Prepared by Jiang Xiao

第十章：流体力学

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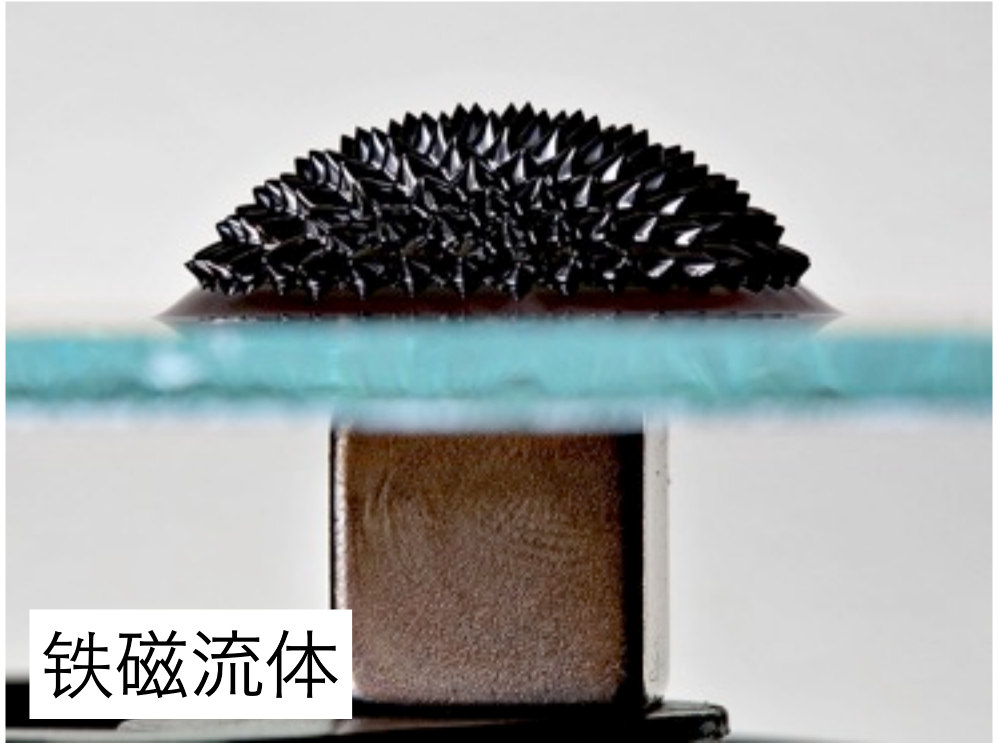
什么是流体？

流体：液体和气体



http://v.youku.com/v_show/id_XMzc3NDIyNTU2.html

http://v.youku.com/v_show/id_XMzc3MzUxNTU2.html

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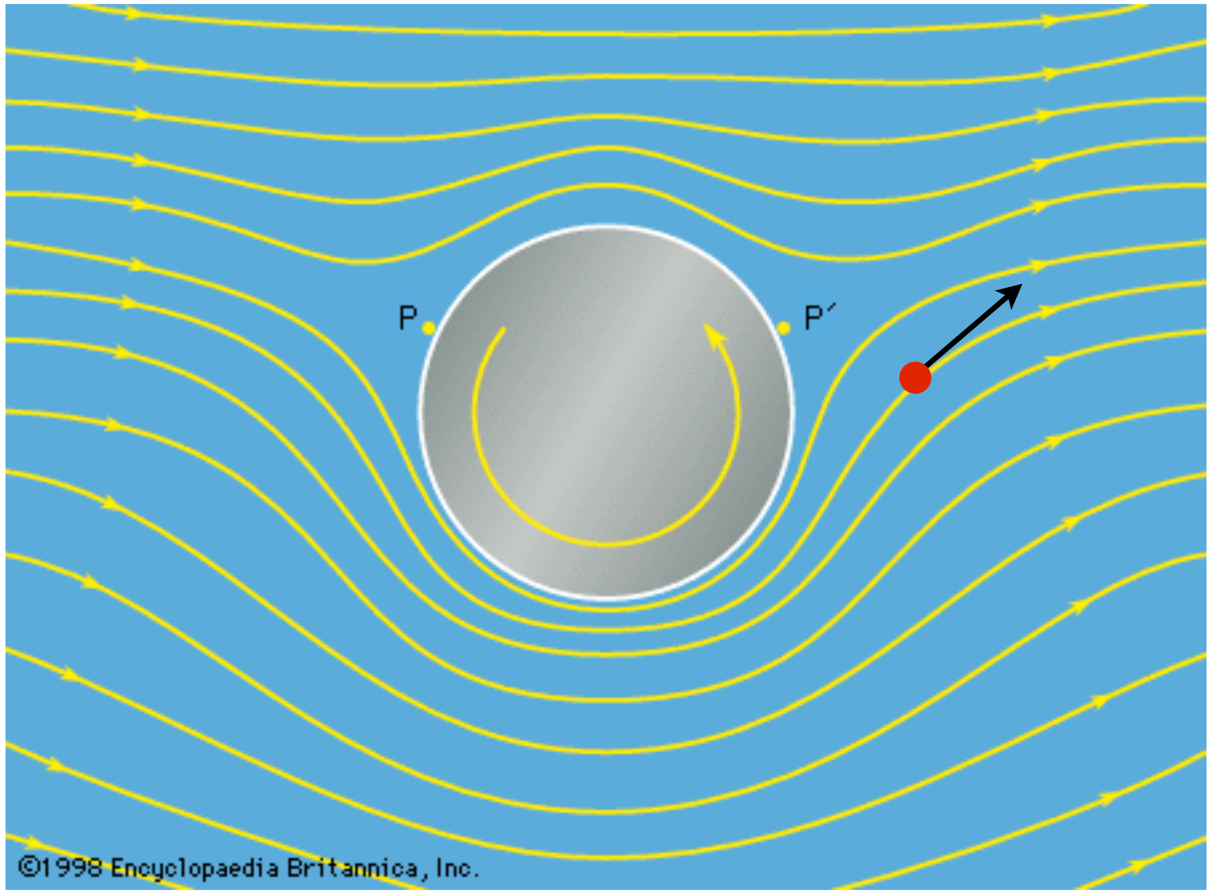
压强、密度、速度



流体内的压强各向同性：

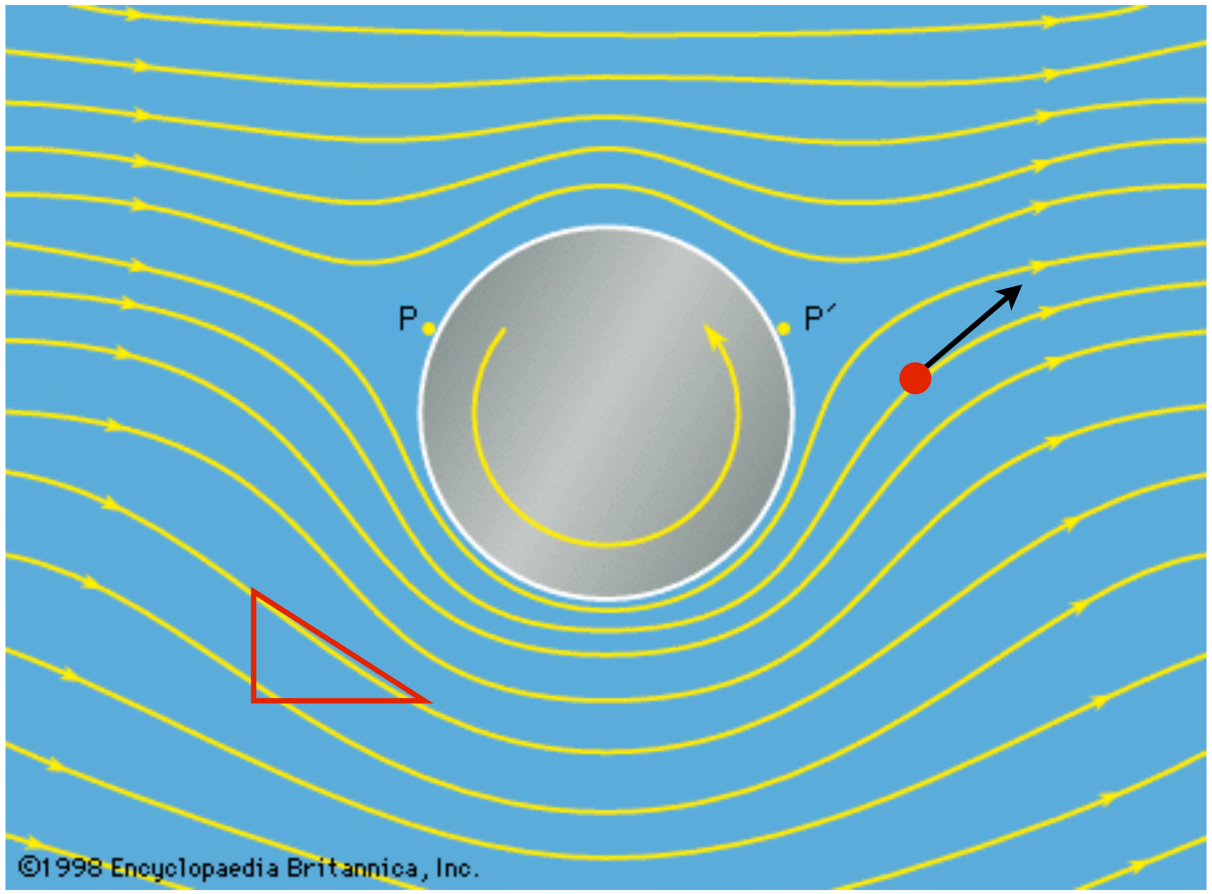
$$P_x(\mathbf{r}) = P_y(\mathbf{r}) = P_z(\mathbf{r})$$

压强：$P(\mathbf{r}, t)$，Pascal = N/m^2

密度：$\rho(\mathbf{r}, t)$

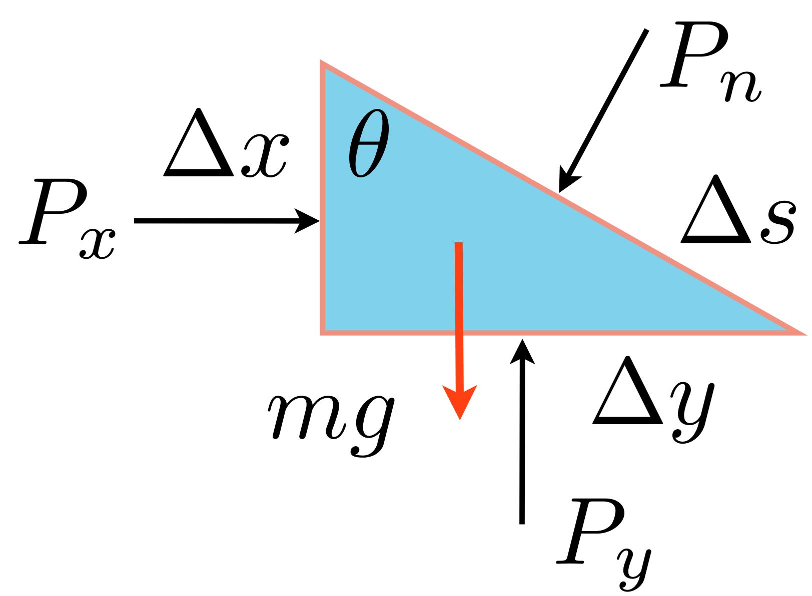
速度：$\mathbf{v}(\mathbf{r}, t)$

流体内的压强



流体内的压强各向同性：

$P_x(\mathbf{r}) = P_y(\mathbf{r}) = P_z(\mathbf{r})$



$$F_x = P_x \Delta x \Delta z - P_n \Delta s \Delta z \cos\theta = m a_x$$

$$F_y = P_y \Delta y \Delta z - P_n \Delta s \Delta z \sin\theta - mg = m a_y$$

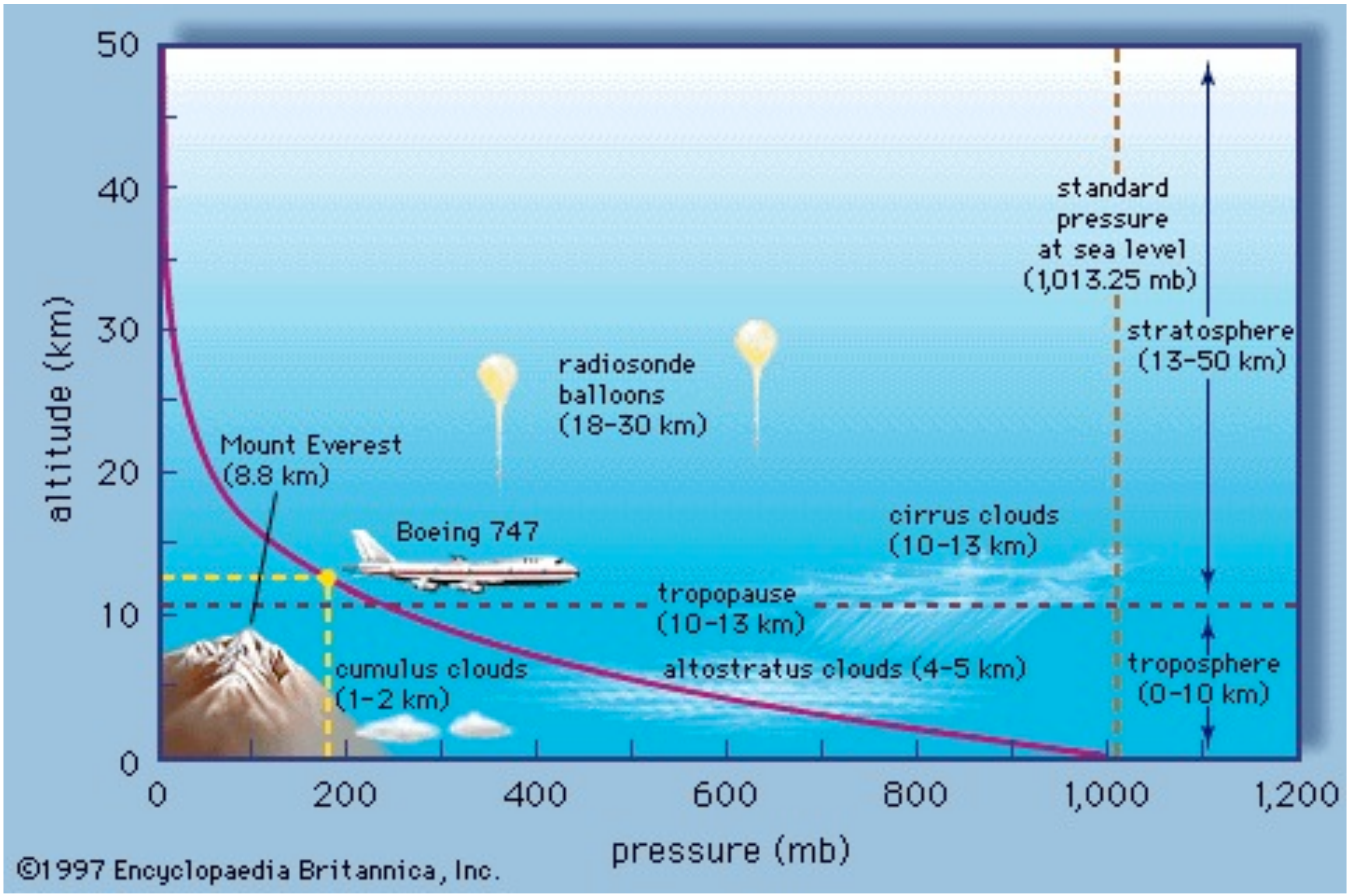
$$m = \frac{1}{2}\rho \Delta x \Delta y \Delta z$$

