - 两端固定



Node (N): 波节, Anti-node (A): 波腹

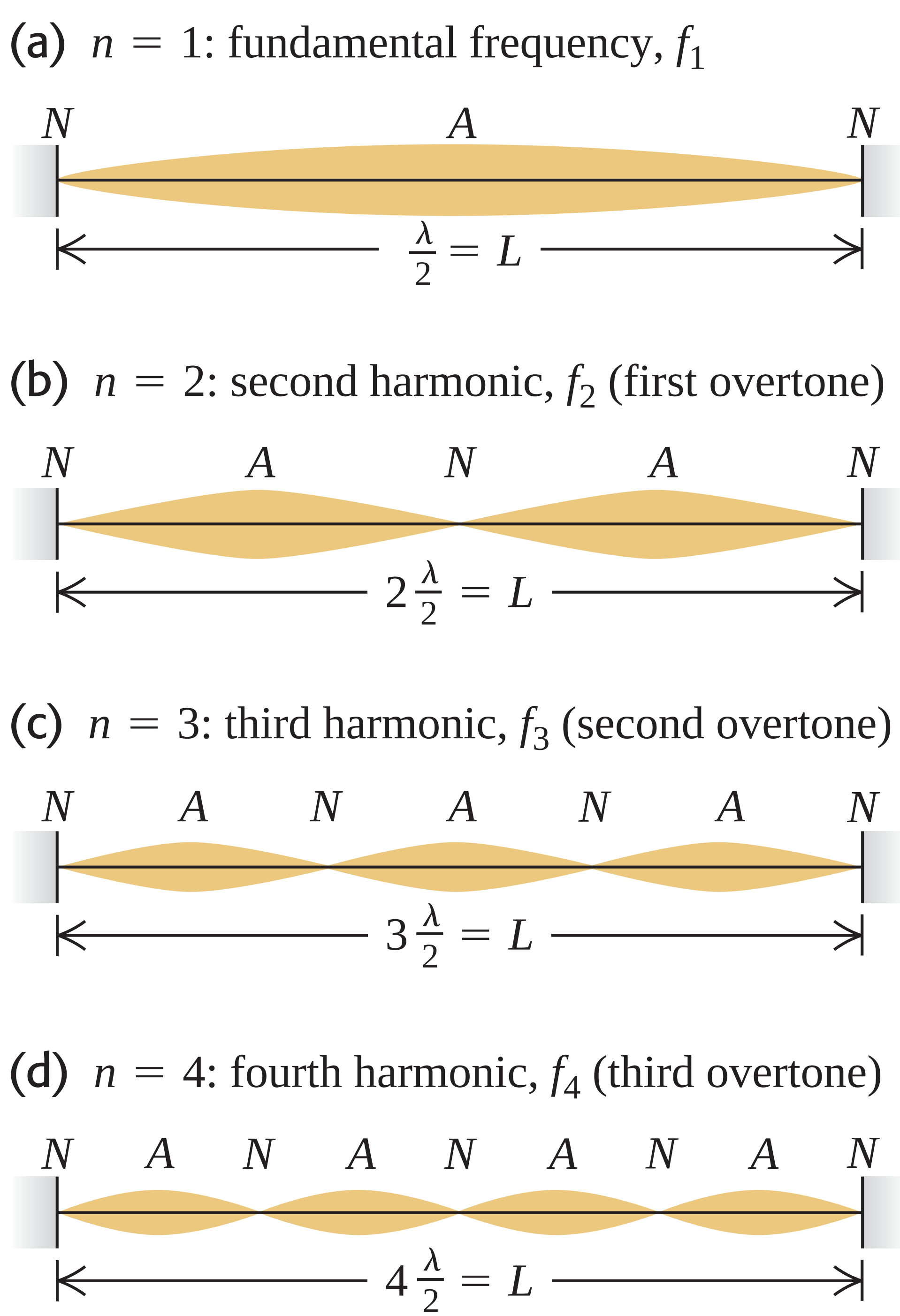




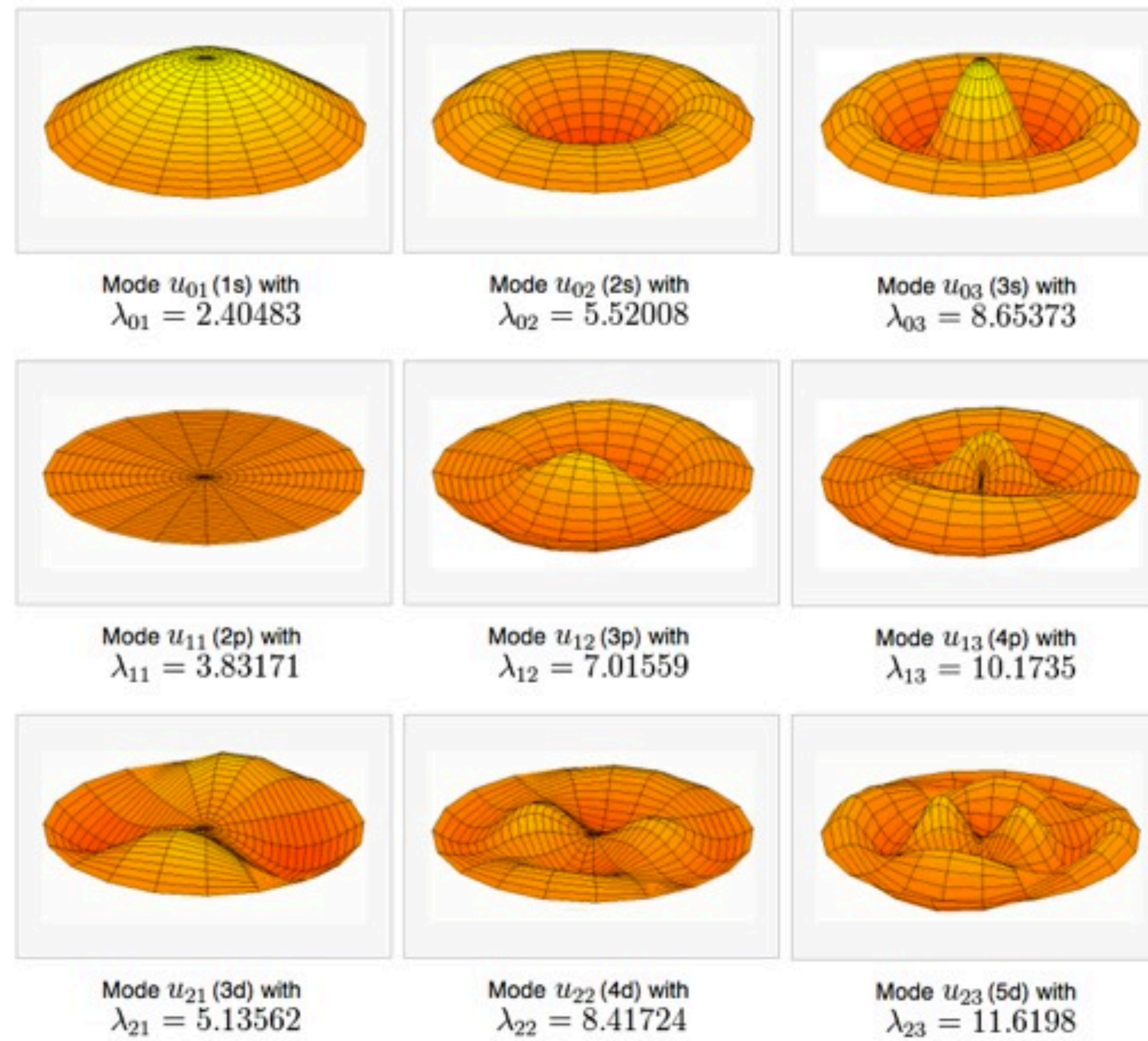
Node (N): 波节, Anti-node (A): 波腹

Prepared by Jiang Xiao

驻波 - 二维

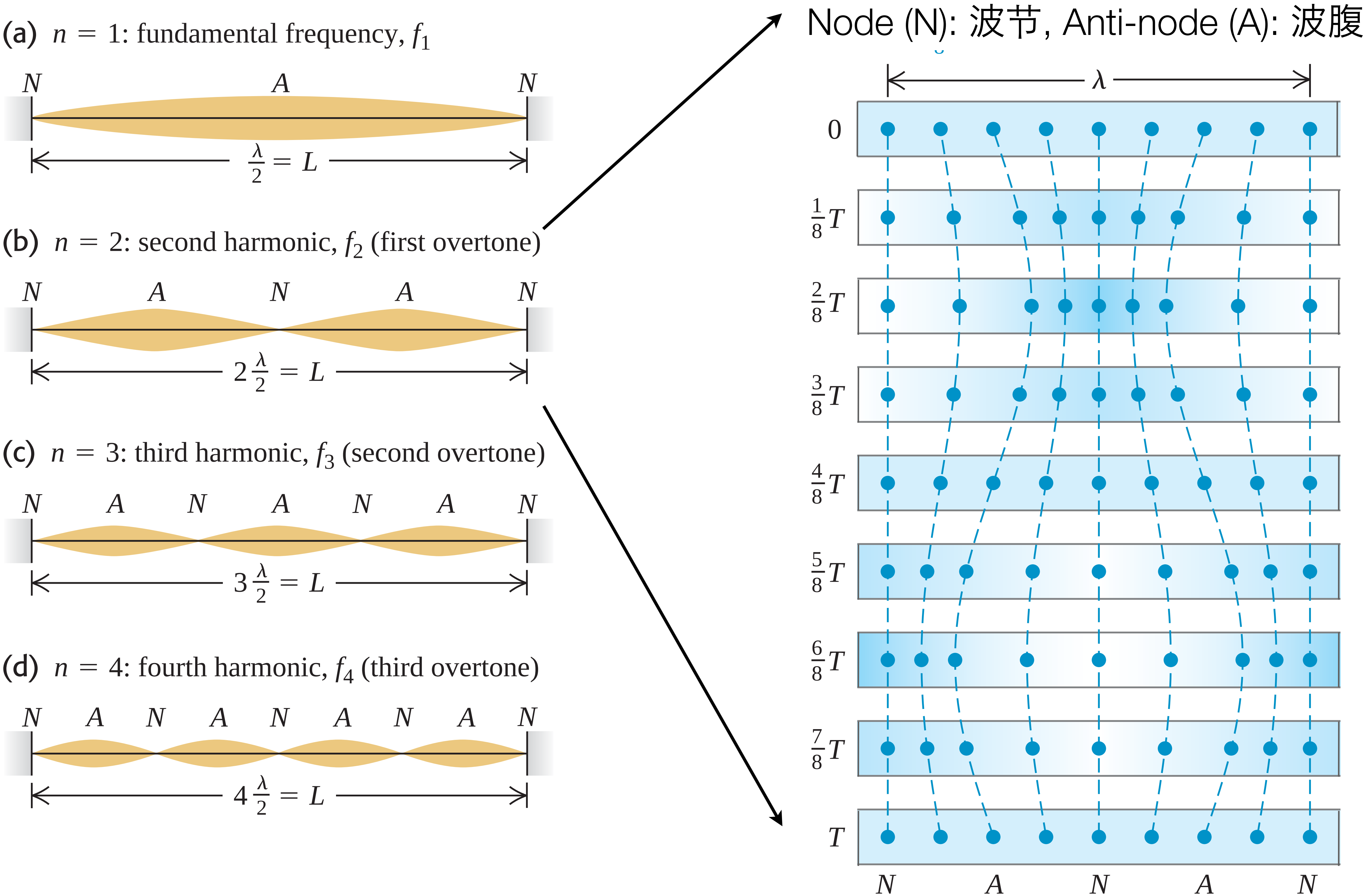


二维圆盘：http://en.wikipedia.org/wiki/Vibrations_of_a_circular_drum#Animations_of_several_vibration_modes

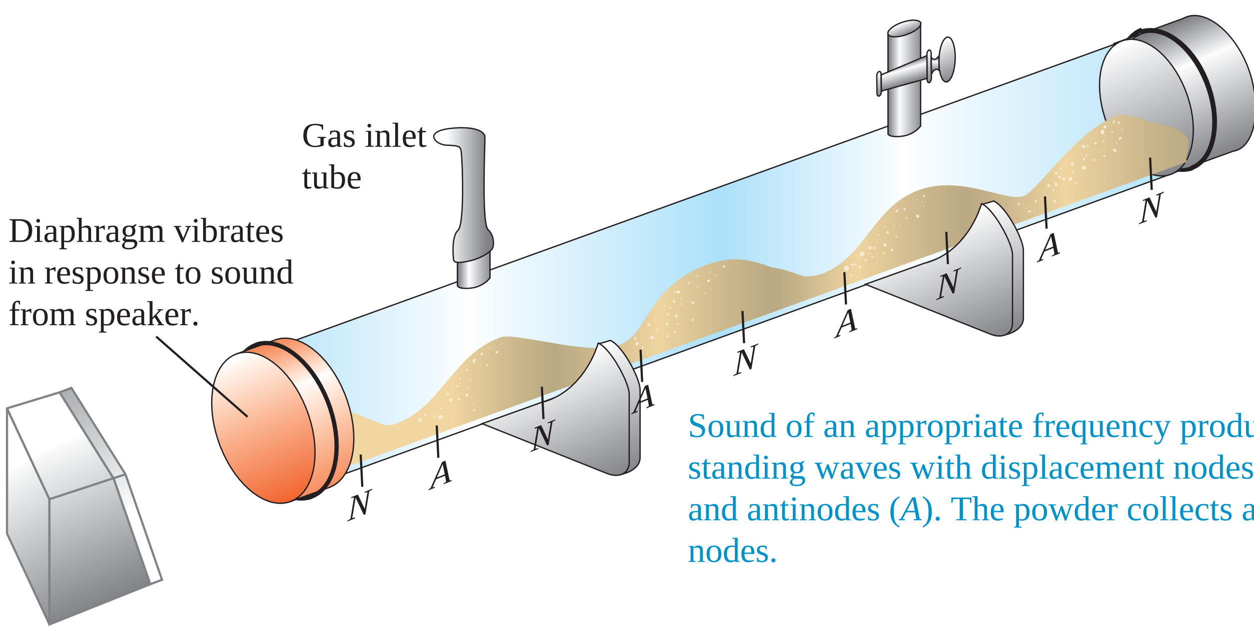


二维方盘：http://v.youku.com/v_show/id_XMzg2ODI1NTI0.html

声驻波

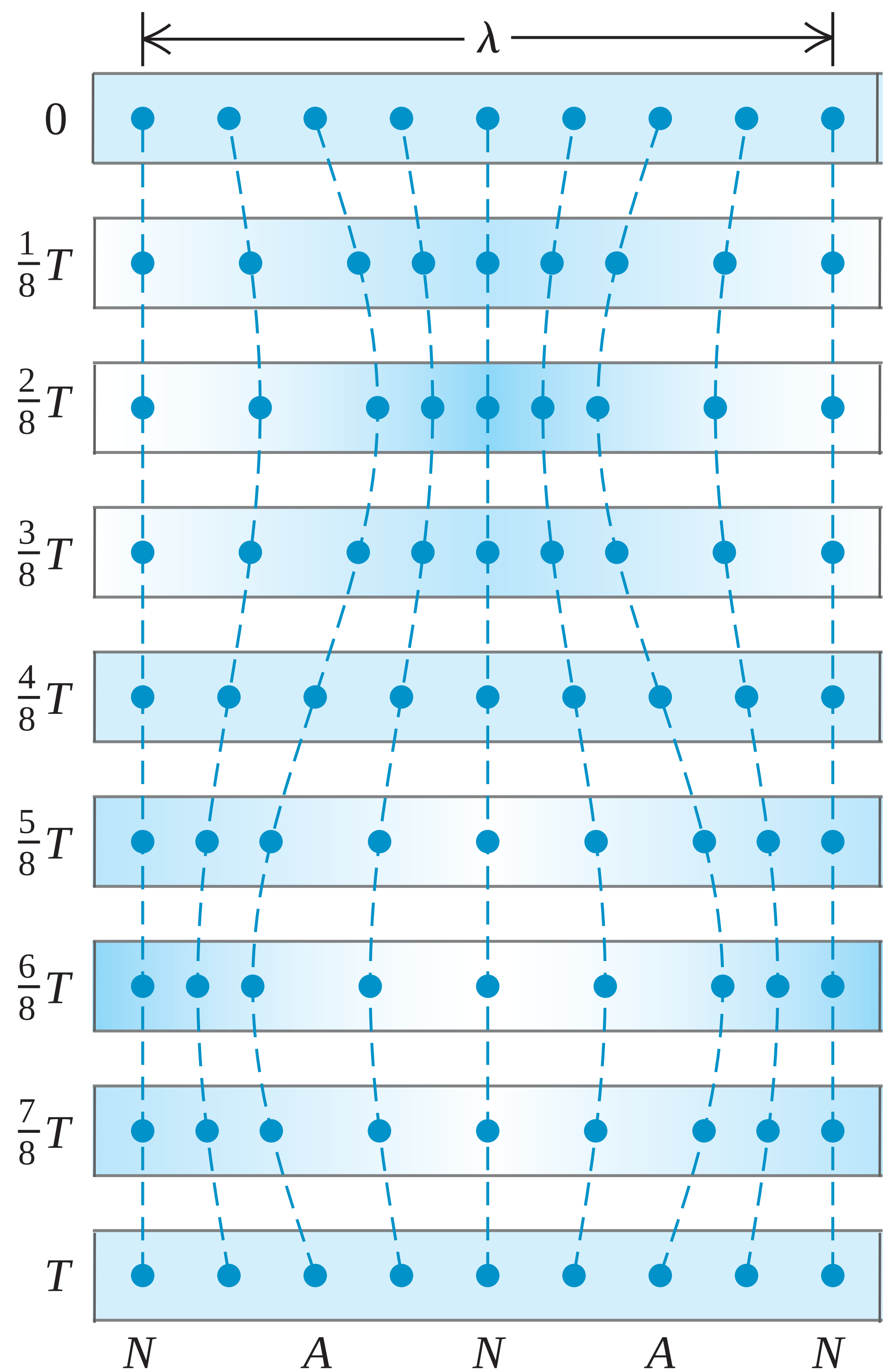

(a) $n = 1$: fundamental frequency, f_1
N A N
$\frac{\lambda}{2} = L$
(b) $n = 2$: second harmonic, f_2 (first overtone)
N A N A N
$2\frac{\lambda}{2} = L$
(c) $n = 3$: third harmonic, f_3 (second overtone)
N A N A N A N
$3\frac{\lambda}{2} = L$
(d) $n = 4$: fourth harmonic, f_4 (third overtone)
N A N A N A N A N
$4\frac{\lambda}{2} = L$
Node (N): 波节, Anti-node (A): 波腹
λ
0
$\frac{1}{8}T$
$\frac{2}{8}T$
$\frac{3}{8}T$
$\frac{4}{8}T$
$\frac{5}{8}T$
$\frac{6}{8}T$
$\frac{7}{8}T$
T
N A N A N

声驻波



Sound of an appropriate frequency produces standing waves with displacement nodes (N) and antinodes (A). The powder collects at the nodes.

Node (N): 波节, Anti-node (A): 波腹



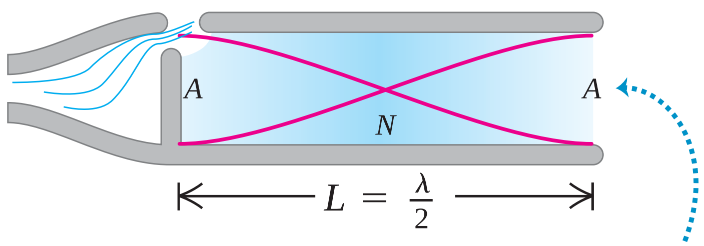
Prepared by Jiang Xiao

声驻波

端点自由：端点为波腹

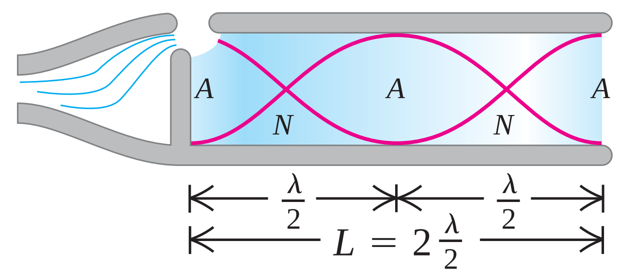
Node (N): 波节, Anti-node (A): 波腹

(a) Fundamental: $f_1 = \frac{v}{2L}$

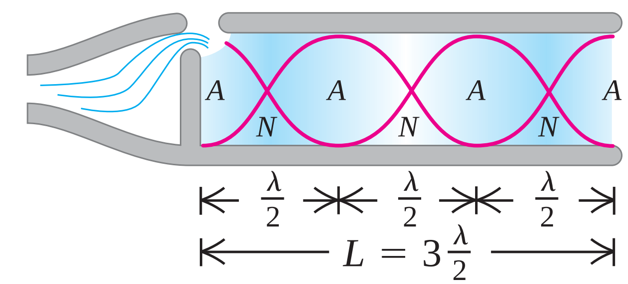


Open end is always a displacement antinode.

(b) Second harmonic: $f_2 = 2\frac{v}{2L} = 2f_1$



(c) Third harmonic: $f_3 = 3\frac{v}{2L} = 3f_1$

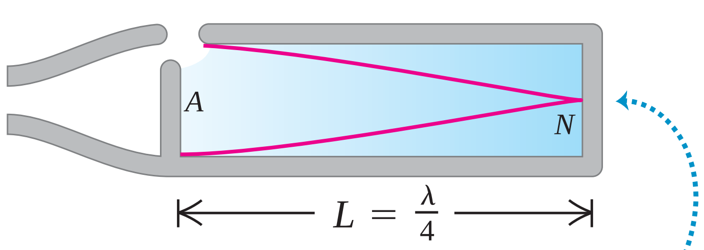


$$L = \frac{n}{2}\lambda \quad \Rightarrow \quad \lambda_n = \frac{2L}{n}, f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1$$

Ruben's tube: http://v.youku.com/v_show/id_XMzg5MDcyMjcy.html
http://v.youku.com/v_show/id_XMzg5MDc0MzQ0.html

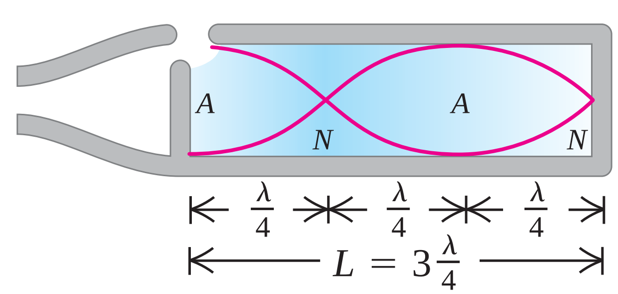
端点固定：端点为波节

(a) Fundamental: $f_1 = \frac{v}{4L}$

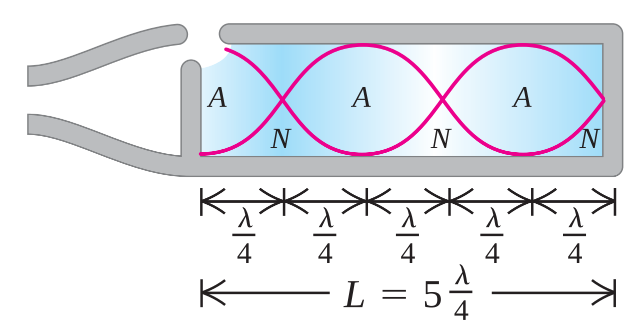


Closed end is always a displacement node.