$$P_x - P_n = \frac{1}{2}\rho a_x \Delta y \to 0$$

$$P_y - P_n = \frac{1}{2}\rho (a_x + g) \Delta x \to 0$$

$$\Rightarrow \quad P_x = P_y = P_n$$

静止流体中的压强：大气

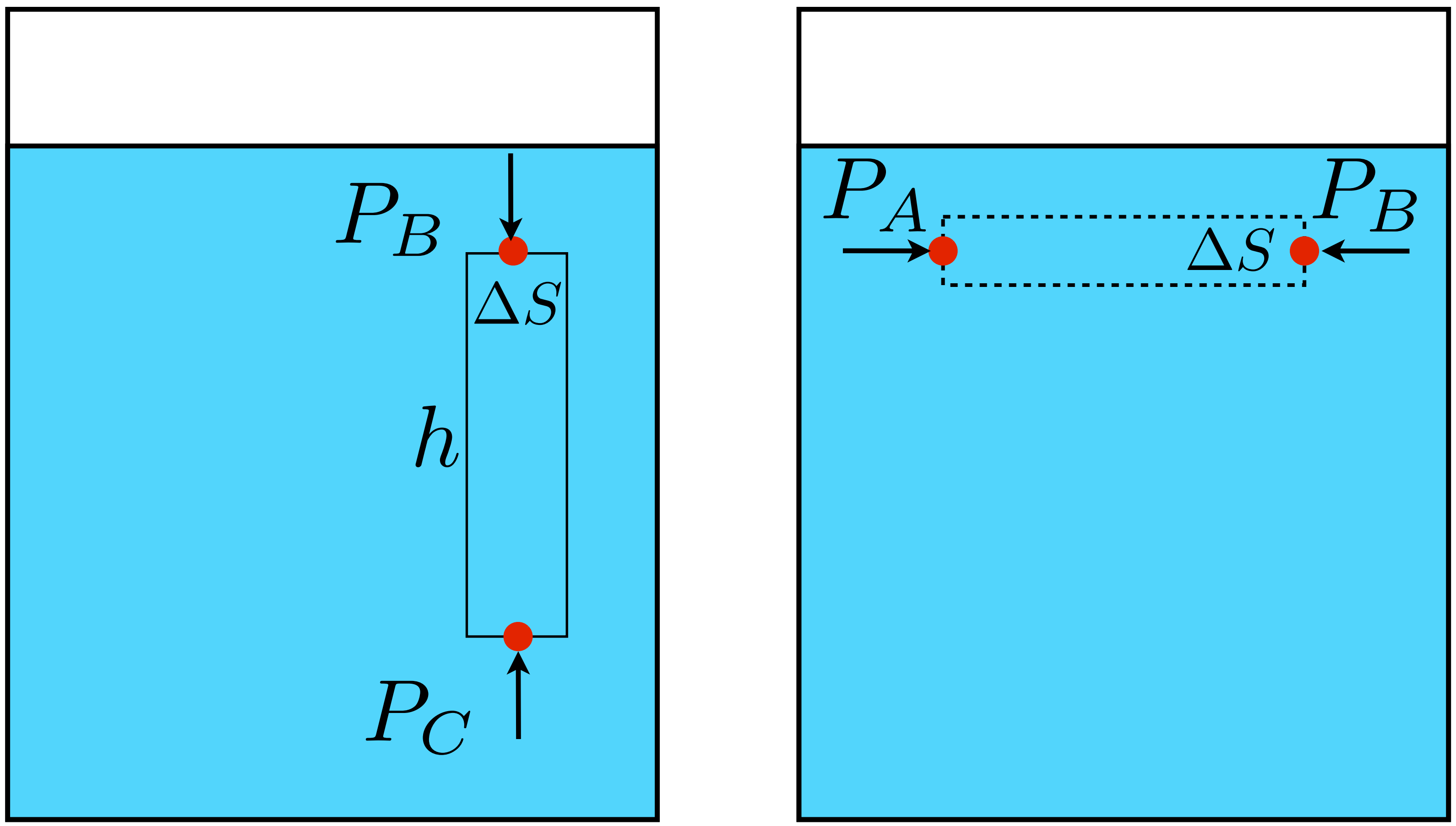


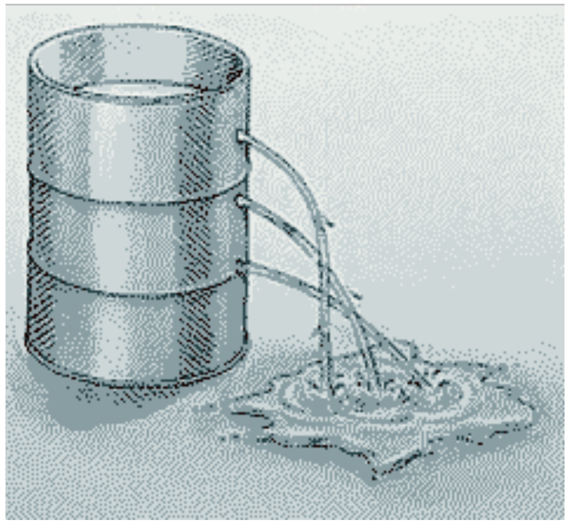
1 mbar = 0.001 bar = 0.1 kPa

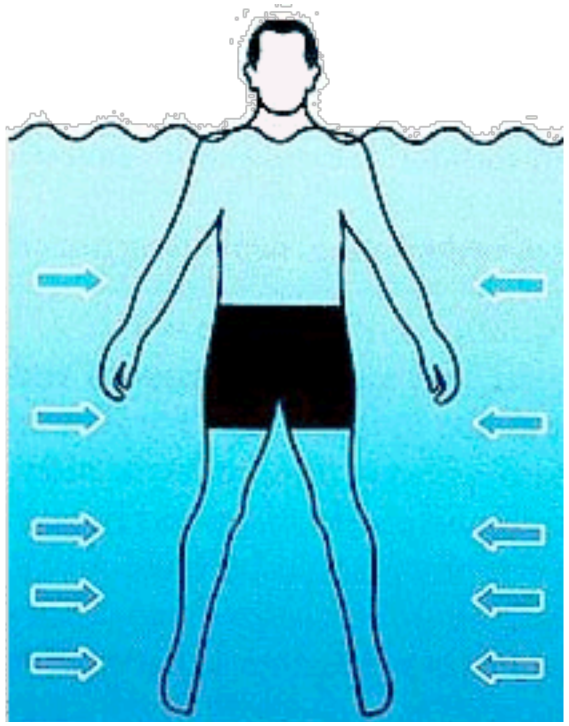
静止流体中的压强：大气

静止流体中的压强：水

$$P_A \Delta S - P_B \Delta S = 0 \quad \Rightarrow \quad P_A = P_B$$



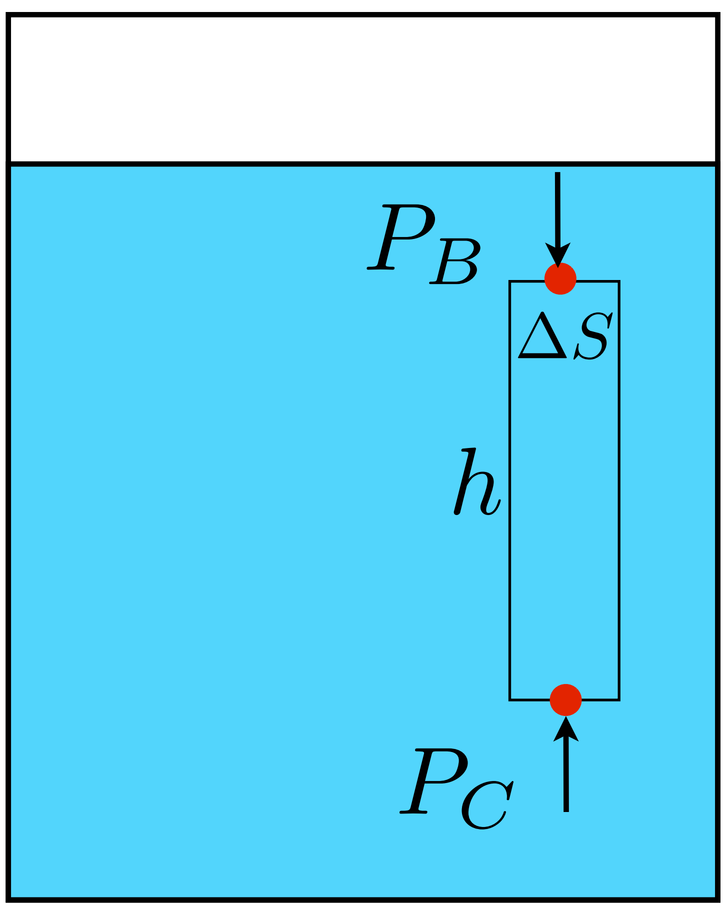




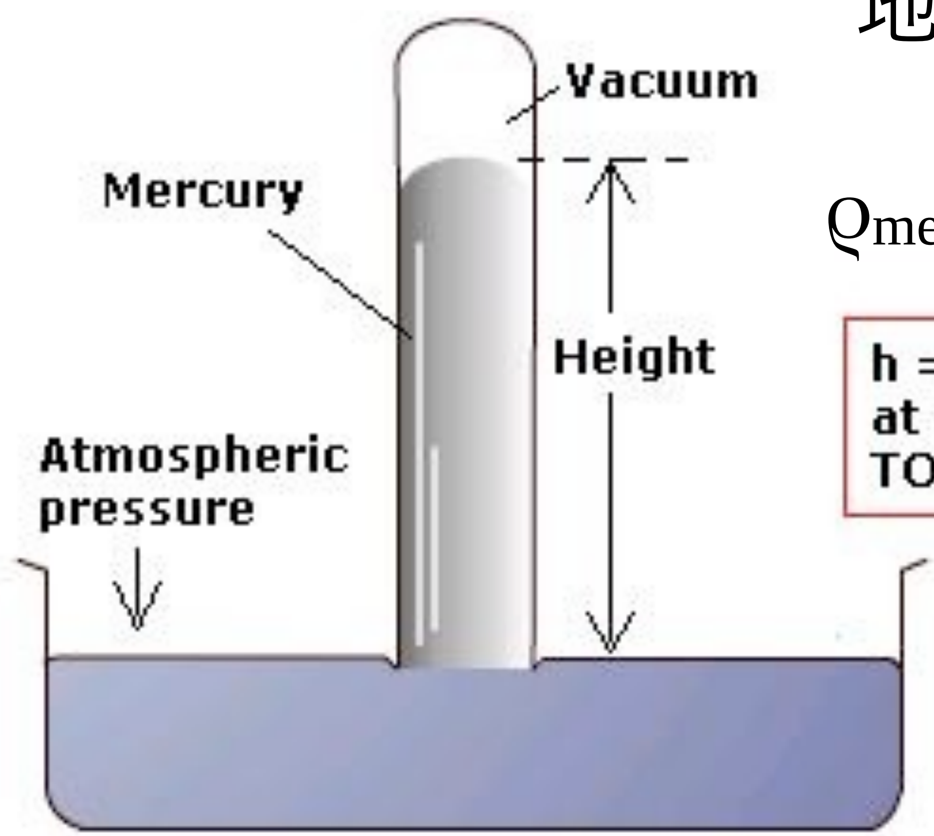
$$P_C \Delta S - P_B \Delta S = (\rho \Delta S) g h \quad \Rightarrow \quad P_C - P_B = \rho g h$$

气压计

$$P_{\text{atomsphere}} = \rho_{\text{mercury}} g h \approx 13.5 \times 10^3 \ \text{kg/m}^3 \times 10 \ \text{m/s}^2 \times 0.76 \ \text{m}$$
$$\approx 10^5 \ \text{kg/m s}^2 = 10^5 \text{N/m}^2 = 100 \ \text{kPa}$$



地球大气质量？



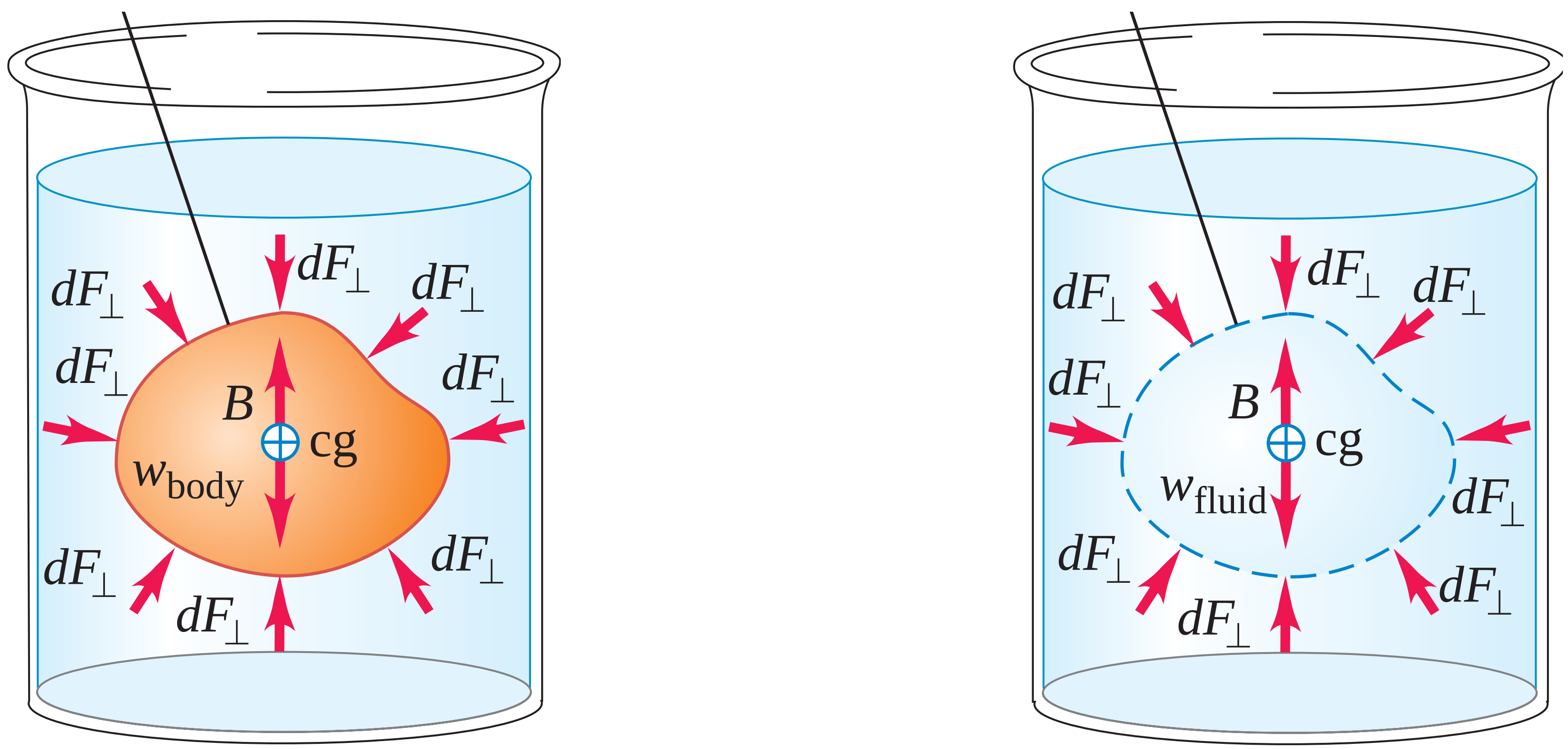
ϱ_{mercury} = 13.534 g·cm^{-3}

h = 76 cm (or 760 mm) at 0°c CORRESPONDS TO 1 ATM

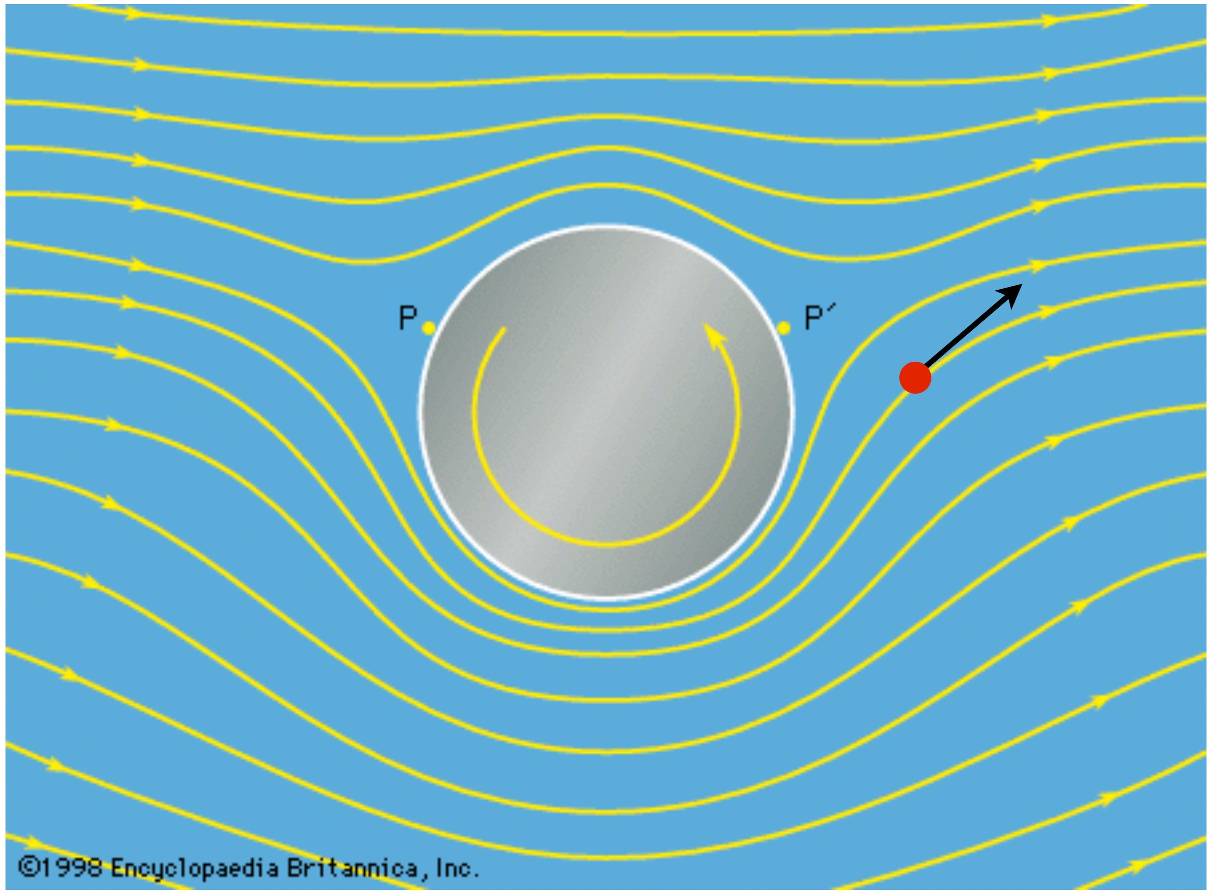
$$P_C \Delta S - P_B \Delta S = (\rho \Delta S) g h \quad \Rightarrow \quad P_C - P_B = \rho g h$$