(b) Third harmonic: $f_3 = 3\frac{v}{4L} = 3f_1$



(c) Fifth harmonic: $f_5 = 5\frac{v}{4L} = 5f_1$

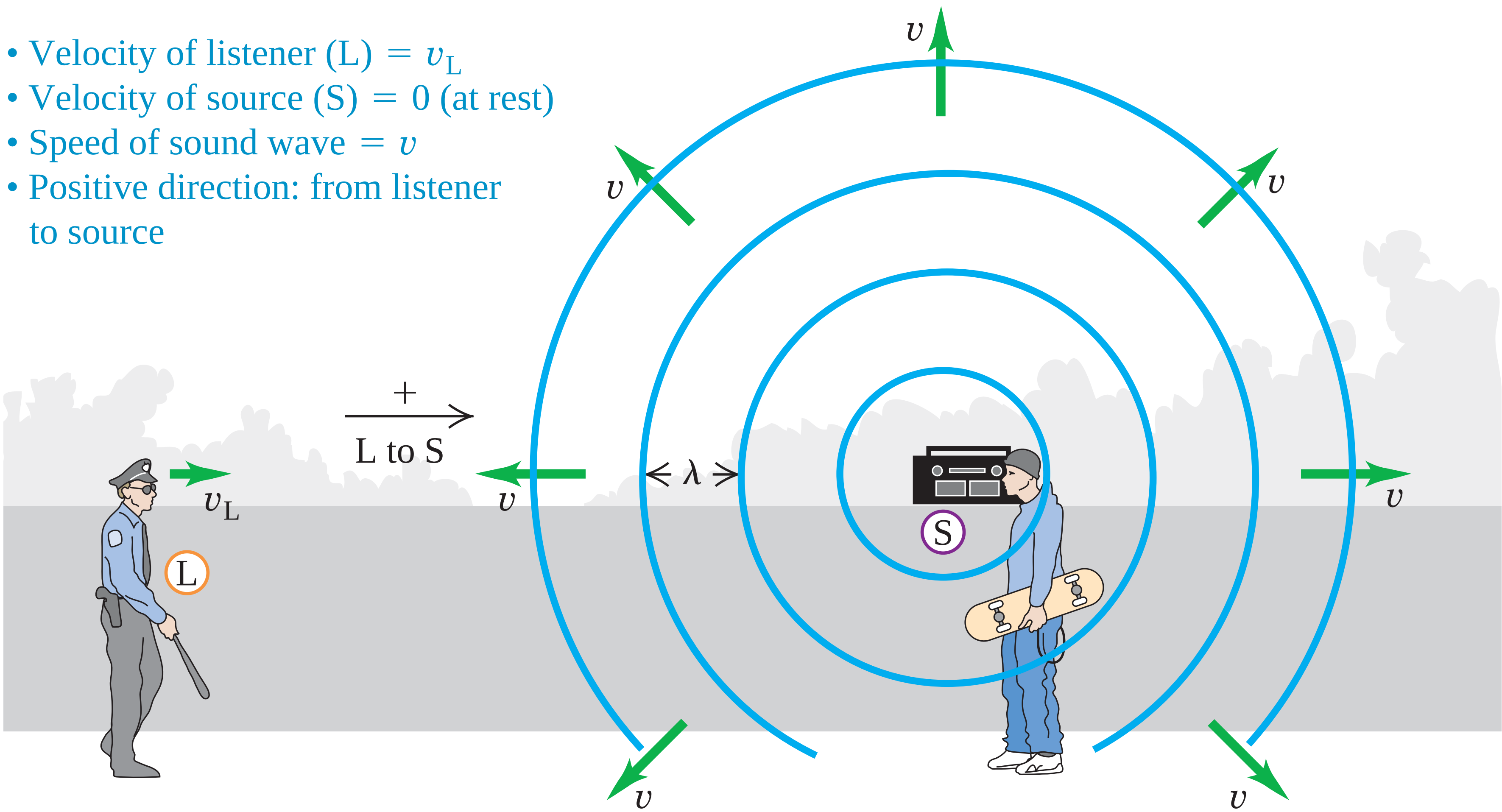


$$L = \left(\frac{n}{2} - \frac{1}{4}\right)\lambda \quad \Rightarrow \quad \lambda_n = \frac{4L}{2n-1}, f_n = \frac{v}{\lambda_n} = (2n-1)\frac{v}{4L} = (2n-1)f_1$$

多普勒效应

源不动，观测者在动

- Velocity of listener (L) $= v_L$
- Velocity of source (S) $= 0$ (at rest)
- Speed of sound wave $= v$
- Positive direction: from listener to source



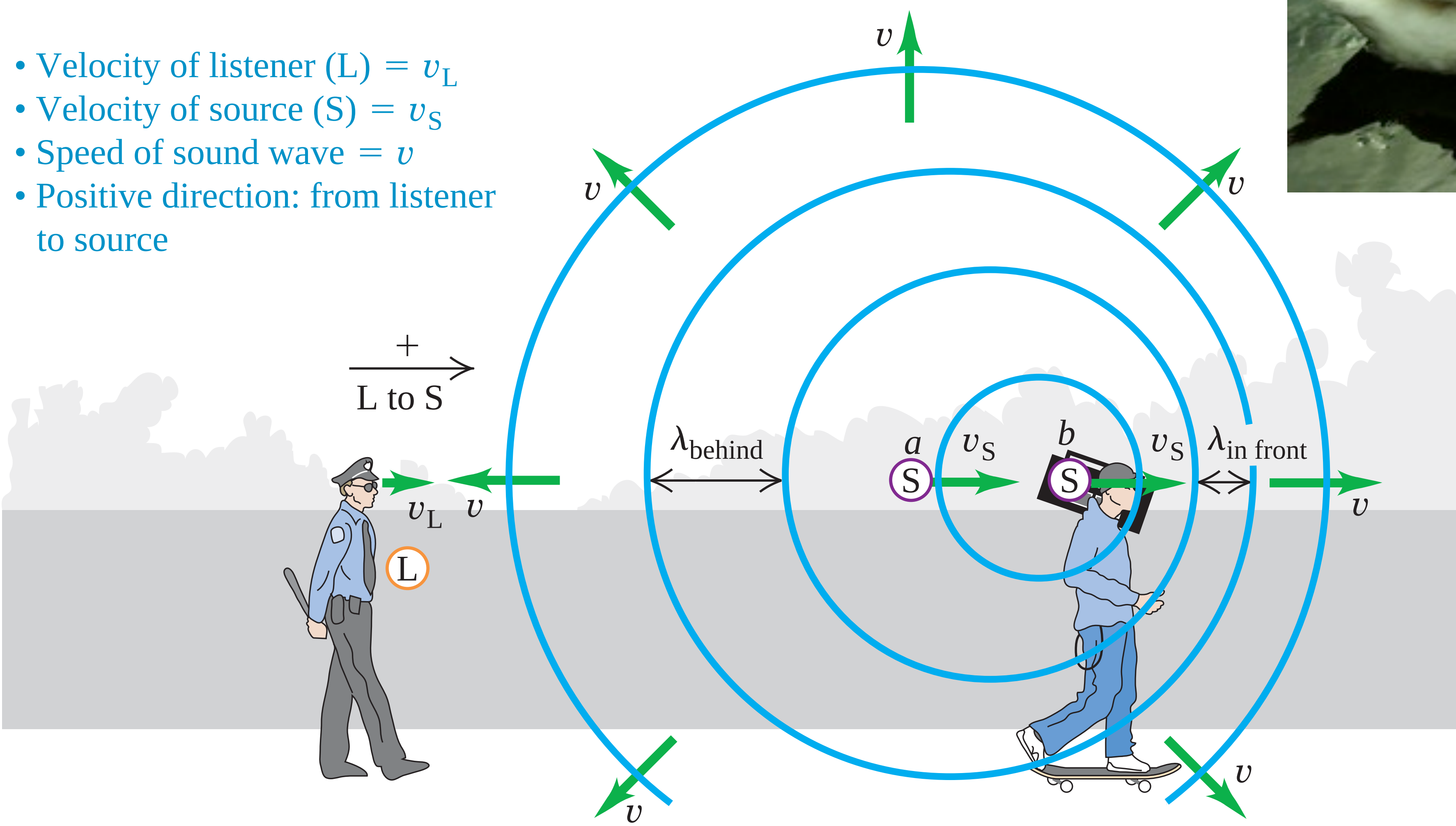
$$f_S = \frac{v}{\lambda}$$

$$f_L = \frac{v + v_L}{\lambda} = \left(1 + \frac{v_L}{v}\right) f_S$$

多普勒效应

源和观测者都在动

- Velocity of listener (L) $= v_L$
- Velocity of source (S) $= v_S$
- Speed of sound wave $= v$
- Positive direction: from listener to source



$$\lambda_{\text{in front}} = \frac{v}{f_S} - \frac{v_S}{f_S} = \frac{v - v_S}{f_S}$$

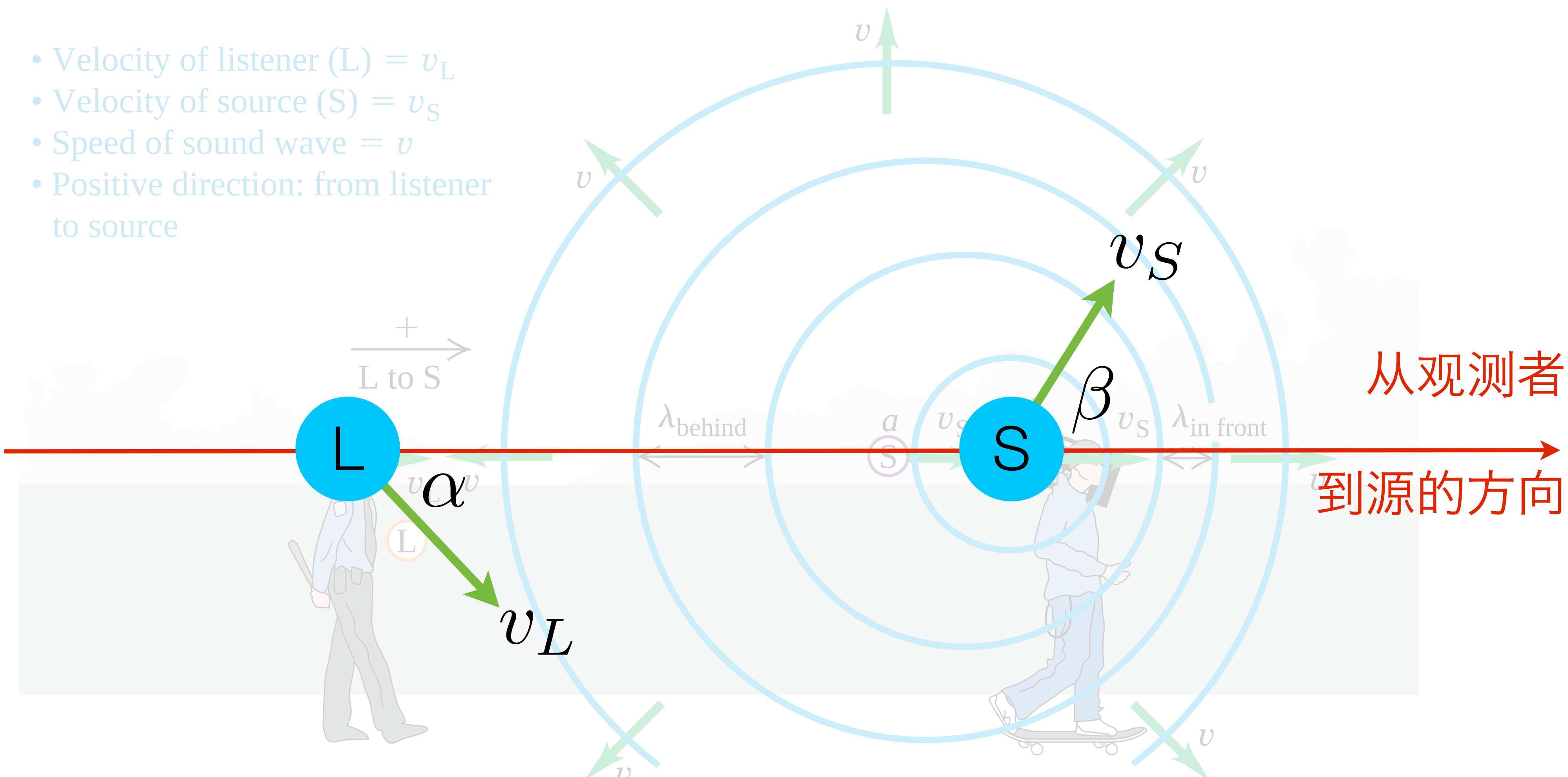
$$\lambda_{\text{behind}} = \frac{v}{f_S} + \frac{v_S}{f_S} = \frac{v + v_S}{f_S}$$

$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} = \frac{v + v_L}{v + v_S} f_S$$

Prepared by Jiang Xiao

多普勒效应

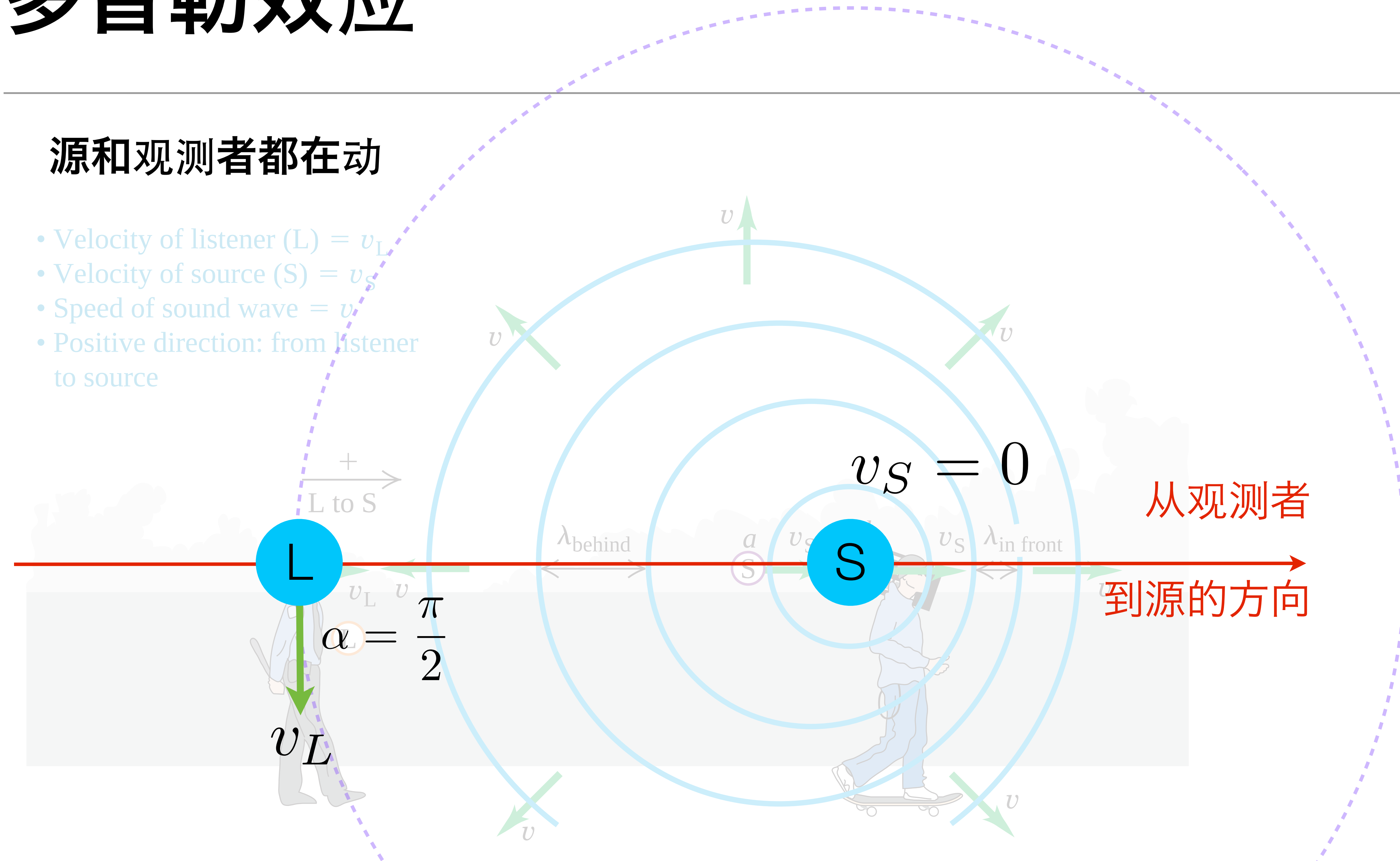
源和观测者都在动



$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} = \frac{v + v_L}{v + v_S} f_S = \frac{v + v_L \cos\alpha}{v + v_S \cos\beta} f_S$$

多普勒效应

源和观测者都在动



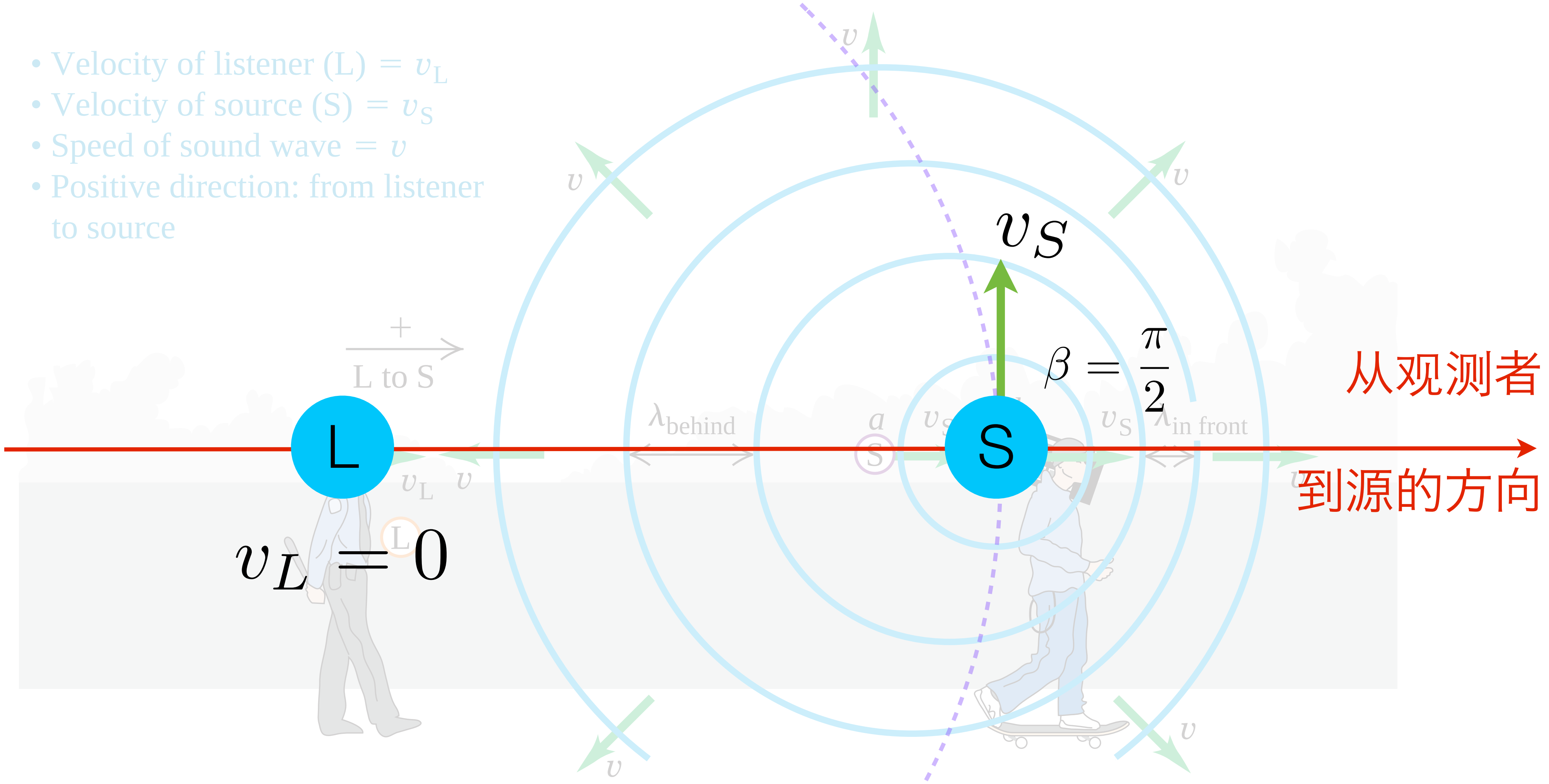
$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} = \frac{v + v_L}{v + v_S} f_S = \frac{v + v_L \cos\alpha}{v + v_S \cos\beta} f_S = f_S$$

Prepared by Jiang Xiao

多普勒效应

源和观测者都在动

- Velocity of listener (L) = v_L
- Velocity of source (S) = v_S
- Speed of sound wave = v
- Positive direction: from listener to source