浮力

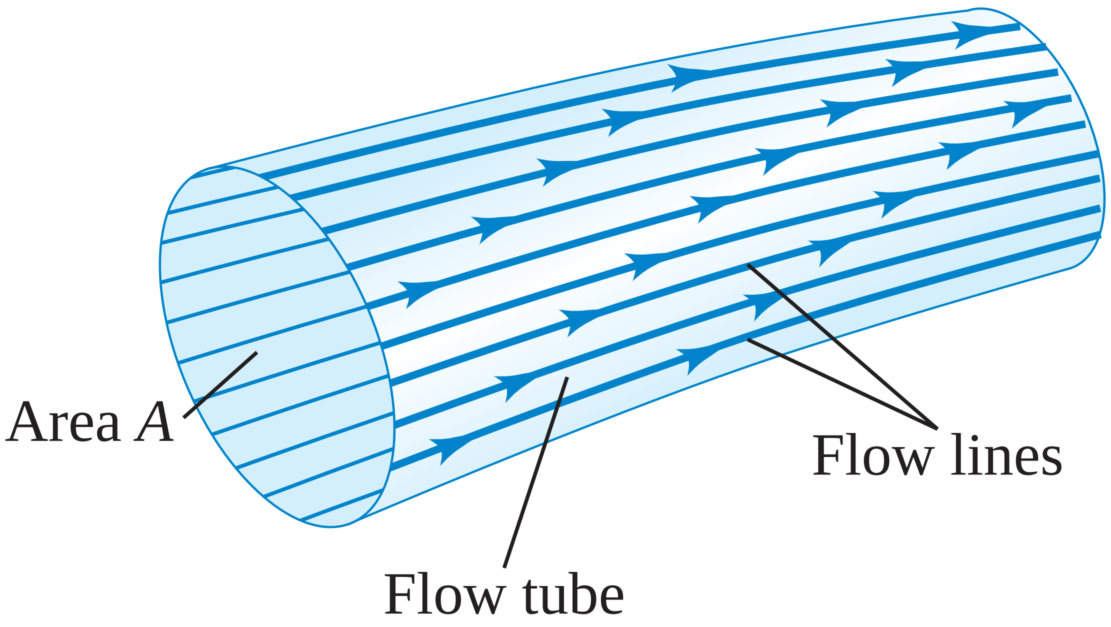
阿基米德定律：浸在液体（或气体）里的物体受到向上的浮力作用，浮力的大小等于被该物体排开的液体的重力。



流场、流线、流管



不可压缩流体： $\rho(\boldsymbol{r}, t) = \rho$



压强：$P(\mathbf{r}, t)$， Pascal = N/m^2

密度：$\rho(\mathbf{r}, t)$

速度：$\mathbf{v}(\mathbf{r}, t)$

定常流动：$P(\boldsymbol{r}), \rho(\boldsymbol{r}), \boldsymbol{v}(\boldsymbol{r})$

不定常流动：$P(\boldsymbol{r}, t), \rho(\boldsymbol{r}, t), \boldsymbol{v}(\boldsymbol{r}, t)$

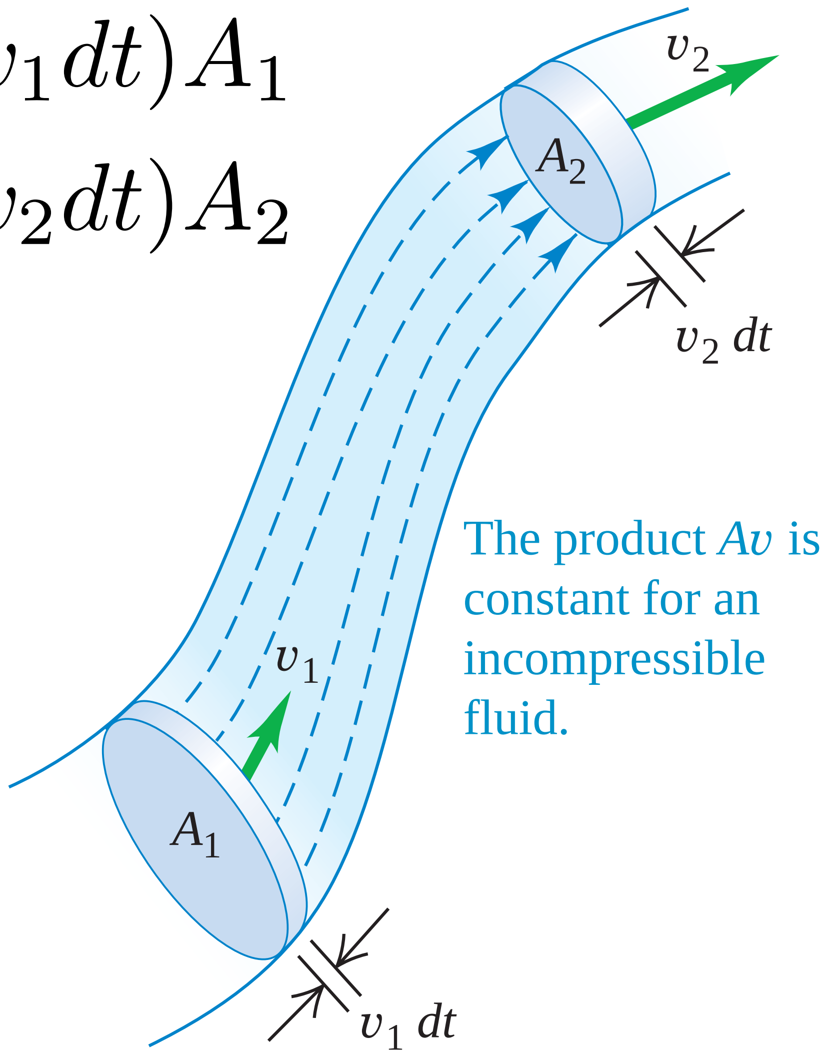
定常流动的连续性方程

定常流动：$P(\boldsymbol{r}), \rho(\boldsymbol{r}), \boldsymbol{v}(\boldsymbol{r})$

不可压缩流体：ρ(**r**, *t*) = ρ

$$m_1 = \rho_1 (v_1 dt) A_1$$
$$m_2 = \rho_2 (v_2 dt) A_2$$

$$m_1 = m_2 \quad \Rightarrow \quad \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$
$$\Rightarrow \quad \rho A v = \text{const.}$$



伯努利方程

$$E_2 - E_1 = \left(\frac{1}{2}mv_2^2 + mgh_2\right) - \left(\frac{1}{2}mv_1^2 + mgh_1\right)$$

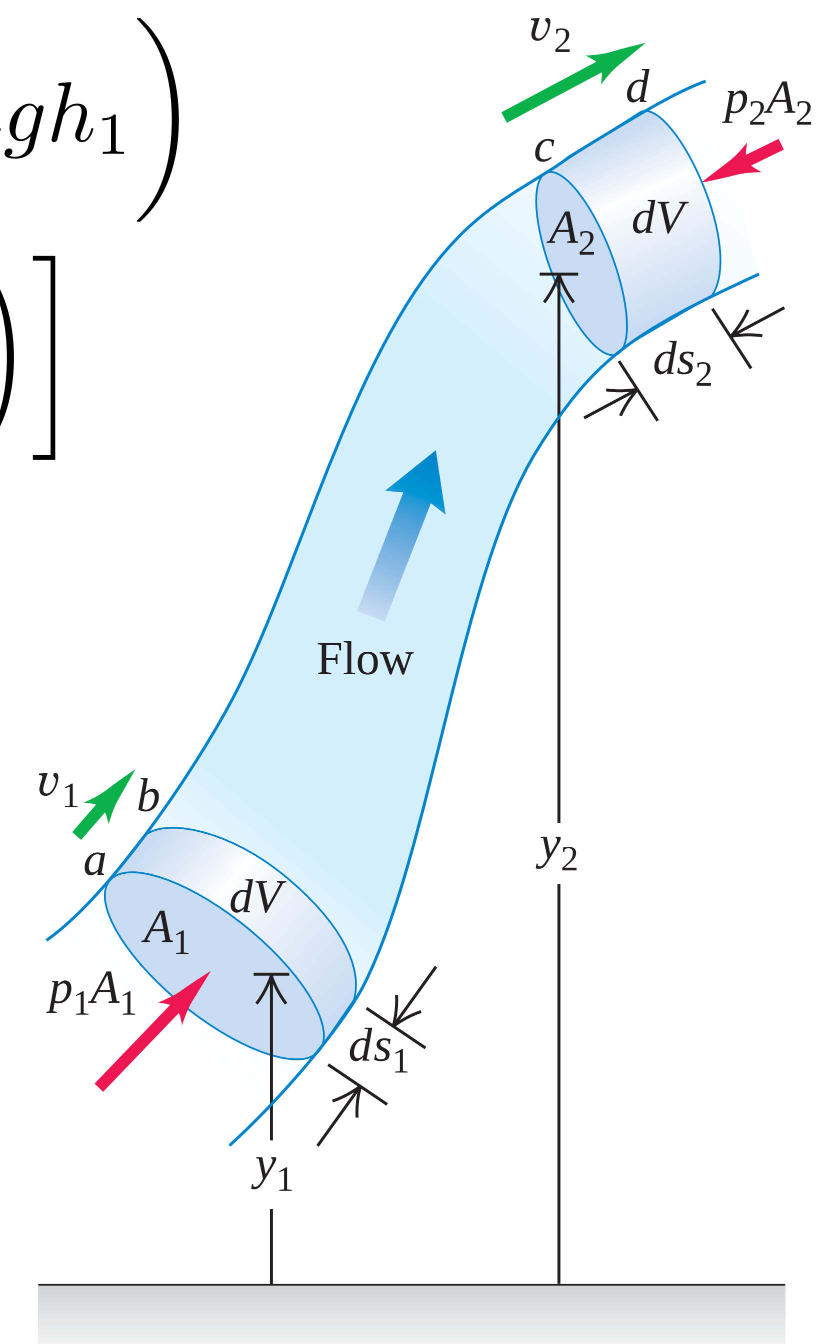
$$= m\left[\left(\frac{1}{2}v_2^2 + gh_2\right) - \left(\frac{1}{2}v_1^2 + gh_1\right)\right]$$

$$W = p_1 A_1 v_1 \Delta t - p_2 A_2 v_2 \Delta t = (p_1 - p_2)\frac{m}{\rho}$$

$$W = E_2 - E_1 \quad \Rightarrow$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

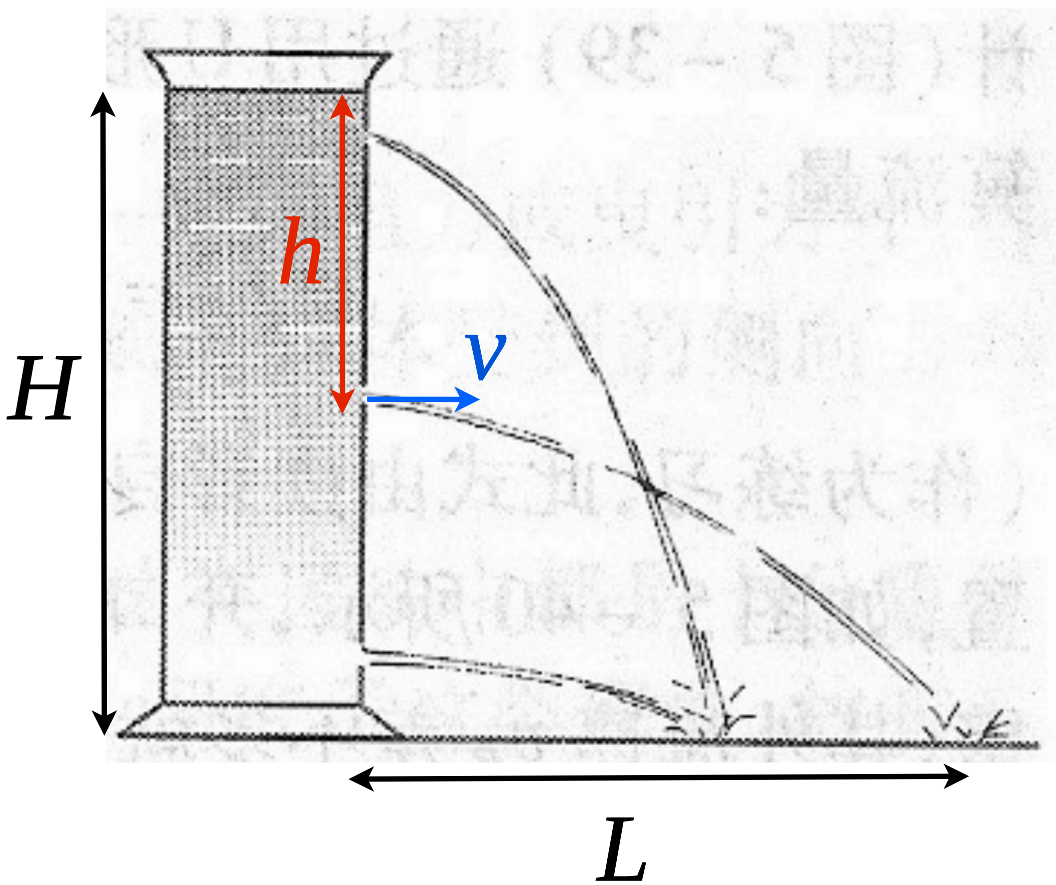
$$\boxed{p + \frac{1}{2}\rho v^2 + \rho g h = \text{const.}}$$



$$m = \rho A_1 v_1 dt = \rho A_2 v_2 dt$$

伯努利方程的应用

$$伯努利方程： p + \frac{1}{2}\rho v^2 + \rho g h = \text{const.}$$



$$\frac{1}{2}\rho v^2 + P_{\text{atomsphere}} = \rho g h + P_{\text{atomsphere}}$$

$$\Rightarrow \quad v = \sqrt{2gh}$$

在何处开孔射程最远？

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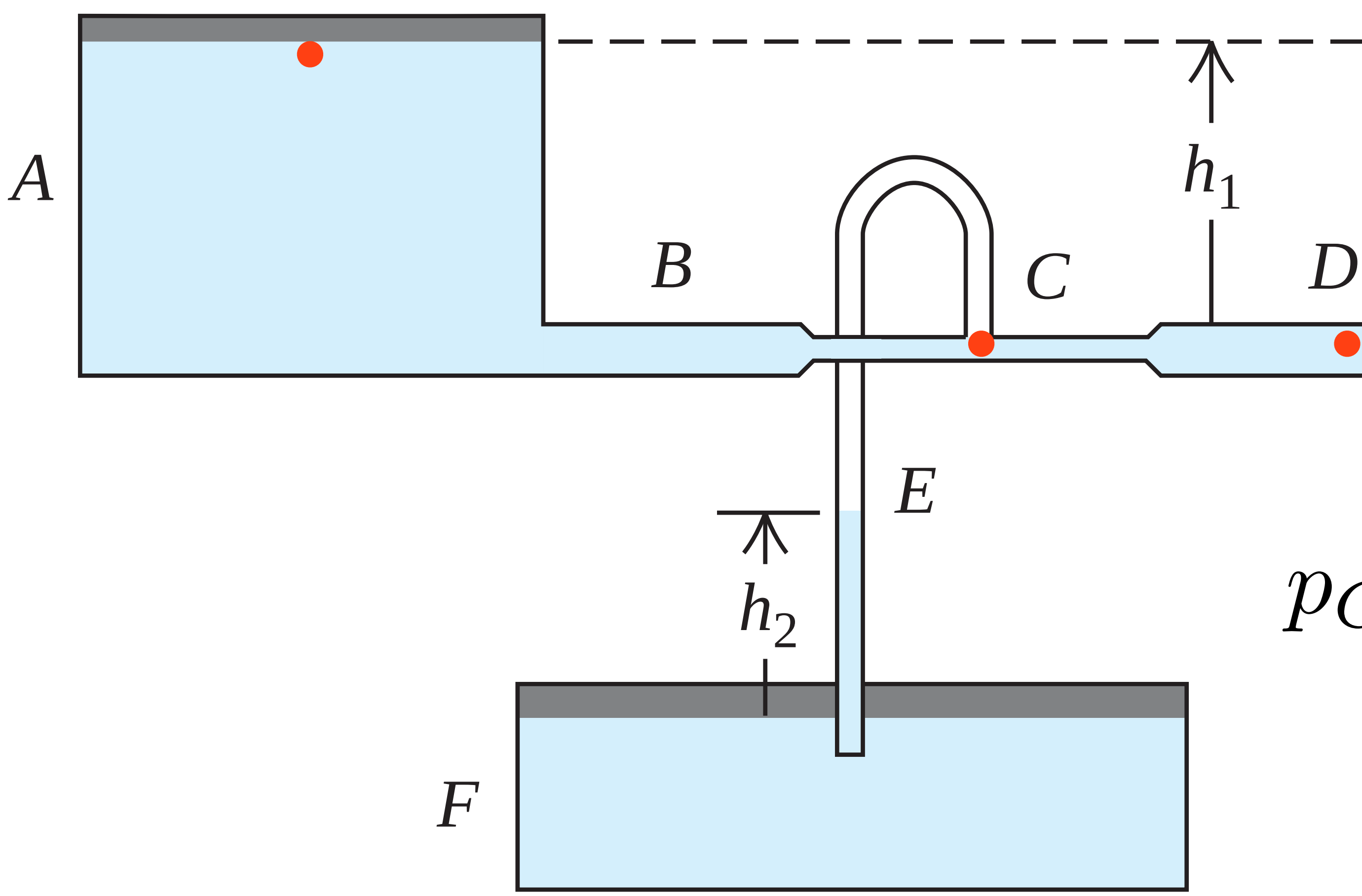
伯努利方程的应用：空吸作用