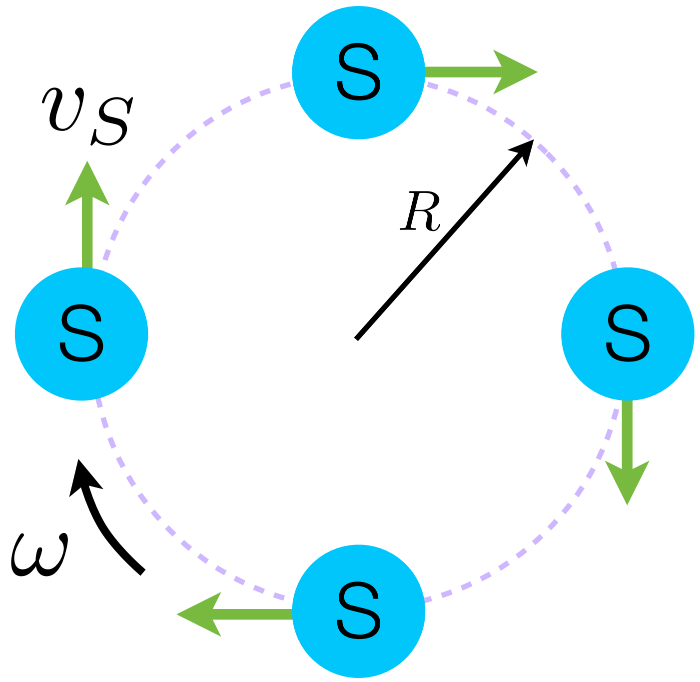
$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} = \frac{v + v_L}{v + v_S} f_S = \frac{v + v_L \cos\alpha}{v + v_S \cos\beta} f_S = f_S$$

Prepared by Jiang Xiao

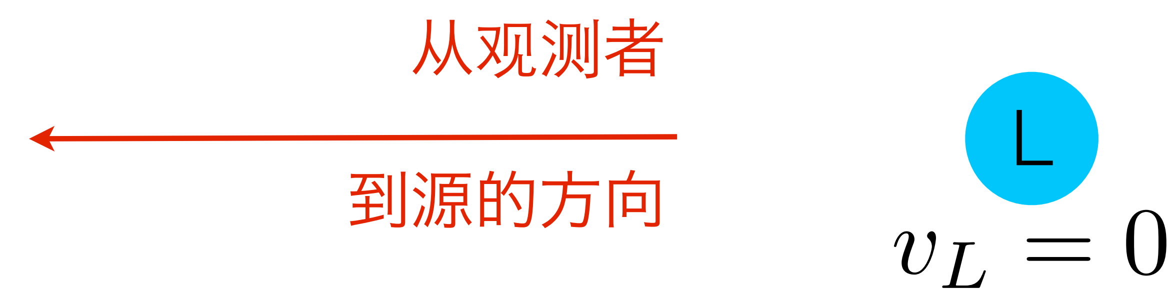
多普勒效应

$$f_L = \frac{v}{v - v_S} f_S > f_S$$

$$\omega = \frac{v_S}{R}, \quad T = \frac{2\pi}{\omega}, \quad \beta = \omega t$$



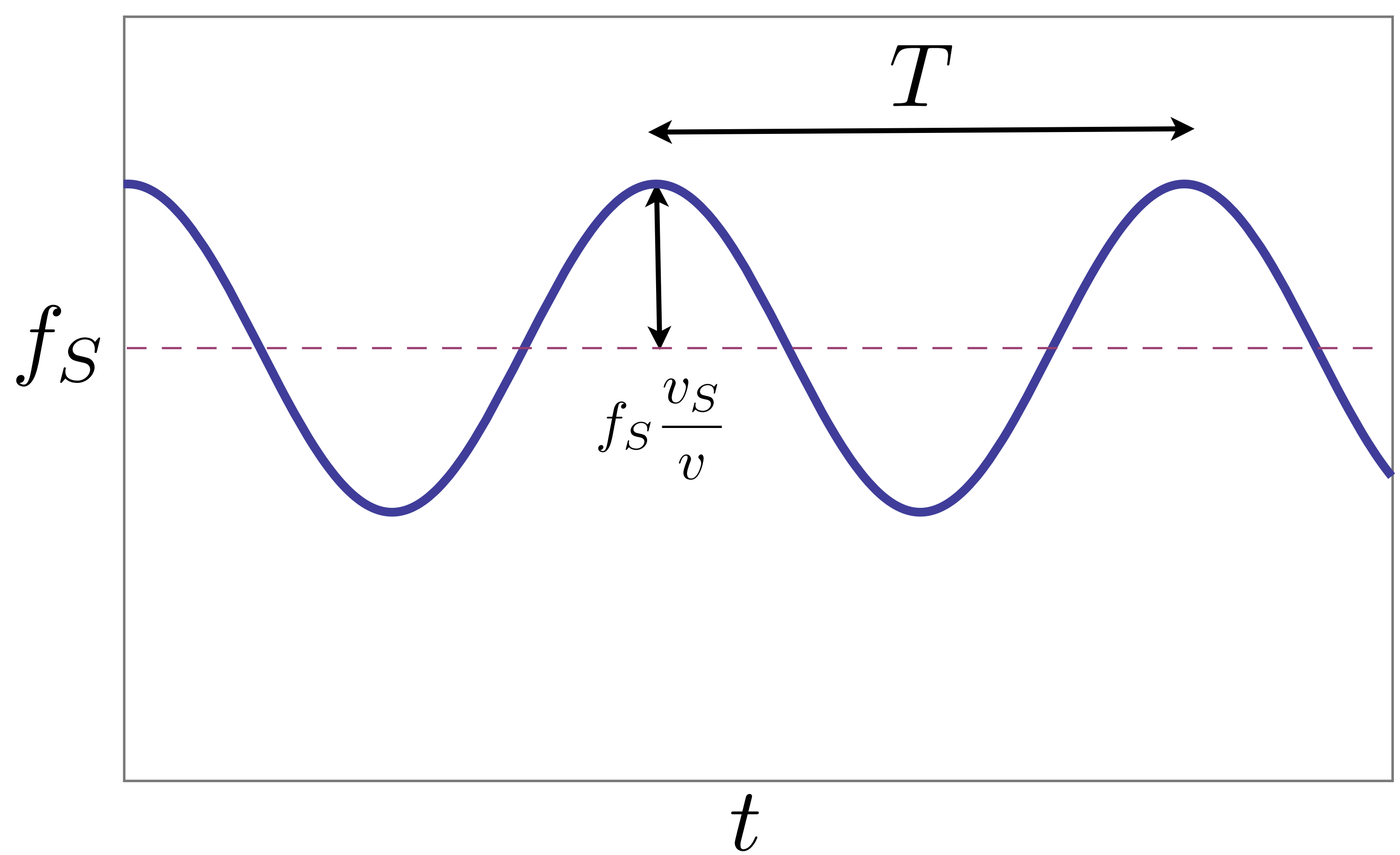
$f_L = f_S$ $\qquad$ $f_L = f_S$



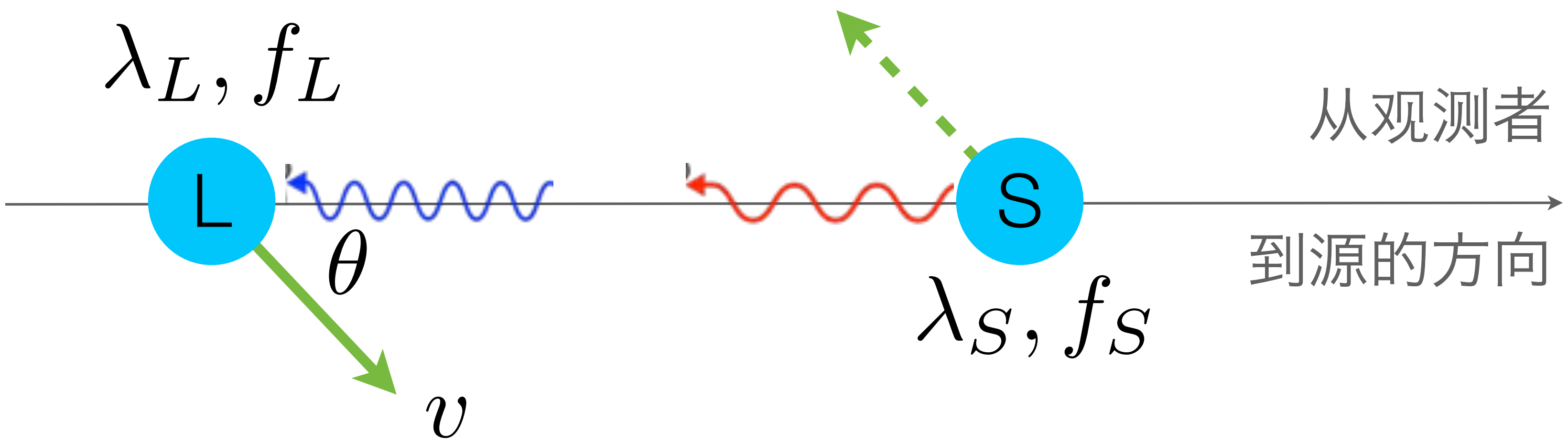
$$f_L = \frac{v}{v + v_S} f_S < f_S$$

$$f_L(t) = \frac{v}{v + v_S \cos(\omega t)} f_S$$

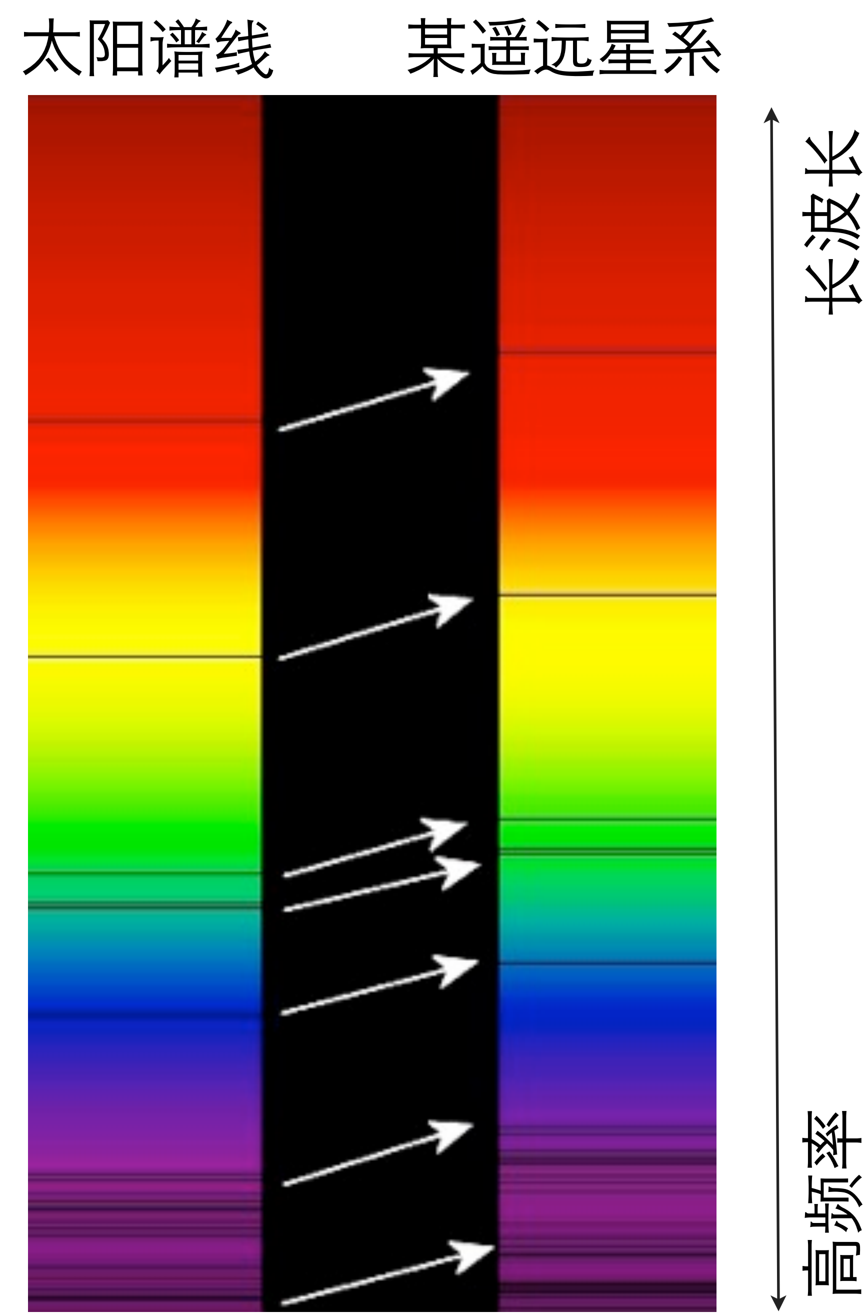
$$\simeq f_S \left[1 - \frac{v_S}{v} \cos(\omega t)\right]$$



光波的多普勒效应



$$\lambda_L = \frac{c}{f_L} \quad \text{and} \quad \lambda_S = \frac{c}{f_S}$$



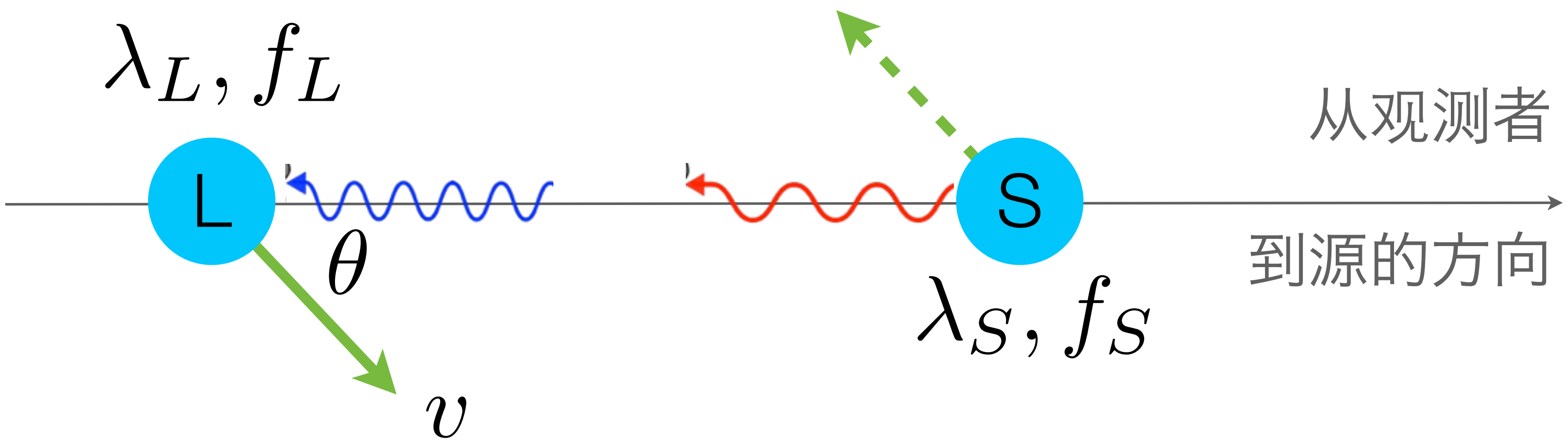
$$\lambda_L = \lambda_S \frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}} \quad \text{with} \quad \beta = \frac{v}{c}$$

$$f_L = f_S \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} \simeq f_S(1+\beta\cos\theta)$$

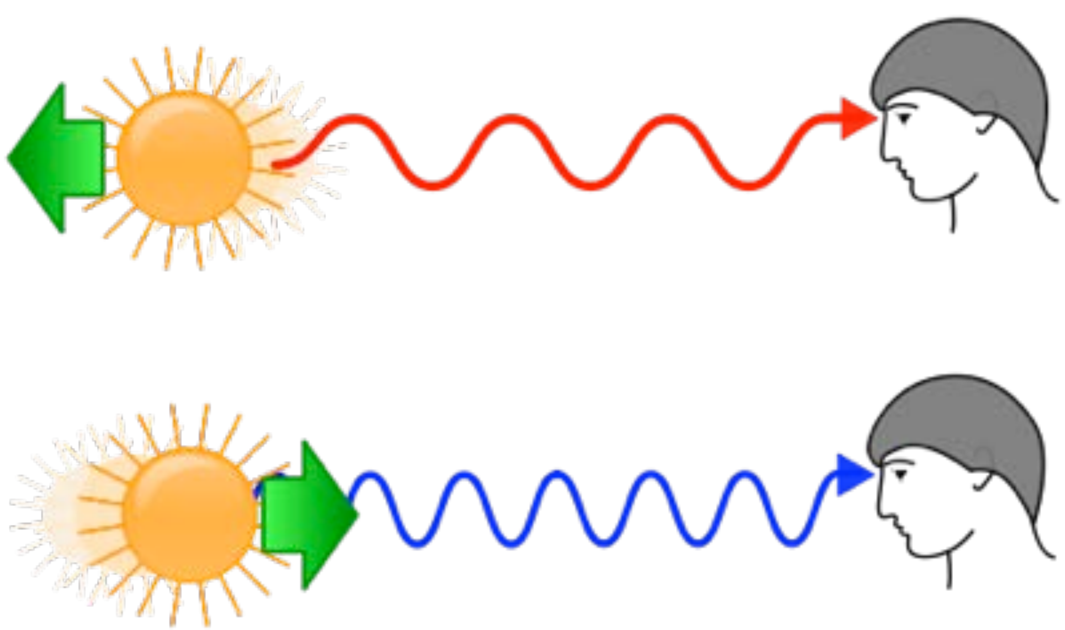
互相靠近：$0 < \theta < \pi/2: \quad f_L > f_S$ 蓝移

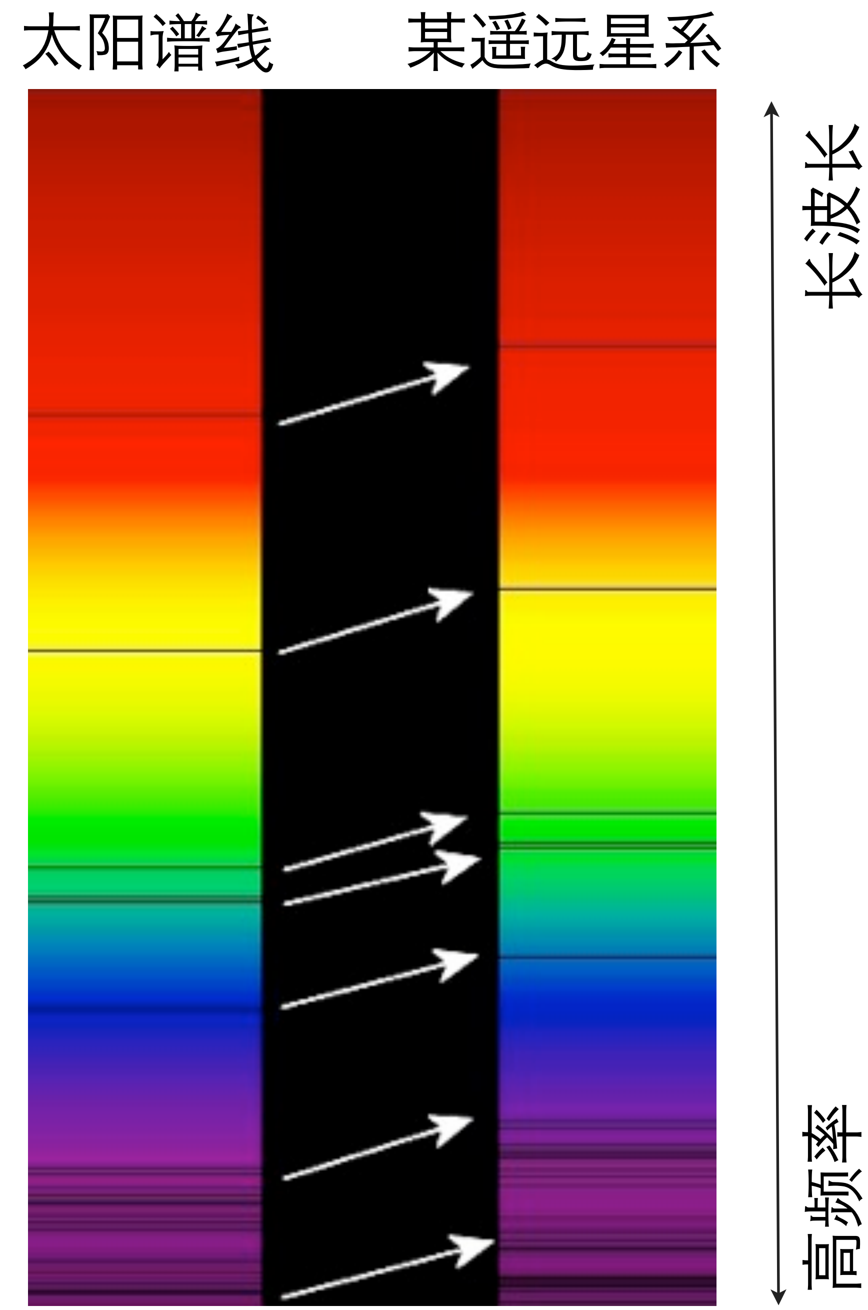
互相远离：$\pi/2 < \theta < \pi: \quad f_L < f_S$ 红移