$$\text{伯努利方程：} p+\frac{1}{2}\rho v^2+\rho gh=\text{const.}$$



$$\begin{aligned} p_0+\rho gh_1 &= p_C+\frac{1}{2}\rho v_C^2 \\ &= p_0+\frac{1}{2}\rho v_D^2 \end{aligned}$$

$$v_C A_C = V_D A_D$$

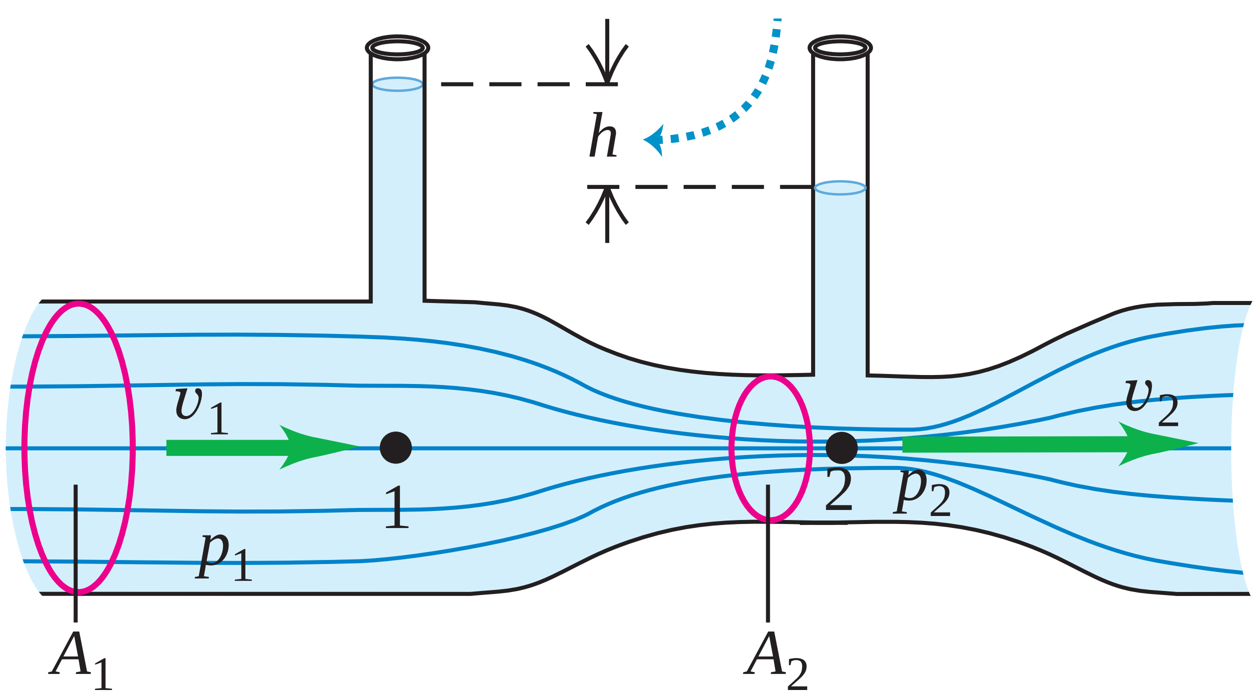
$$\begin{aligned} p_C-p_0 &= \rho gh_1\left[1-\left(\frac{A_D}{A_C}\right)^2\right] \\ &= \rho gh_2 \end{aligned}$$

文丘里流量计和测速皮托管

$$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = p_1 - p_2 = \rho g h$$

$$Q = v_1 A_1 = v_2 A_2$$

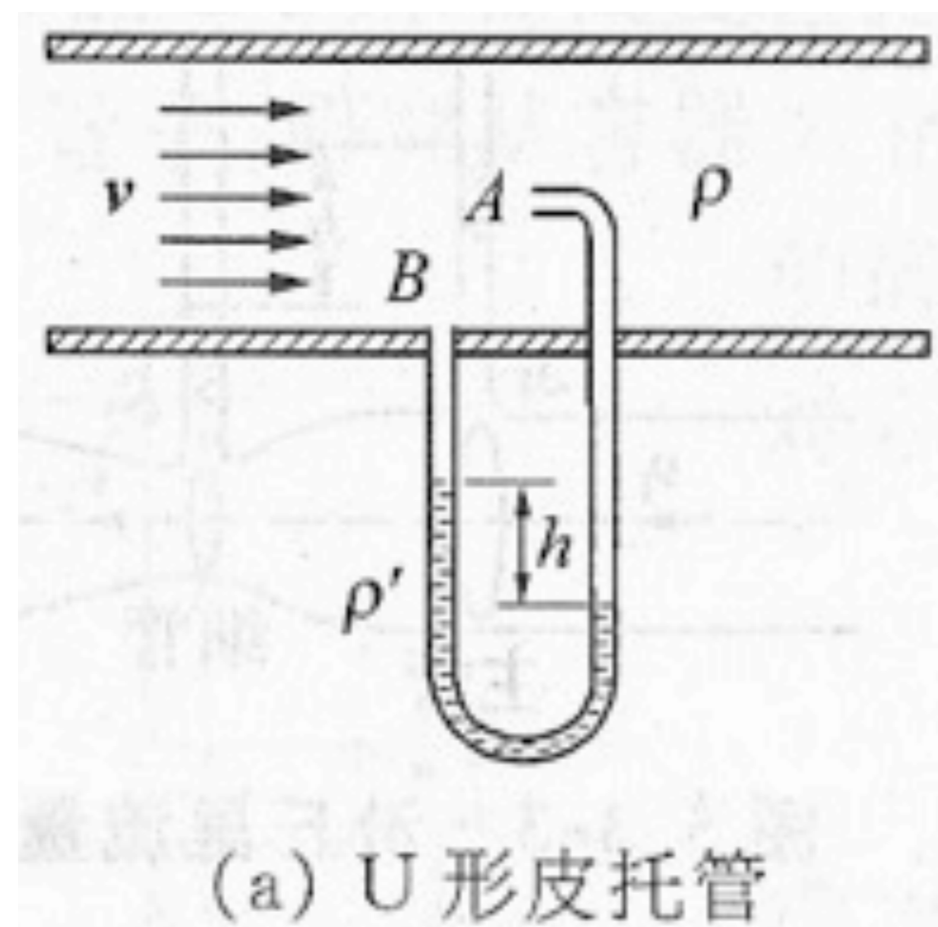
$$\Rightarrow Q = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$



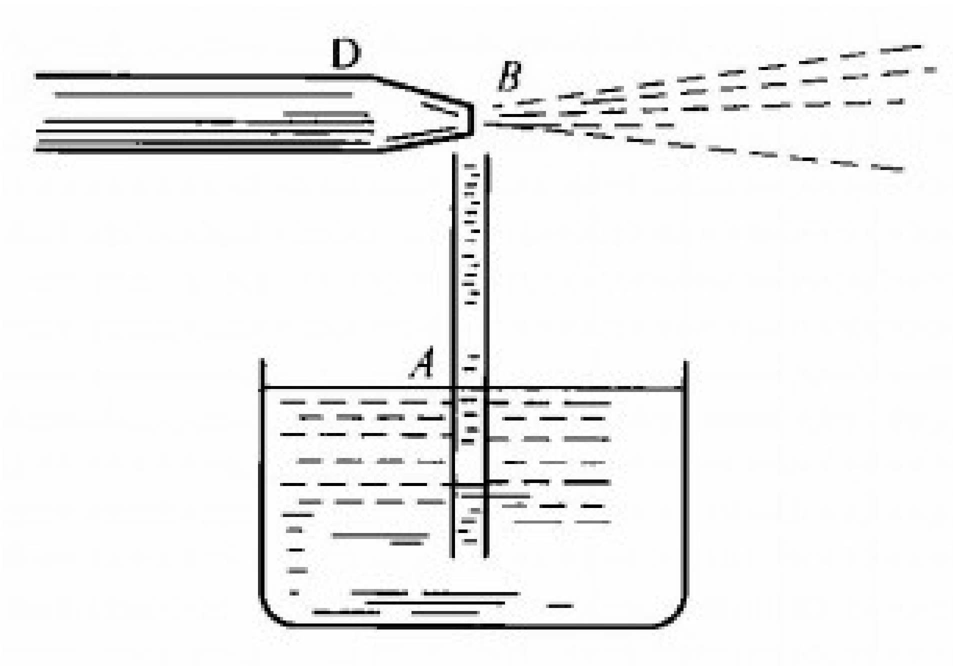
$$p_A = p_B + \frac{1}{2}\rho v^2$$

$$p_A - p_B = (\rho' - \rho) g h$$

$$\Rightarrow \quad v = \sqrt{\frac{2gh(\rho' - \rho)}{\rho}}$$



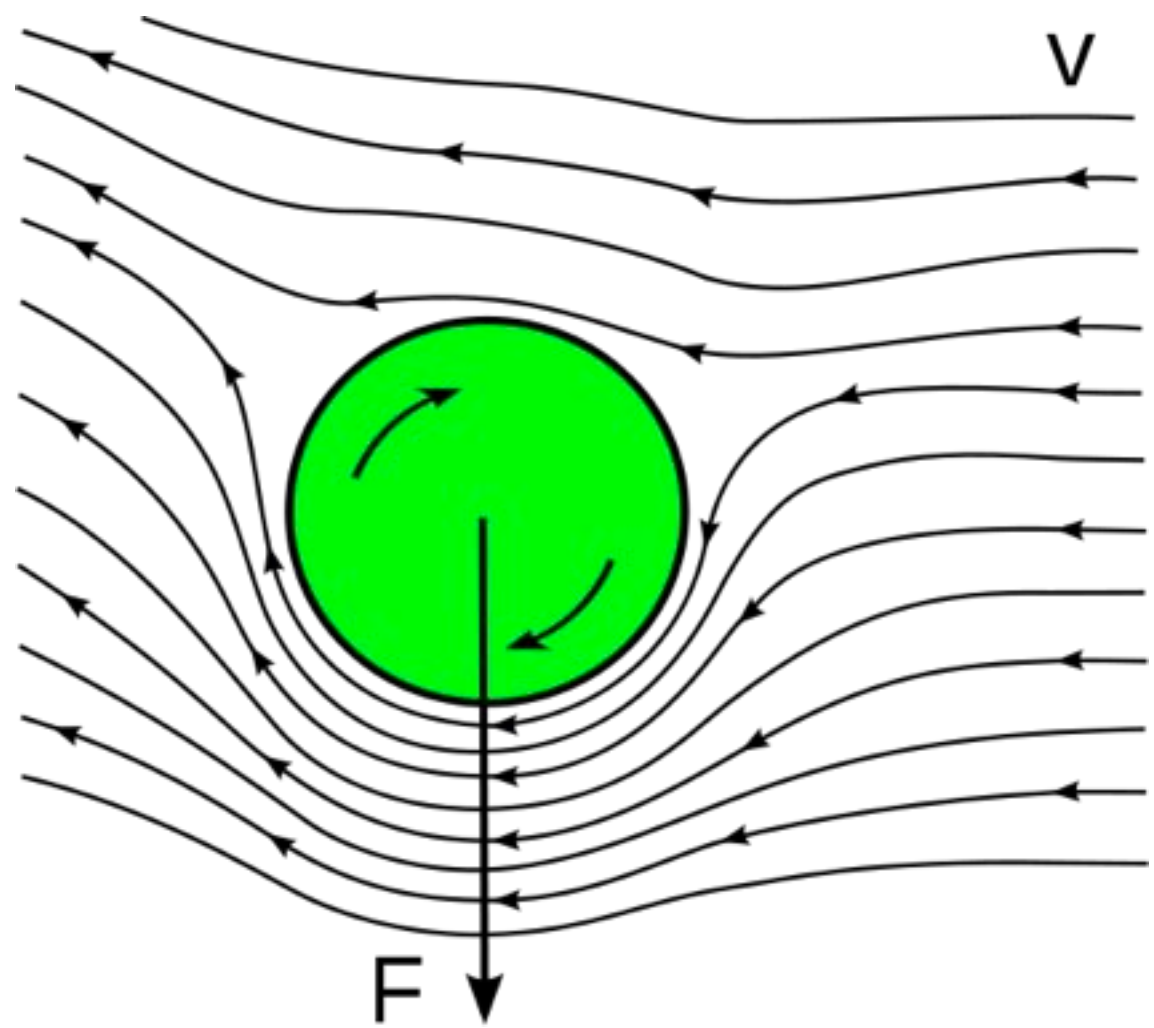
伯努利方程的应用



喷水装置：

http://v.youku.com/v_show/id_XMzkyMDM1OTQ0.html

马格纳斯效应



香蕉球：

http://v.youku.com/v_show/id_XMzc3MzY1OTgw.html

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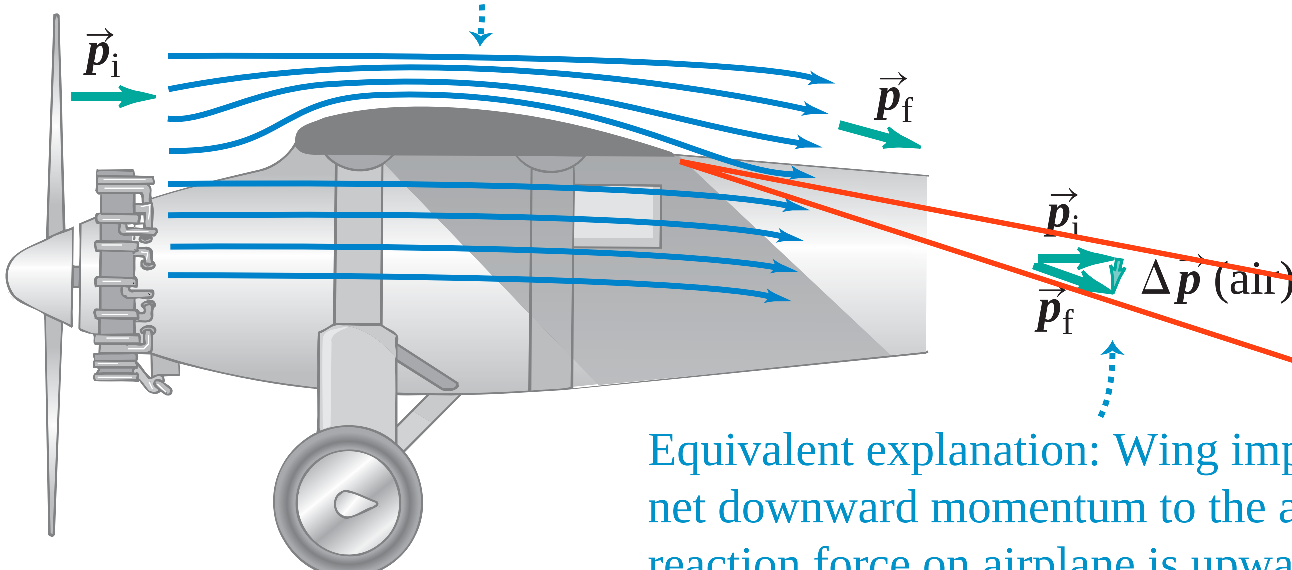
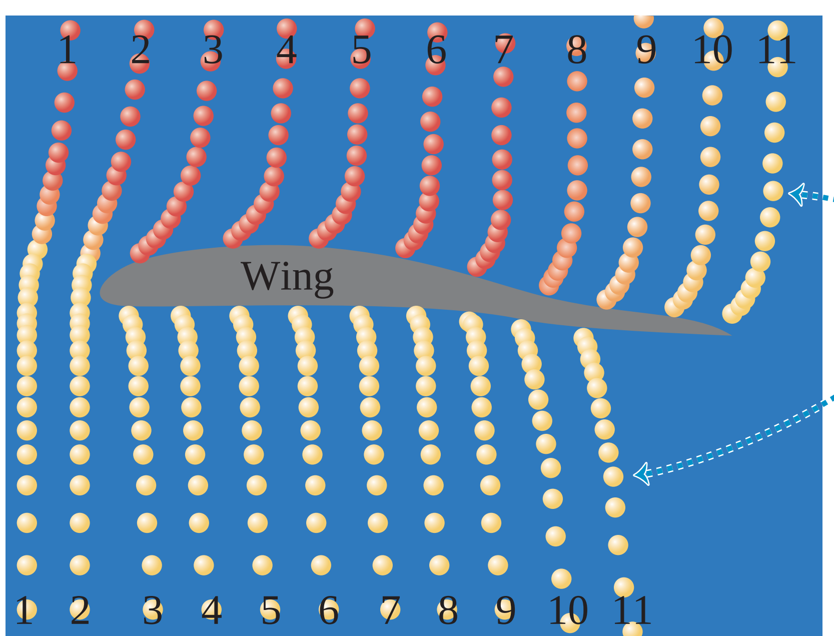
伯努利方程的应用：飞行

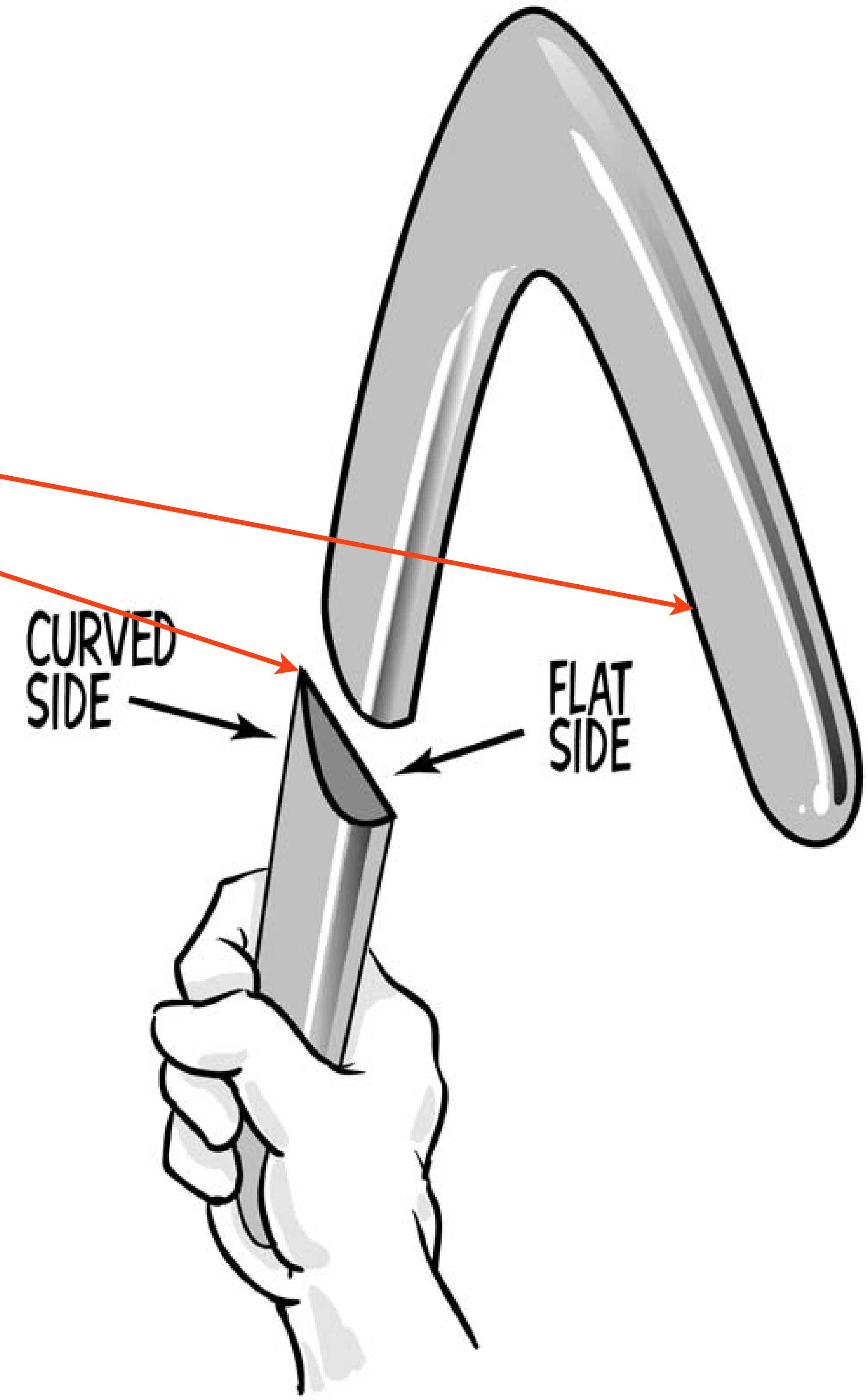
Flow lines are crowded together above the wing, so flow speed is higher there and pressure is lower.



Equivalent explanation: Wing imparts a net downward momentum to the air, so reaction force on airplane is upward.



Notice that air particles that are together at the leading edge of the wing do *not* meet up at the trailing edge!



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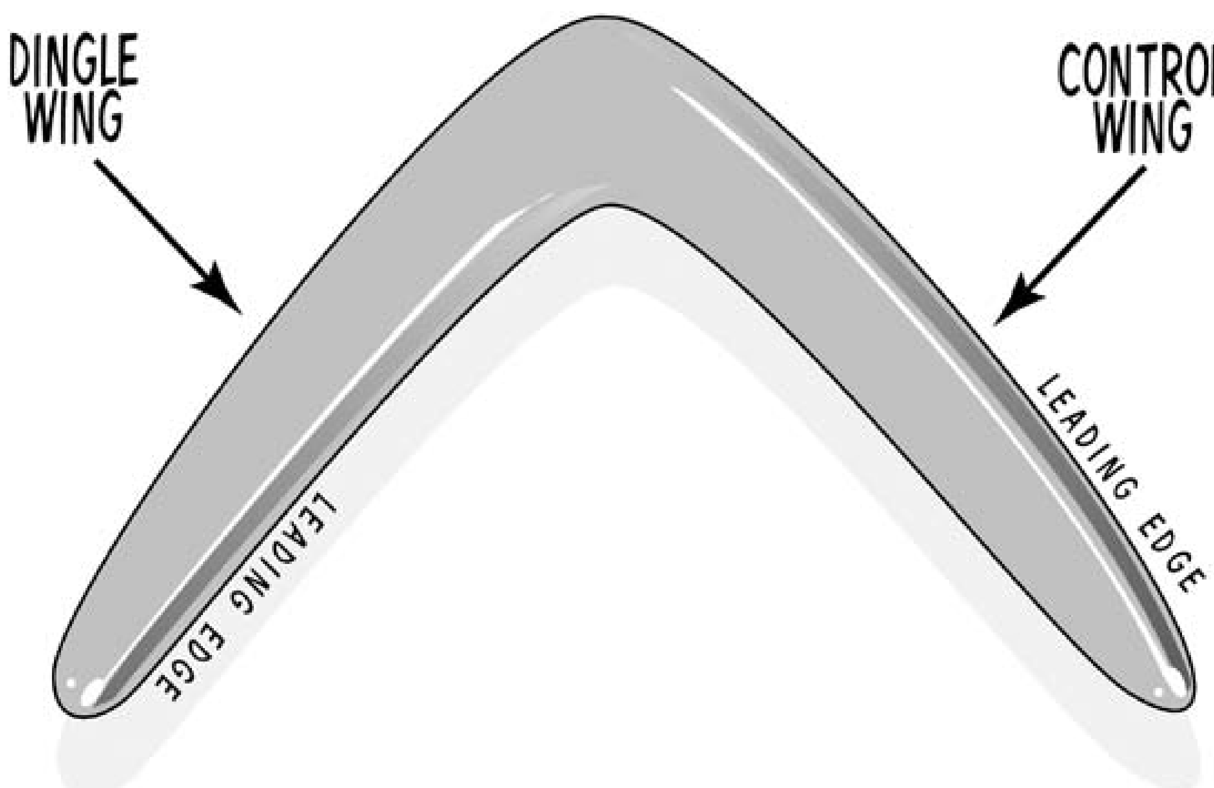
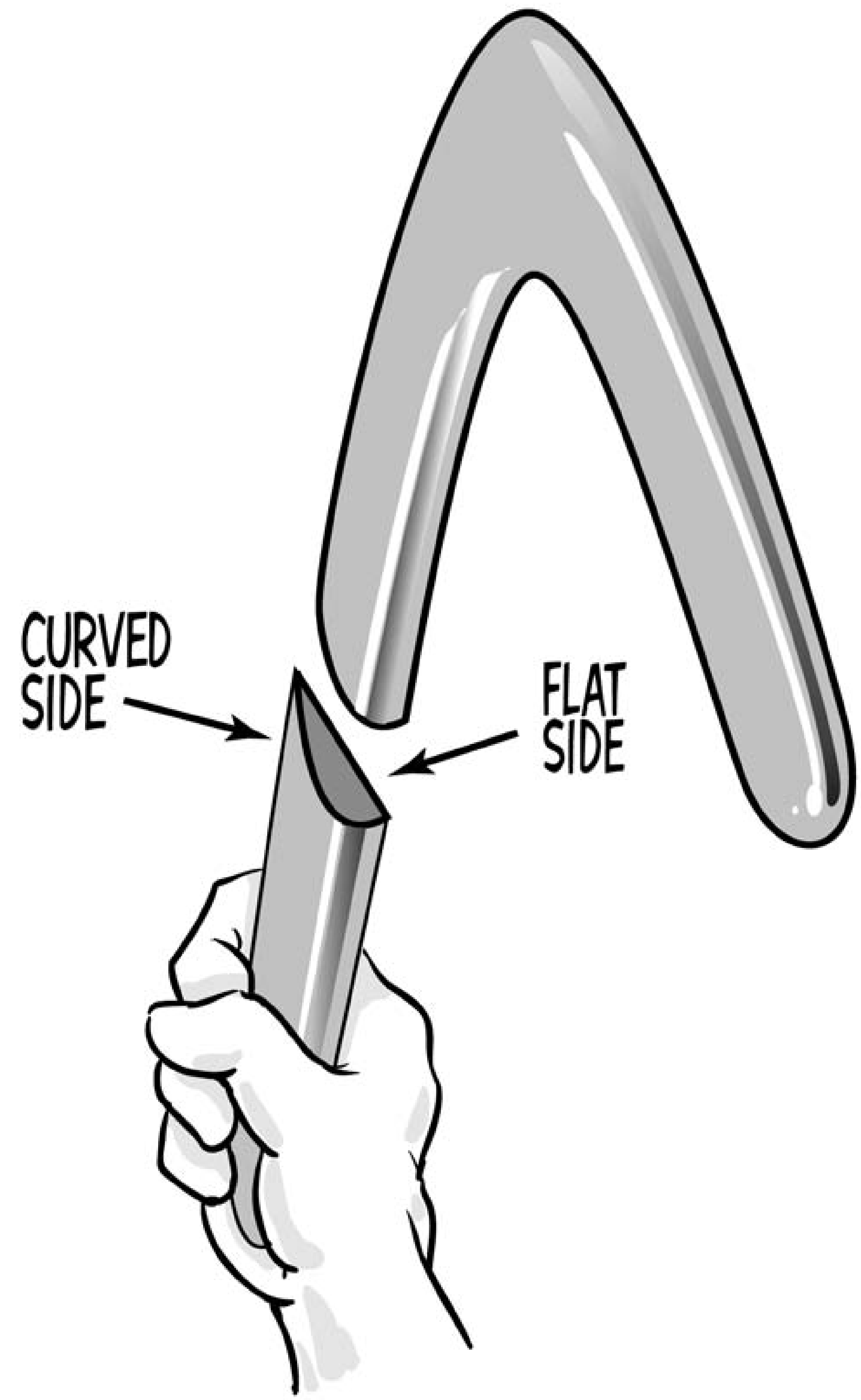
伯努利方程的应用：飞去来





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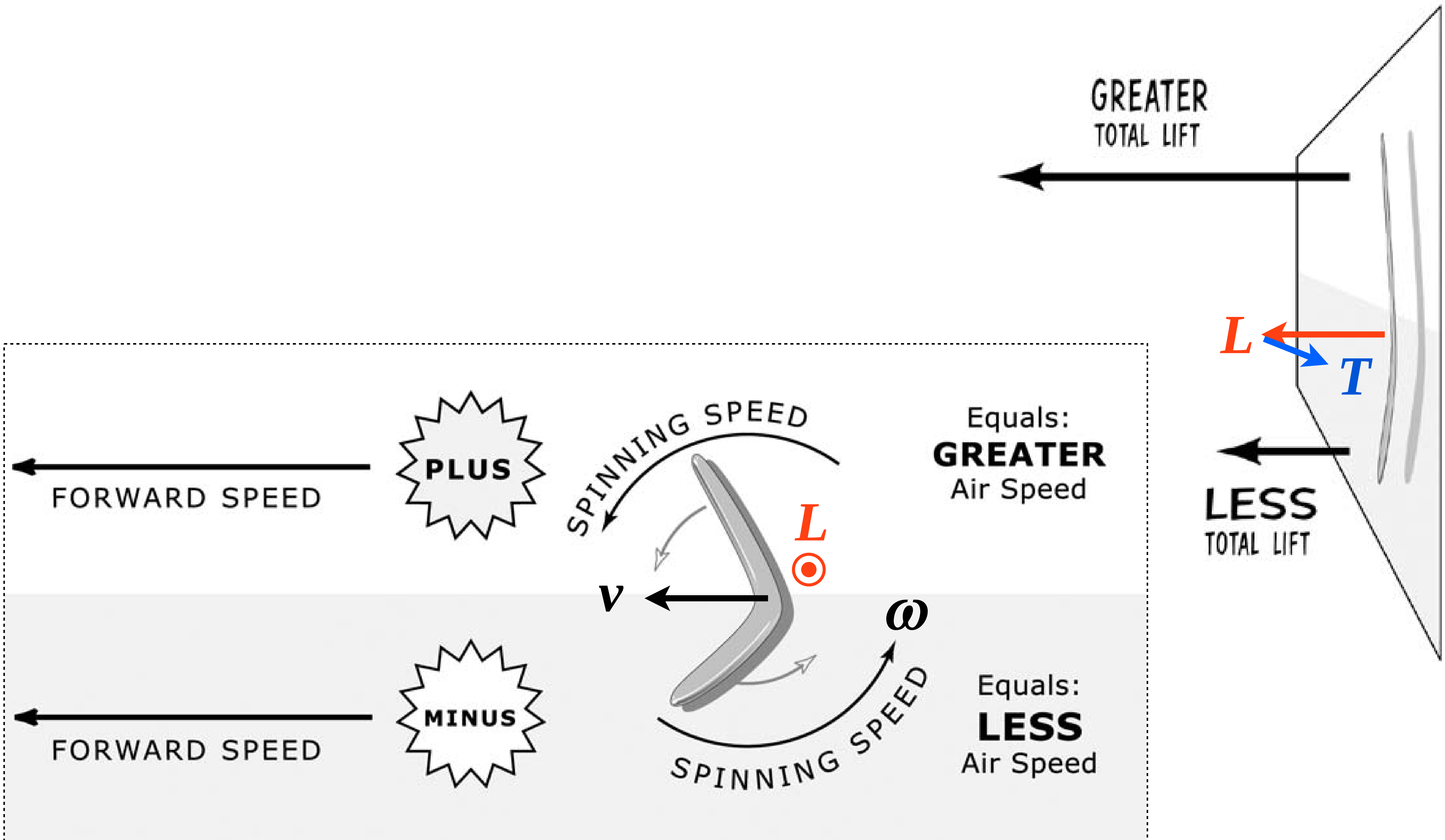
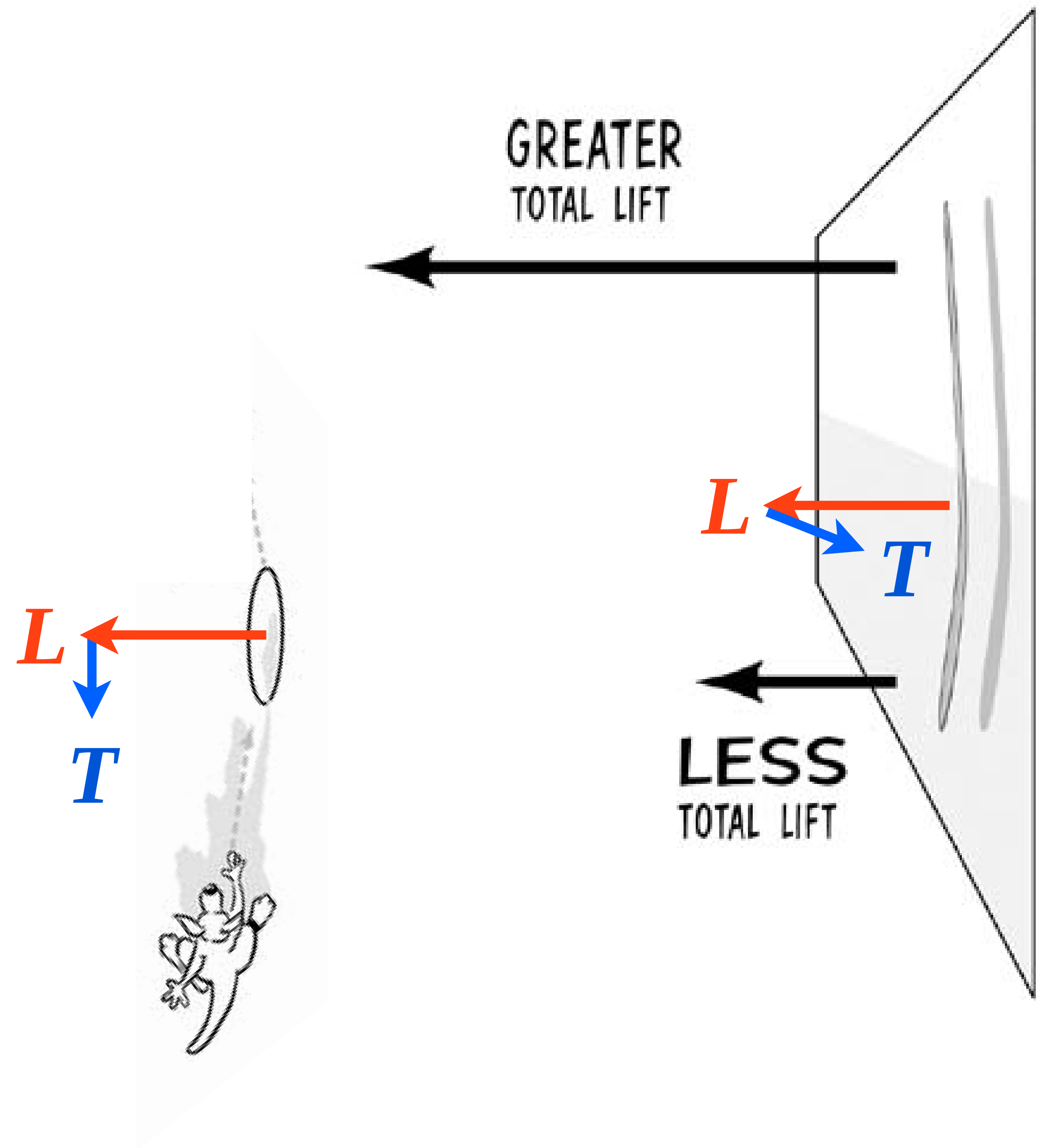
伯努利方程的应用：飞去来