光波的多普勒效应



$$\lambda_L = \frac{c}{f_L} \quad \text{and} \quad \lambda_S = \frac{c}{f_S}$$

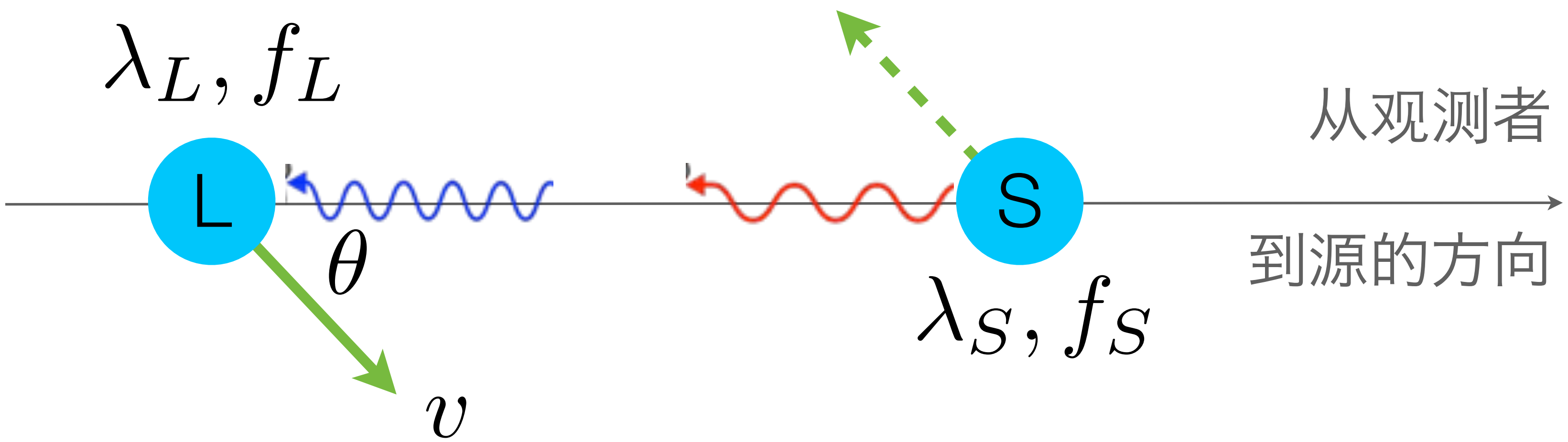




互相靠近：$0 < \theta < \pi/2$: $f_L > f_S$ 蓝移

互相远离：$\pi/2 < \theta < \pi$: $f_L < f_S$ 红移

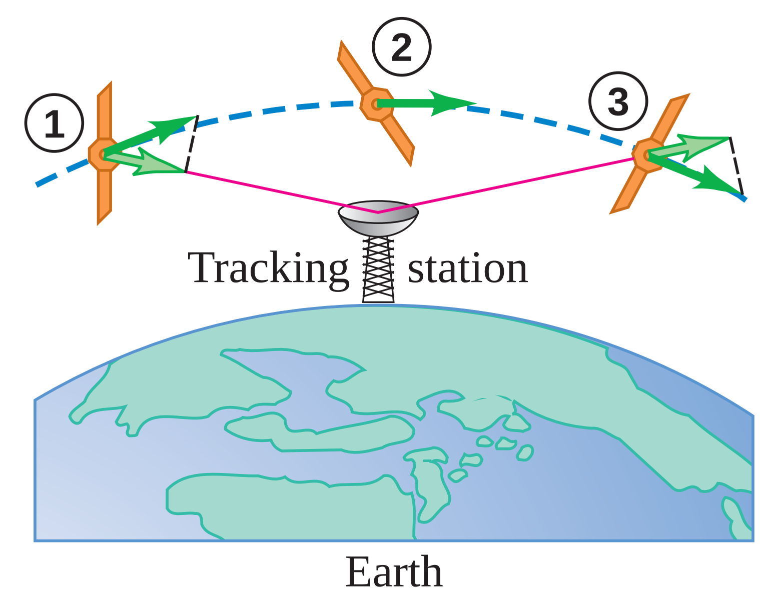
光波的多普勒效应



$$\lambda_L = \frac{c}{f_L} \quad \text{and} \quad \lambda_S = \frac{c}{f_S}$$

$$\lambda_L = \lambda_S \frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}} \quad \text{with} \quad \beta = \frac{v}{c}$$

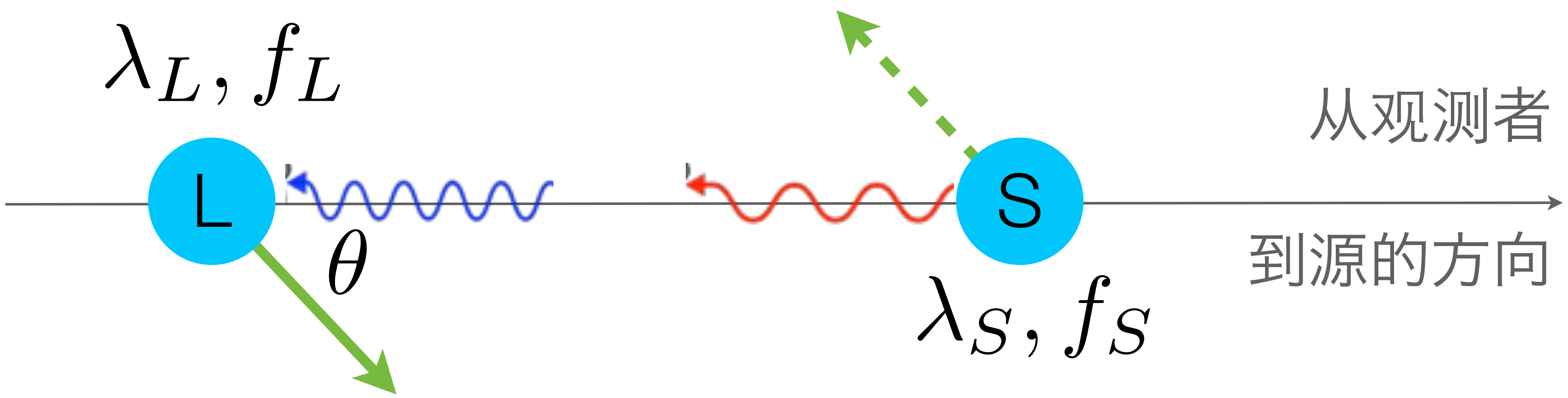
$$f_L = f_S \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} \simeq f_S(1+\beta\cos\theta)$$



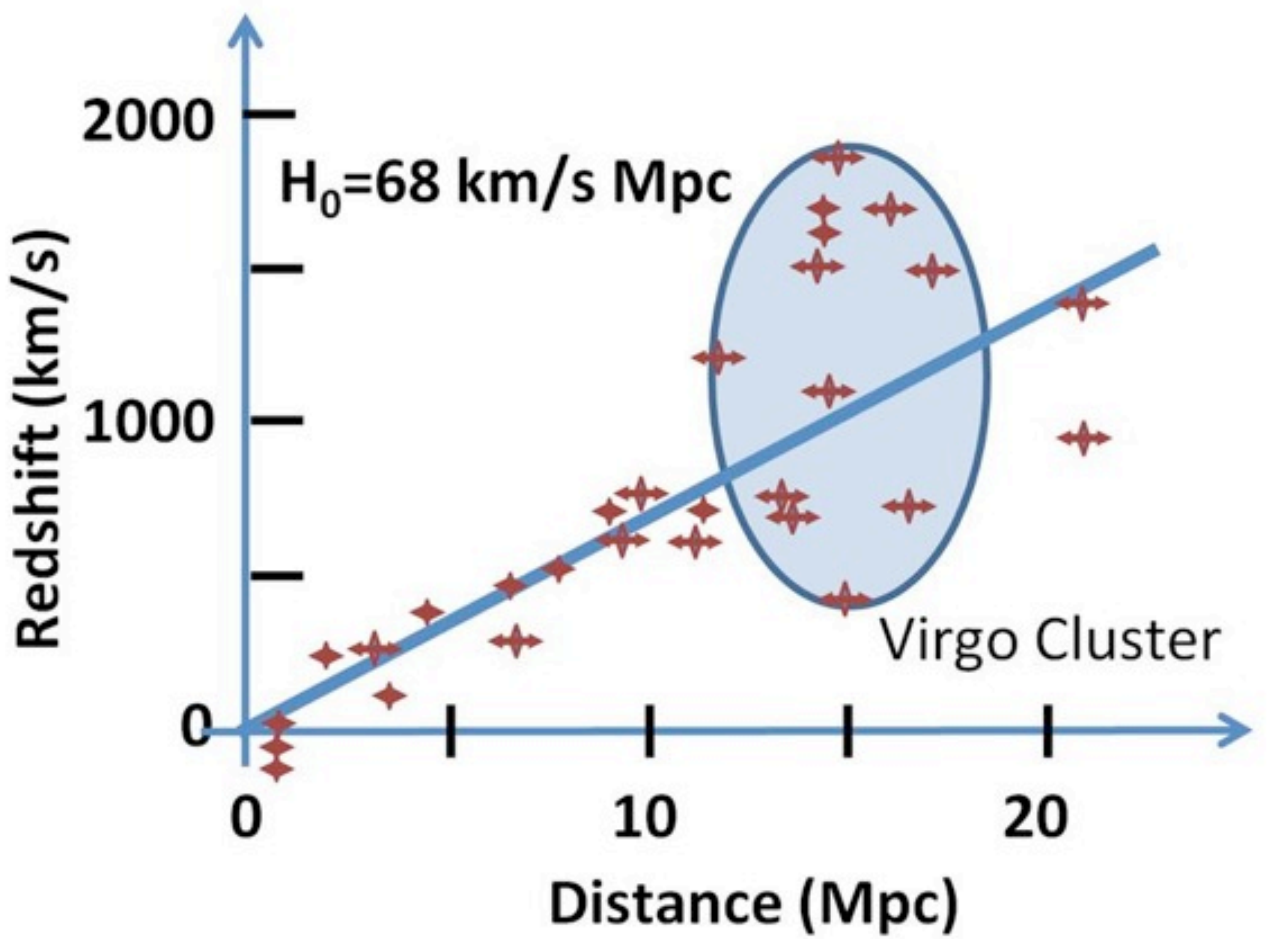
互相靠近：$0 < \theta < \pi/2: \quad f_L > f_S$ 蓝移

互相远离：$\pi/2 < \theta < \pi: \quad f_L < f_S$ 红移

光波的多普勒效应



$$\lambda_L = \frac{c}{f_L} \quad \text{and} \quad \lambda_S = \frac{c}{f_S}$$



Delta Leporis Ca (K line)