伯努利方程的应用：飞去来



流体的粘滞性

牛奶

蜂蜜

沥青

http://v.youku.com/v_show/id_XMzkyMDMyOTk2.html

沥青滴漏实验

沥青滴漏实验时间表

年份	状况	到达此状态所用时间	从切开封口所用总时间	从架设实验所用总时间
1927年	架设实验	/	/	/
1930年	切开封口	3年	/	3年
1938年12月	第一滴	8年11个月	8年11个月	11年11个月
1947年2月	第二滴	8年3个月	17年1个月	20年1个月
1954年4月	第三滴	7年2个月	24年3个月	27年3个月
1962年5月	第四滴	8年1个月	32年4个月	35年4个月
1970年8月	第五滴	8年3个月	40年7个月	43年7个月
1979年4月	第六滴	8年8个月	49年3个月	52年3个月
1988年7月	第七滴	9年3个月	58年6个月	61年6个月
2000年11月28日	第八滴	12年5个月	70年11个月	73年11个月

全球持续时间最长的实验

沥青比水粘性大2.3×10^{11}倍

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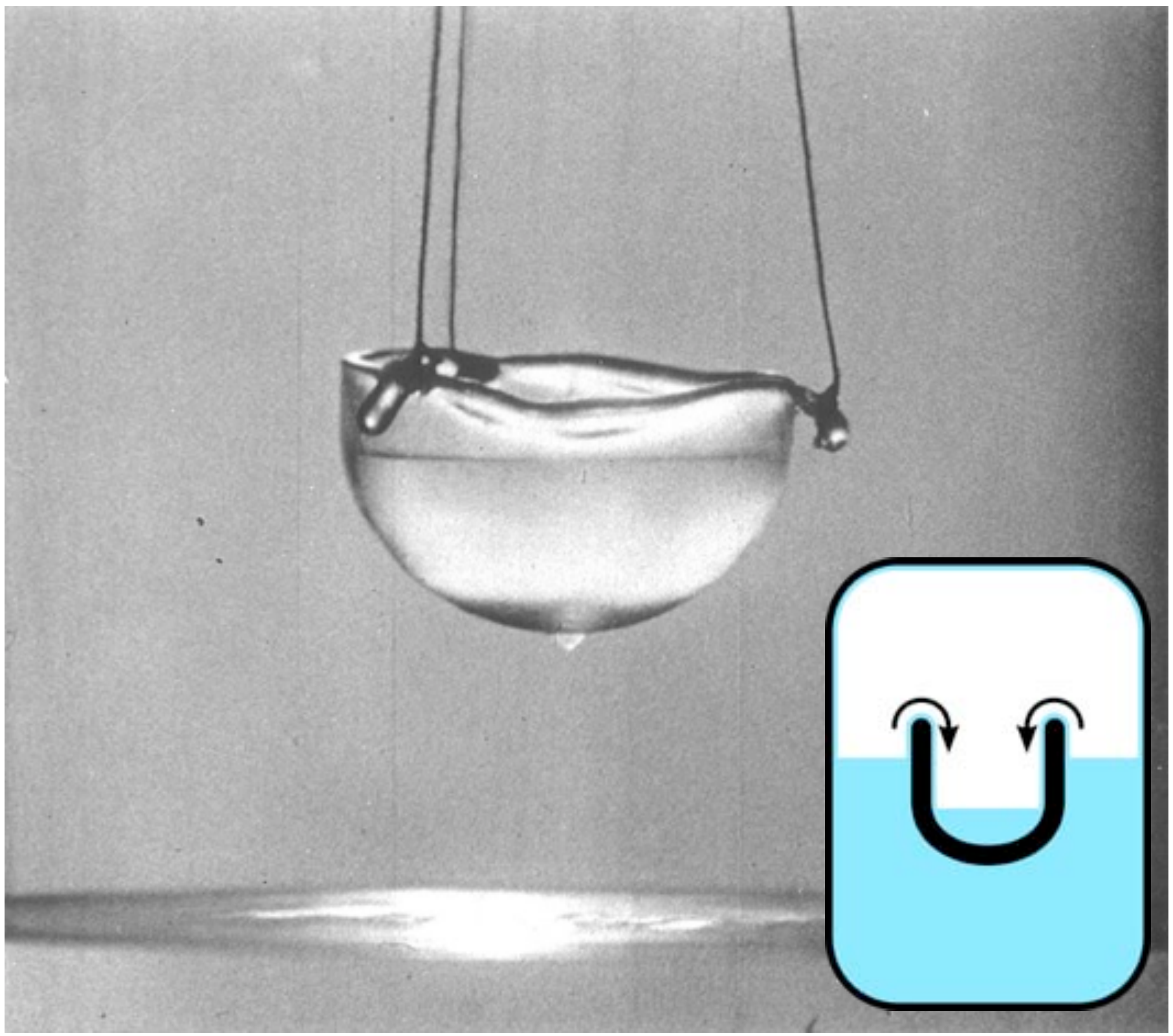
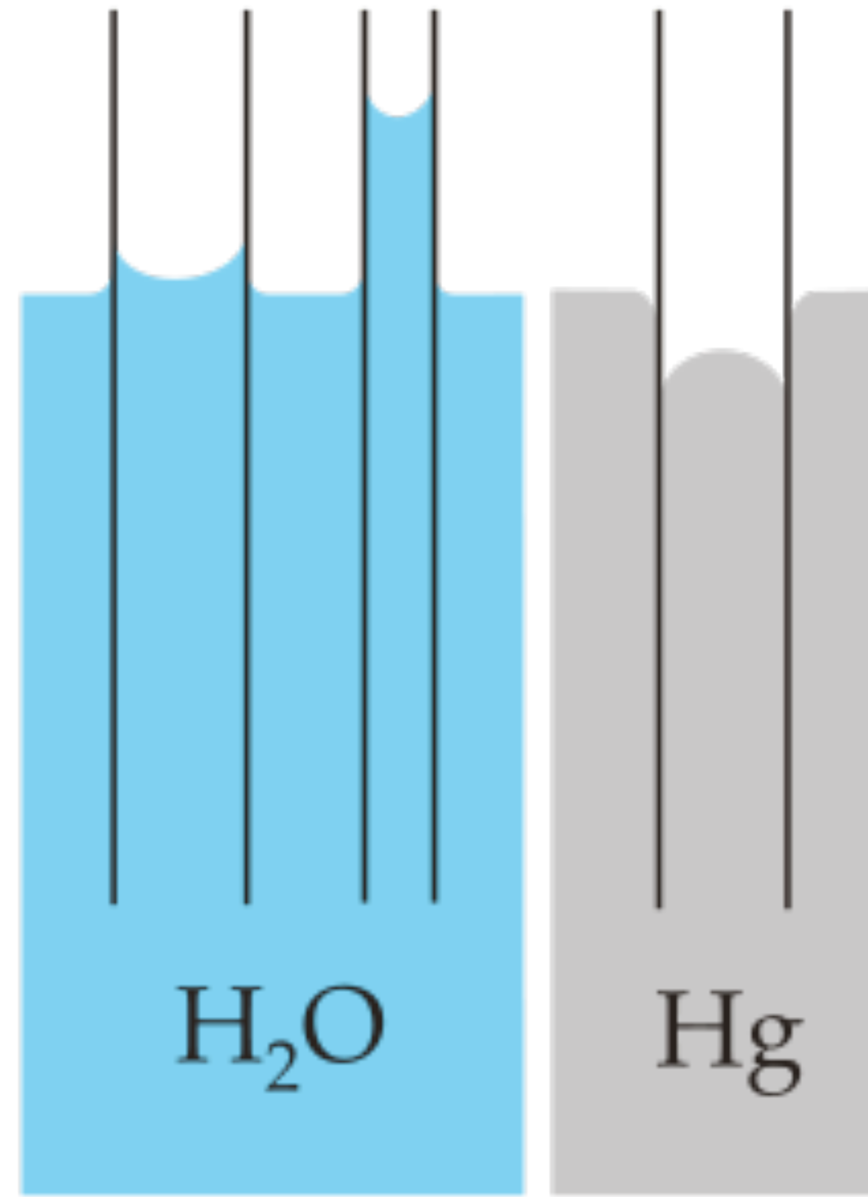
氦超流



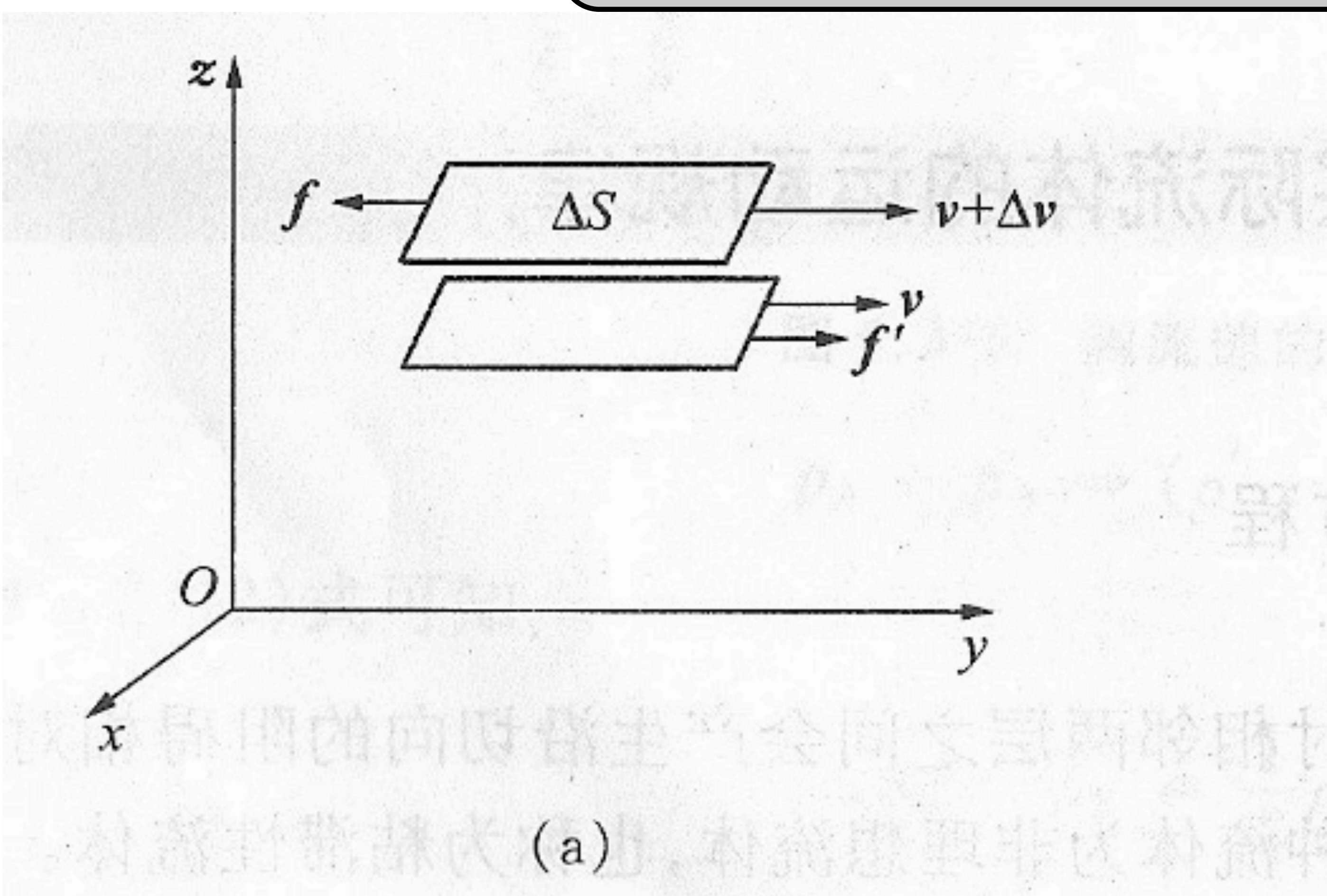


液氦超流：http://v.youku.com/v_show/id_XMzkyMDAwMjI0.html

流体的粘滞性

层间粘滞力： $$f = \eta \frac{dv}{dz} \Delta S$$

粘滞系数，粘度 (单位： $\mathrm{Pa \cdot s = N \cdot s \cdot m^{-2}}$)



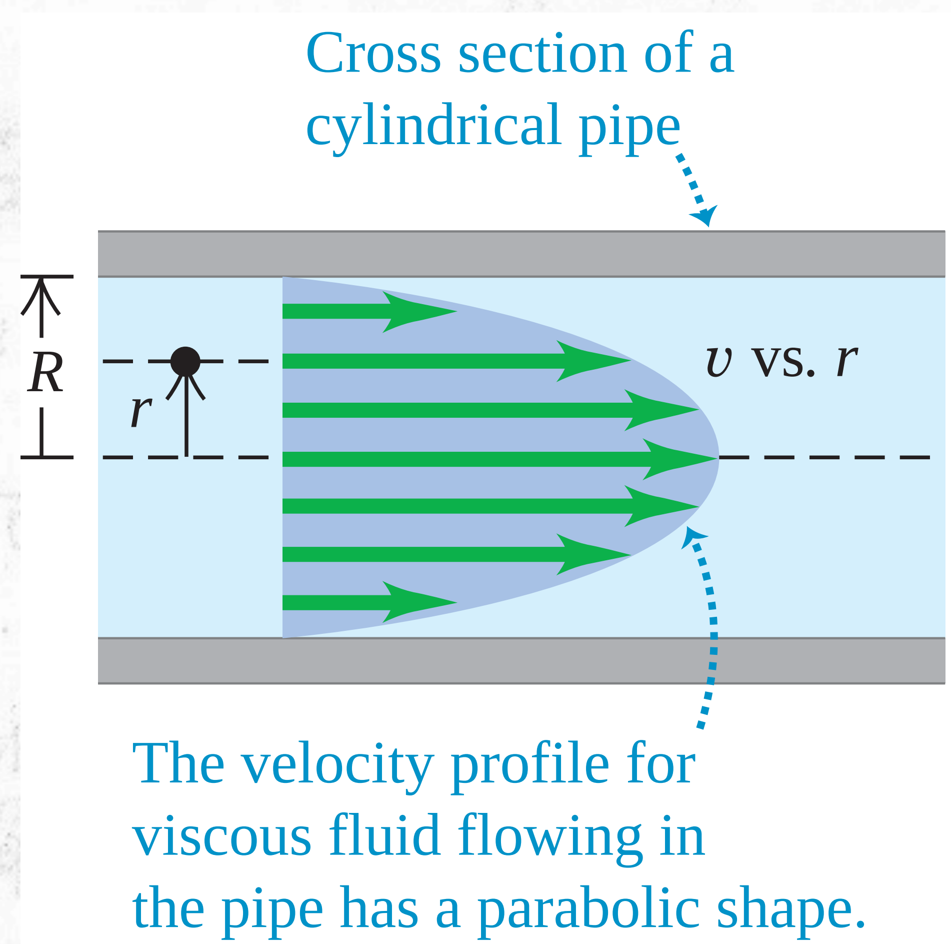


图 5.4-1 流体的粘滞性

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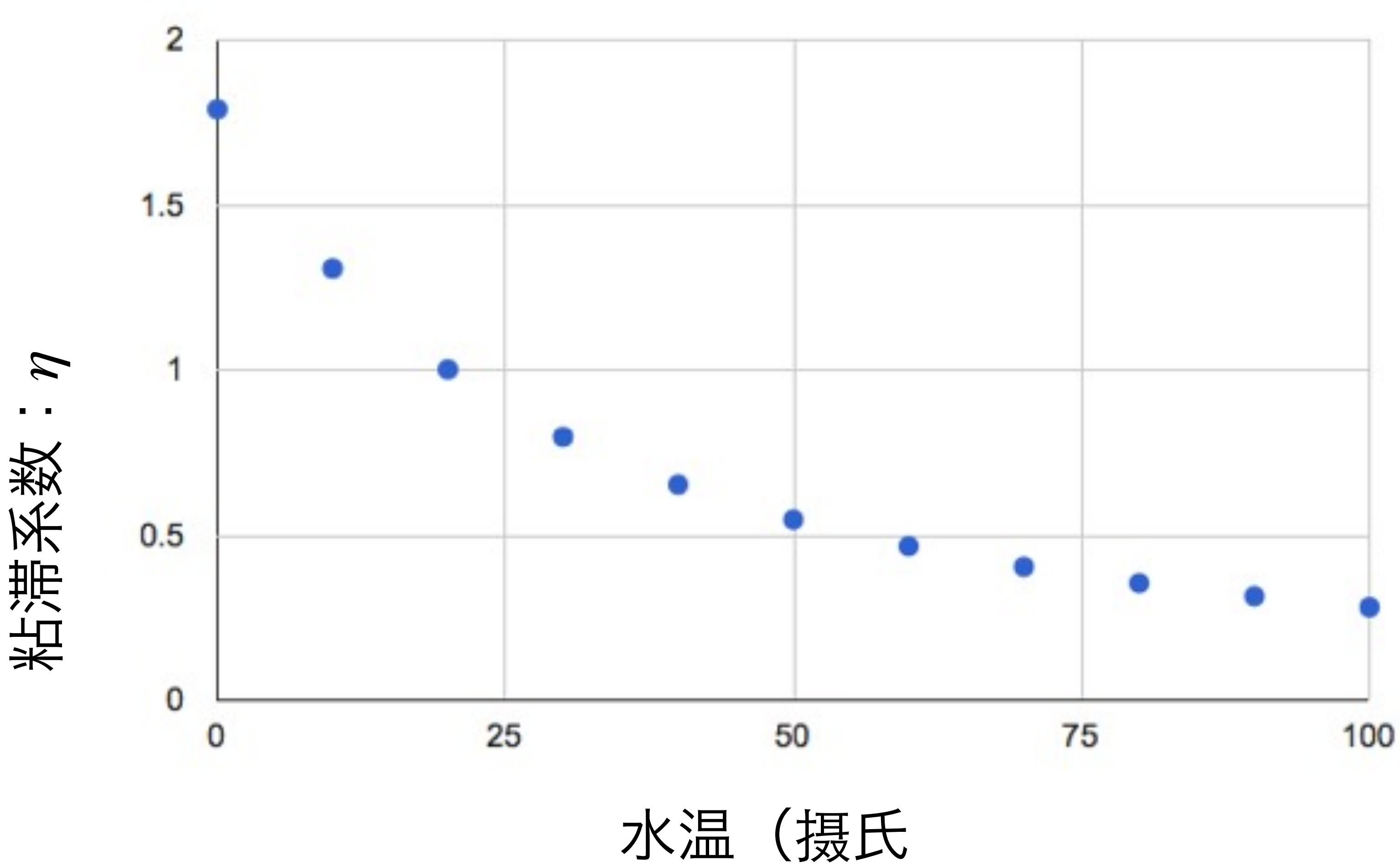
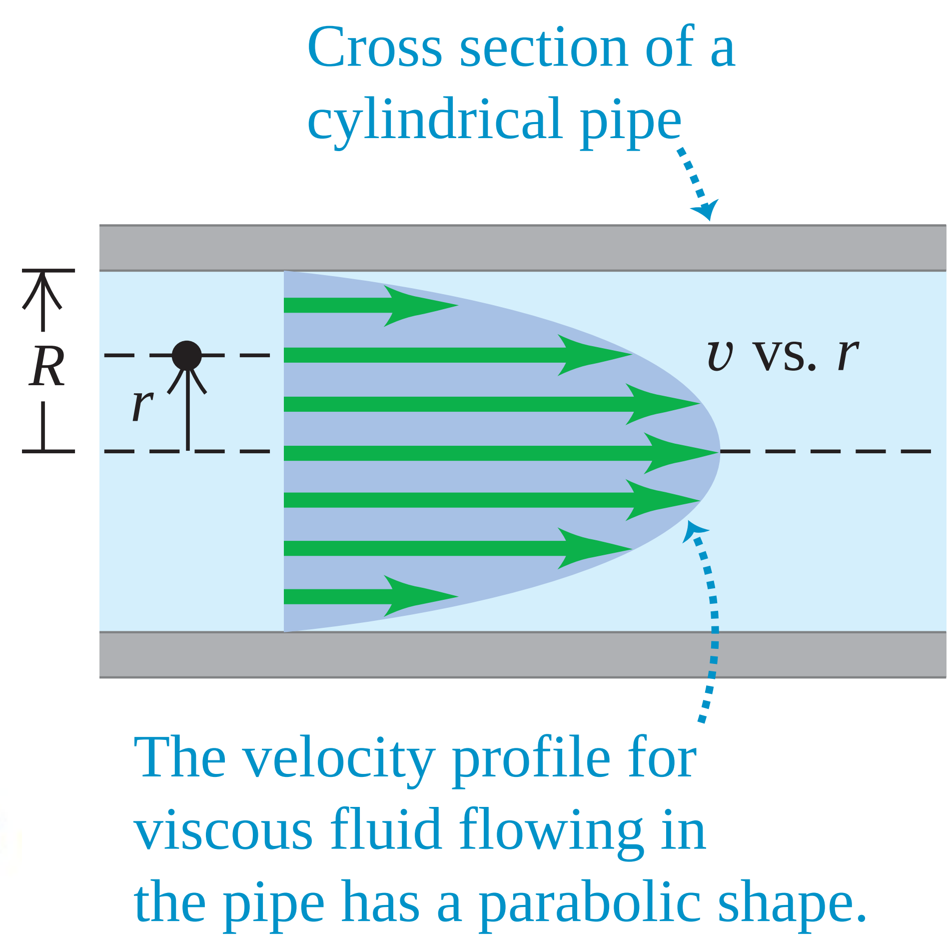
流体的粘滞性

层间粘滞力： $f = \eta \dfrac{dv}{dz} \Delta S = \tau \Delta S$

粘滞系数，粘度 (单位： $\mathrm{Pa \cdot s = N \cdot s \cdot m^{-2}}$)





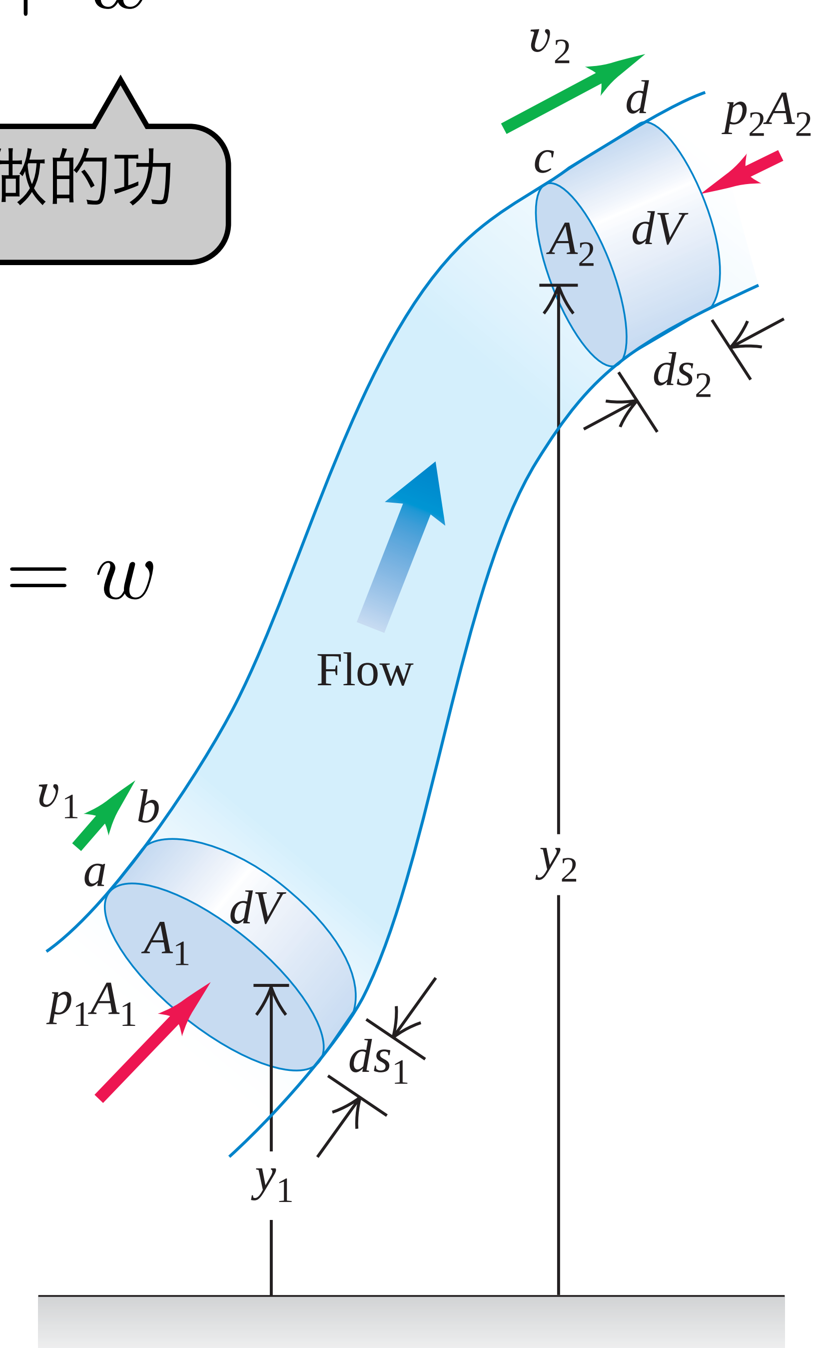
粘滞流体的伯努利方程

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 + w$$

单位体积内粘滞力做的功

$$v_1 = v_2 \quad \text{and} \quad h_1 = h_2 \quad \Rightarrow \quad p_1 - p_2 = w$$

$$v_1 = v_2 \quad \text{and} \quad p_1 = p_2 \quad \Rightarrow \quad \rho g h_1 - \rho g h_2 = w$$



$$Av = \text{const.}$$

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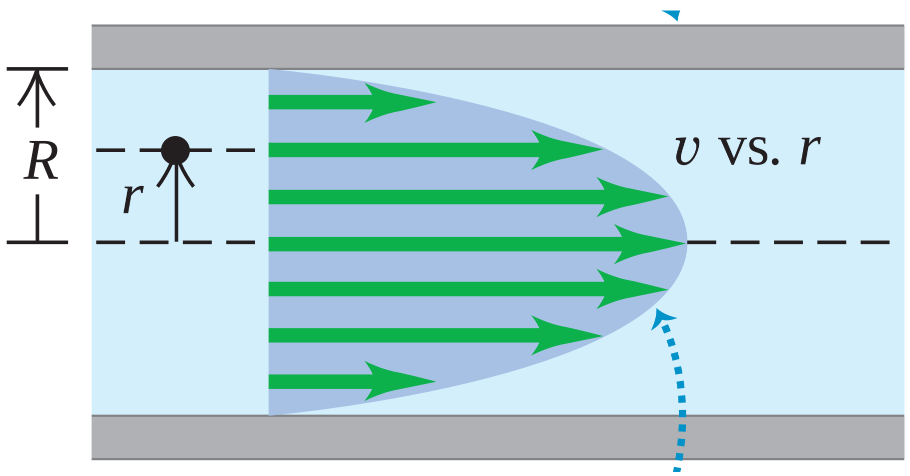
泊肃叶定律



$$Q = kR^x \eta^y \left(\frac{\Delta p}{l}\right)^z$$

流量 Q

管道半径 R　　粘度 η　　压强梯度 $\dfrac{\Delta p}{l}$

$$\frac{m^3}{s} = m^x \cdot \left(\frac{N \cdot s}{m^2}\right)^y \cdot \left(\frac{N}{m^3}\right)^z$$

$$\frac{m^3}{s} = m^x \cdot \left(\frac{kg}{m \cdot s}\right)^y \cdot \left(\frac{kg}{m^2 \cdot s^2}\right)^z$$

$$\begin{aligned} 3 &= x - y - 2z \\ -1 &= -y - 2z \\ 0 &= y + z \end{aligned}$$

$$\begin{aligned} x &= 4 \\ y &= -1 \\ z &= 1 \end{aligned}$$

$$Q = k\frac{R^4}{\eta}\frac{\Delta p}{l} = k\frac{R^4}{\eta l}\Delta p$$

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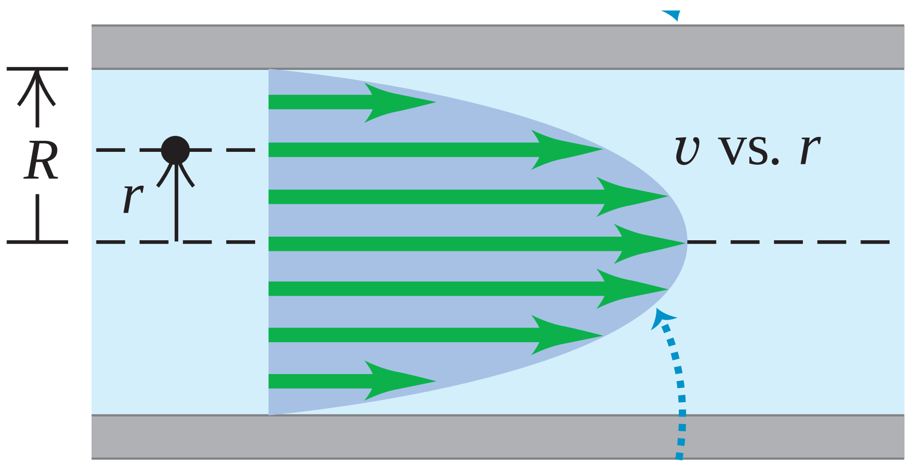
泊肃叶定律



泊肃叶定律：

$$\boxed{Q = \frac{\pi R^4}{8 l \eta}(p_1 - p_2)}$$

高血压

流量 Q

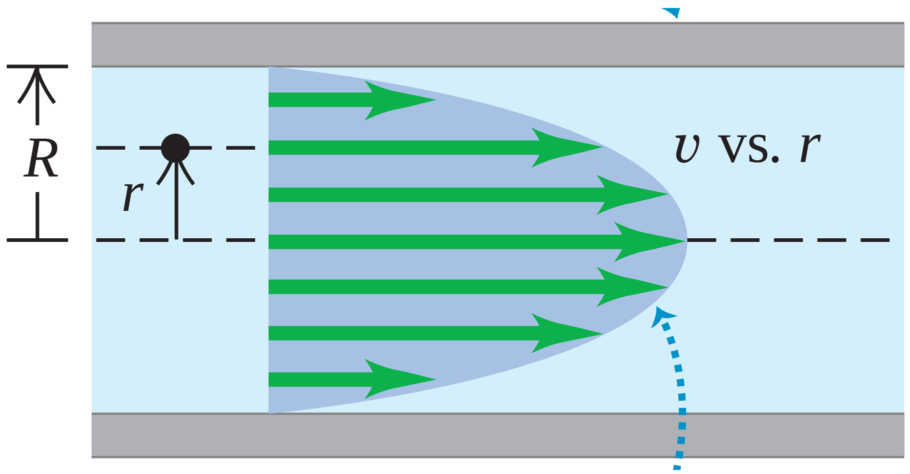
管道半径	粘度	压强梯度
R	η	$\frac{\Delta p}{l}$

$$\frac{m^3}{s} = m^x \cdot \left(\frac{N \cdot s}{m^2}\right)^y \cdot \left(\frac{N}{m^3}\right)^z$$

$$\frac{m^3}{s} = m^x \cdot \left(\frac{kg}{m \cdot s}\right)^y \cdot \left(\frac{kg}{m^2 \cdot s^2}\right)^z$$

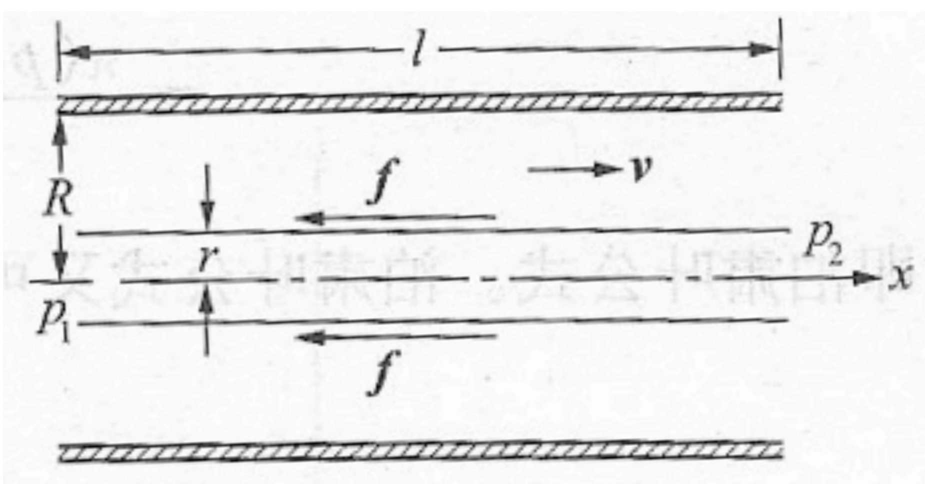
$$\begin{aligned} 3 &= x - y - 2z \\ -1 &= -y - 2z \\ 0 &= y + z \end{aligned} \quad \Rightarrow \quad \begin{aligned} x &= 4 \\ y &= -1 \\ z &= 1 \end{aligned} \quad \Rightarrow \quad Q = k\frac{R^4}{\eta}\frac{\Delta p}{l} = k\frac{R^4}{\eta l}\Delta p$$

泊肃叶定律



泊肃叶定律：

$$Q = \frac{\pi R^4}{8 l \eta}(p_1 - p_2)$$



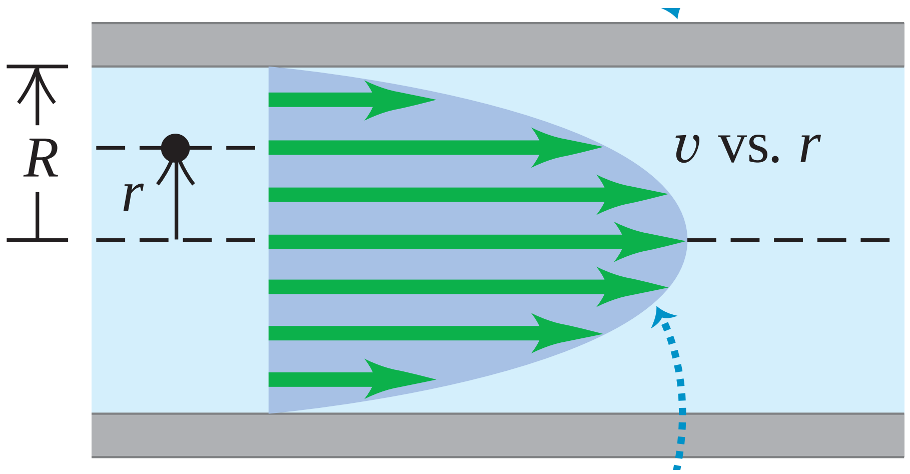
$$F = (p_1 - p_2)\pi r^2$$

$$f = \eta \cdot 2\pi r l \cdot \frac{dv}{dr}$$

定常流动 $\Rightarrow \quad F + f = 0 \quad \Rightarrow \quad dv = \frac{p_1 - p_2}{2 l \eta} r dr$