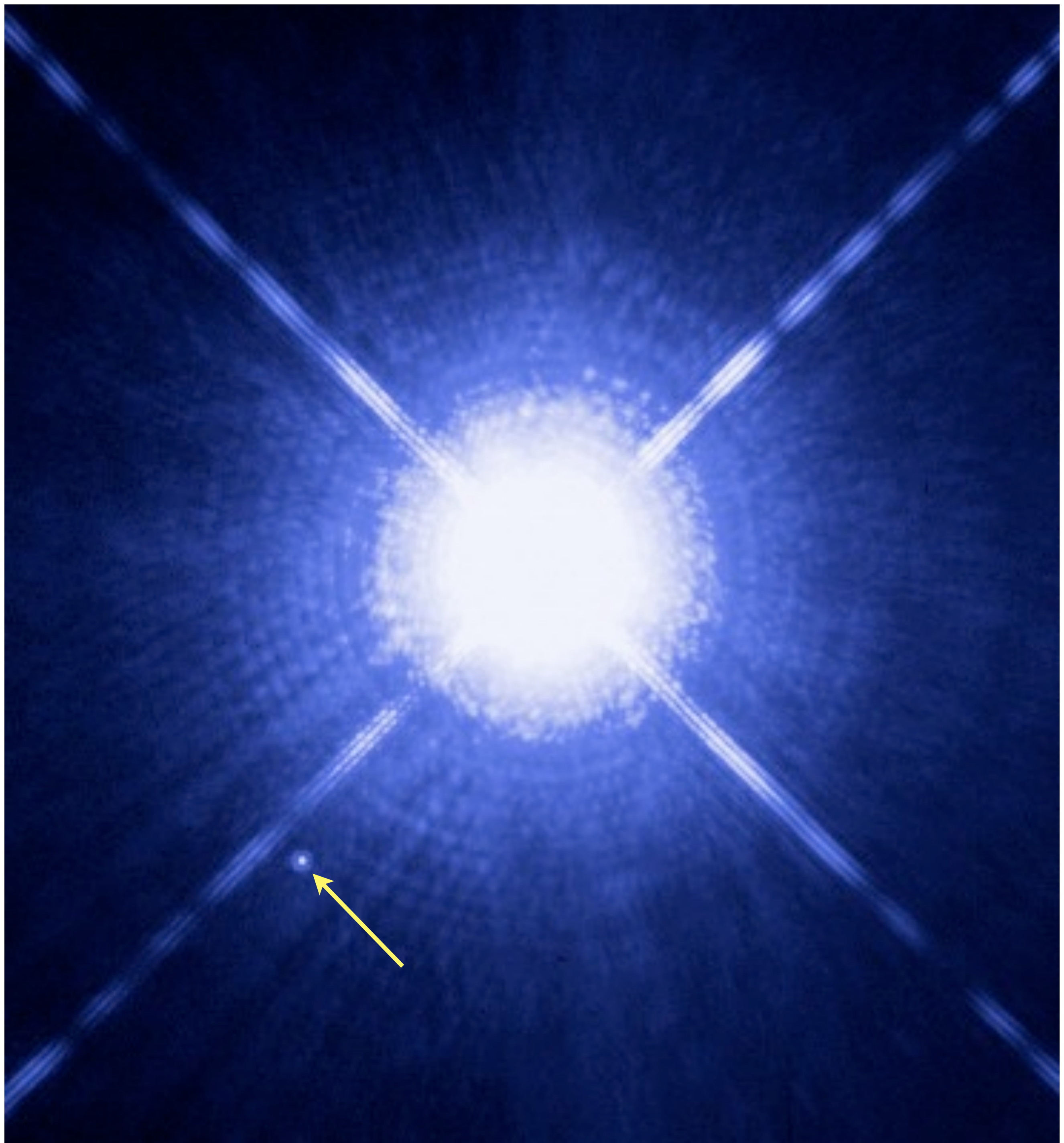
in Lab: $\lambda = 393.3664 \text{ nm}$

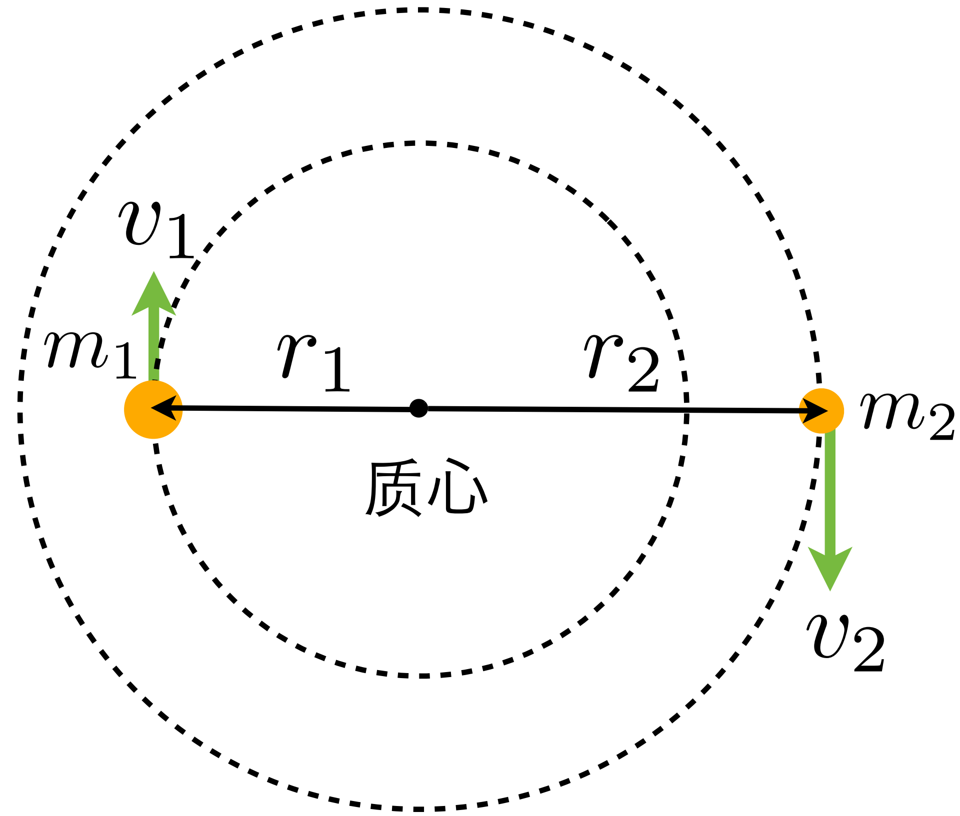
observed: $\lambda' = \lambda + 0.1298 \text{ nm}$

$$-\beta \cos\theta = 3.3 \times 10^{-4}$$

$$\Rightarrow \quad v \cos\theta \simeq -99 \text{ km/s}$$

双星



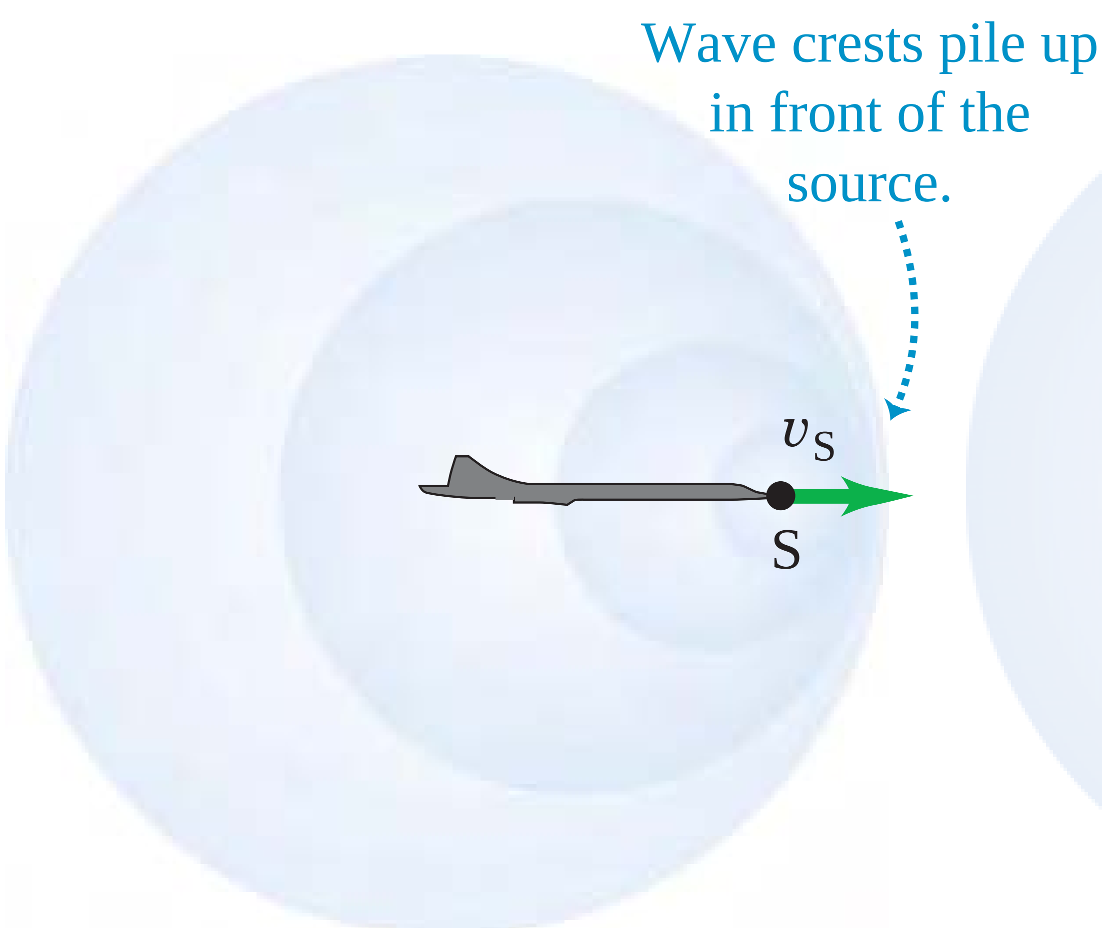


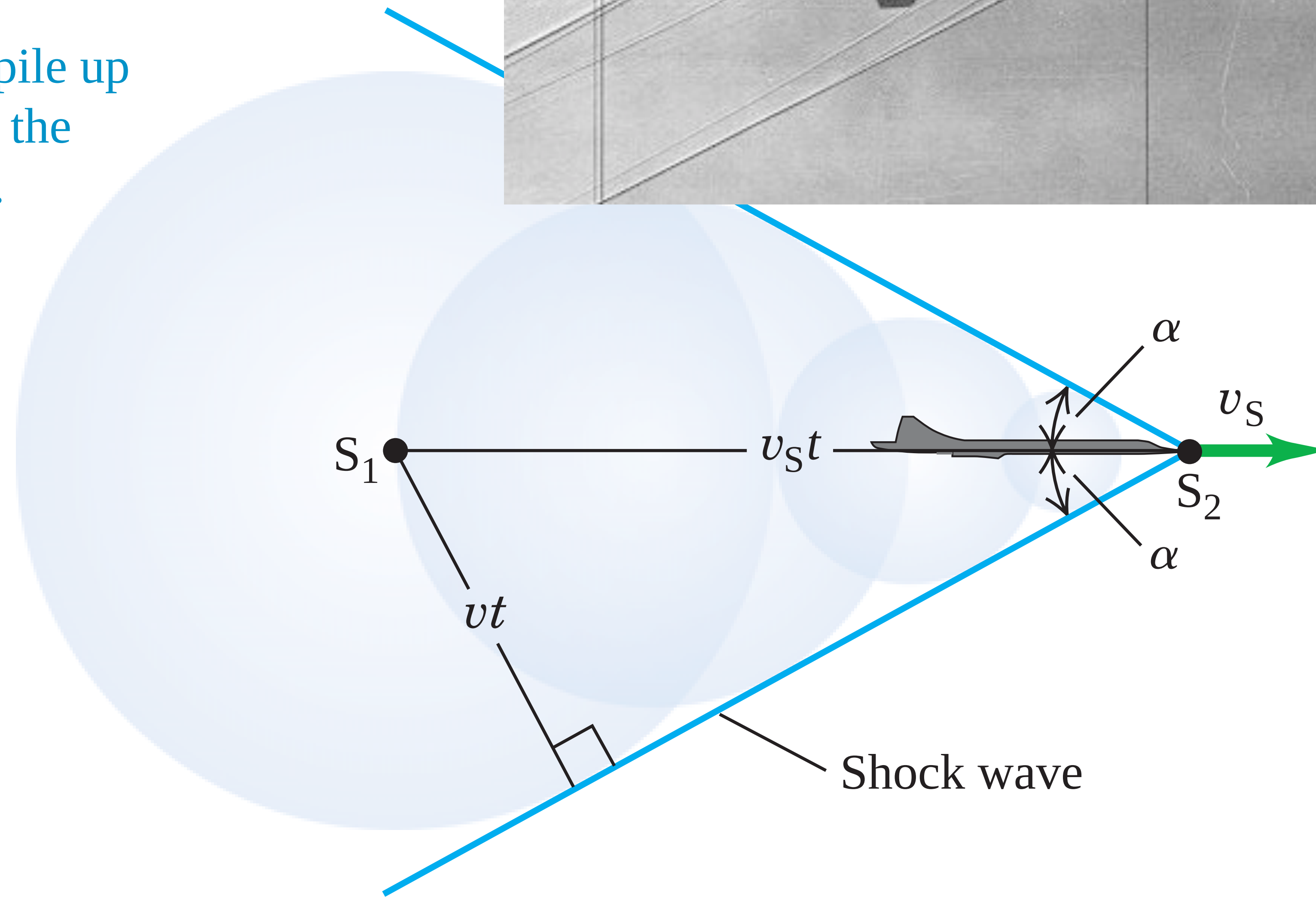
质心：$m_1 r_1 = m_2 r_2$

多普勒效应：$T, v_1, r_1 \quad \text{and} \quad T, v_2, r_2$

万有引力：$T^2 = \frac{4\pi^2}{G}\frac{(r_1+r_2)^3}{m_1+m_2}$

冲击波





马赫数： $Ma = \dfrac{v_S}{v}$

$$\sin\alpha = \frac{vt}{v_S t} = \frac{v}{v_S} = \frac{1}{Ma}$$

声爆

声爆：http://v.youku.com/v_show/id_XMzkxNjk4OTgw.html