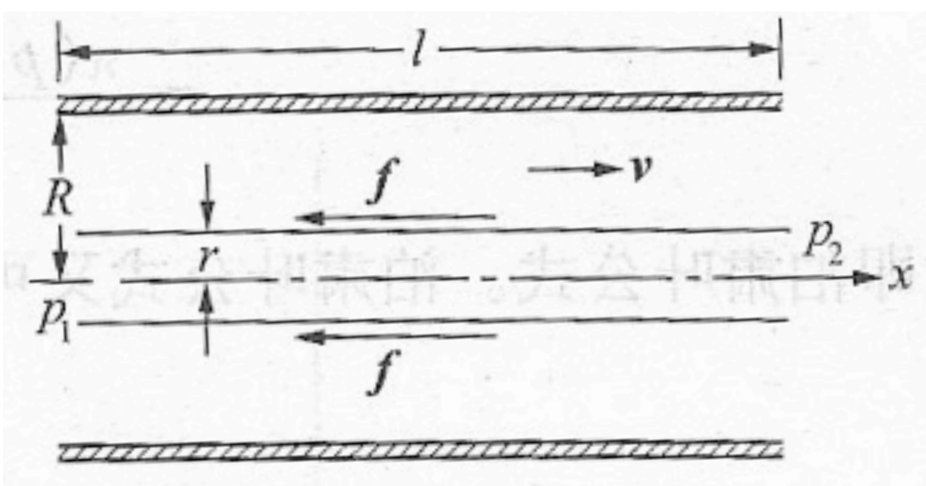
$$\Rightarrow \quad v(r) - v_0 = -\frac{p_1 - p_2}{4 l \eta} r^2$$

泊肃叶定律



泊肃叶定律：

$$\boxed{Q = \frac{\pi R^4}{8l\eta}(p_1 - p_2)}$$



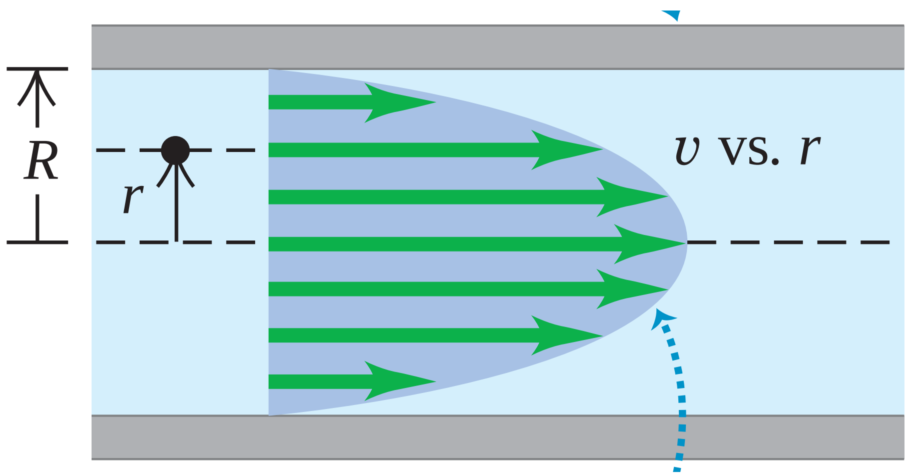
$$v(R) = 0 \quad \Rightarrow \quad v_0 = \frac{p_1 - p_2}{4l\eta} R^2$$

$$\Rightarrow \quad v(R) = \frac{p_1 - p_2}{4l\eta}(R^2 - r^2)$$

定常流动 $\Rightarrow \quad F + f = 0 \quad \Rightarrow \quad dv = \frac{p_1 - p_2}{2l\eta} r dr$

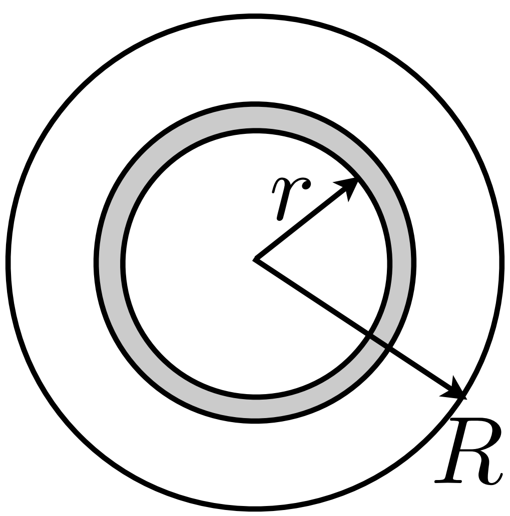
$$\Rightarrow \quad v(r) - v_0 = -\frac{p_1 - p_2}{4l\eta} r^2$$

泊肃叶定律



泊肃叶定律：

$$\boxed{Q = \frac{\pi R^4}{8 l \eta}(p_1 - p_2)}$$



$$v(R) = 0 \quad \Rightarrow \quad v_0 = \frac{p_1 - p_2}{4 l \eta} R^2$$

$$\Rightarrow \quad v(R) = \frac{p_1 - p_2}{4 l \eta}(R^2 - r^2)$$

$$dQ = v(r) \cdot 2\pi r dr \quad \Rightarrow \quad Q = \int dQ = \int_0^R v(r) \cdot 2\pi r dr$$

$$= \frac{\pi R^4}{8 l \eta}(p_1 - p_2)$$

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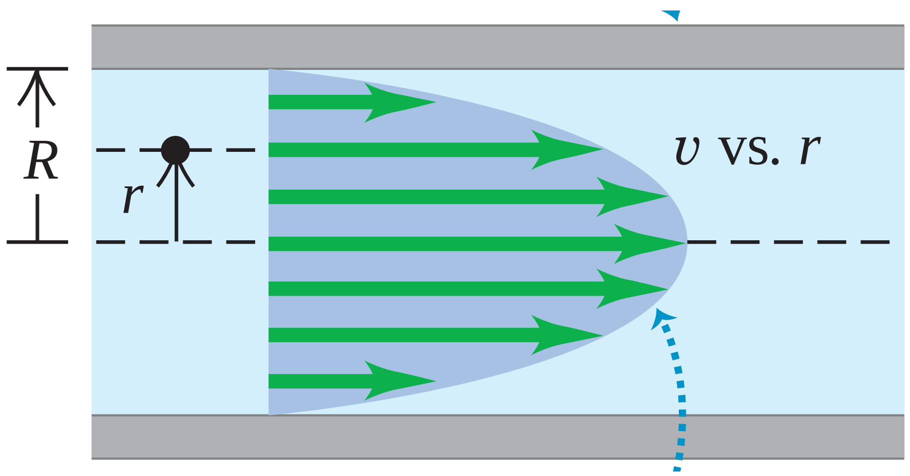
泊肃叶定律



泊肃叶定律：

$$\boxed{Q = \frac{\pi R^4}{8 l \eta}(p_1 - p_2)}$$

$$v(r) = \frac{p_1 - p_2}{4\eta l}\left(R^2 - r^2\right)$$

$$v_0 = v(0) = \frac{p_1 - p_2}{4\eta l} R^2$$

$$\bar{v} = \frac{Q}{\pi R^2} = \frac{1}{2} v_0$$

$$\tau(r) = \eta \frac{dv}{dr} = -\frac{p_1 - p_2}{2l} r$$

$$\tau_M = \tau(R) = \frac{4\eta \bar{v}}{R}$$

$$f_M = \tau_M 2\pi R l = 8\pi\eta l \bar{v}$$

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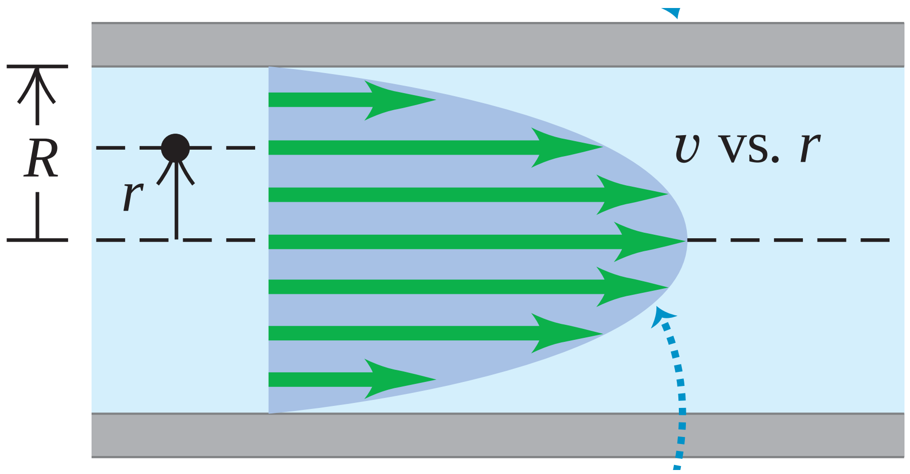
泊肃叶定律



泊肃叶定律：

$$Q = \frac{\pi R^4}{8 l \eta}(p_1 - p_2)$$

电路

$$I = \frac{V}{R}$$

电阻

管路

$$Q = \frac{\Delta p}{Z} \quad \text{with} \quad Z = \frac{8 l \eta}{\pi R^4}$$

流阻

串联：

$$R = R_1 + R_2 + \cdots + R_n$$

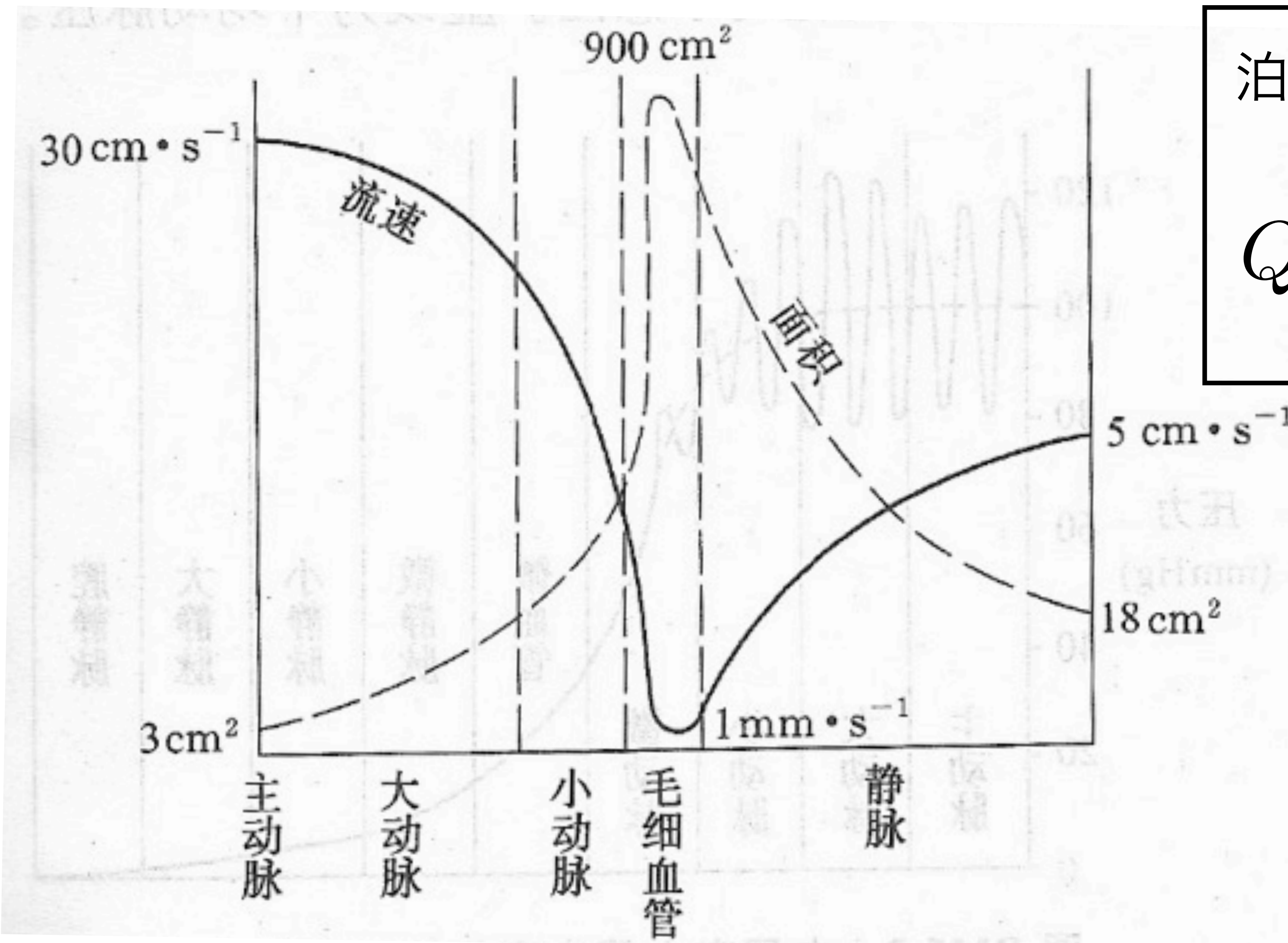
$$Z = Z_1 + Z_2 + \cdots + Z_n$$

并联：

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

泊肃叶定律



泊肃叶定律：

$$Q = \frac{\pi R^4}{8 l \eta}(p_1 - p_2)$$

串联管道：

$$Z = Z_1 + Z_2 + \cdots + Z_n$$

并联管道：

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

泊肃叶定律

泊肃叶定律：

$$Q = \frac{\pi R^4}{8l\eta}(p_1 - p_2)$$

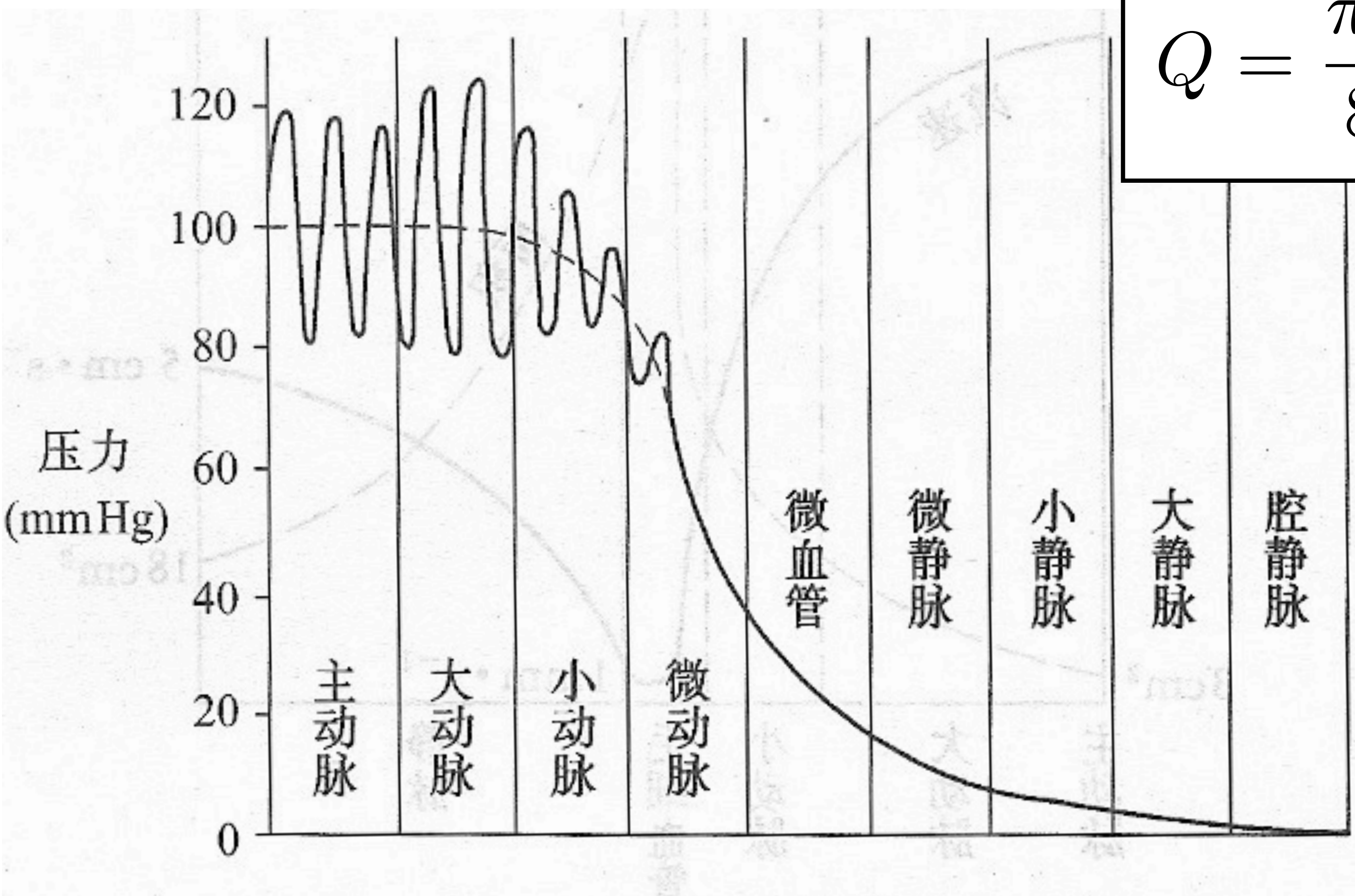


图 RM5-2 血压在血管中变化的示意图

动脉硬化

泊肃叶定律 - 粘滞系数的测量

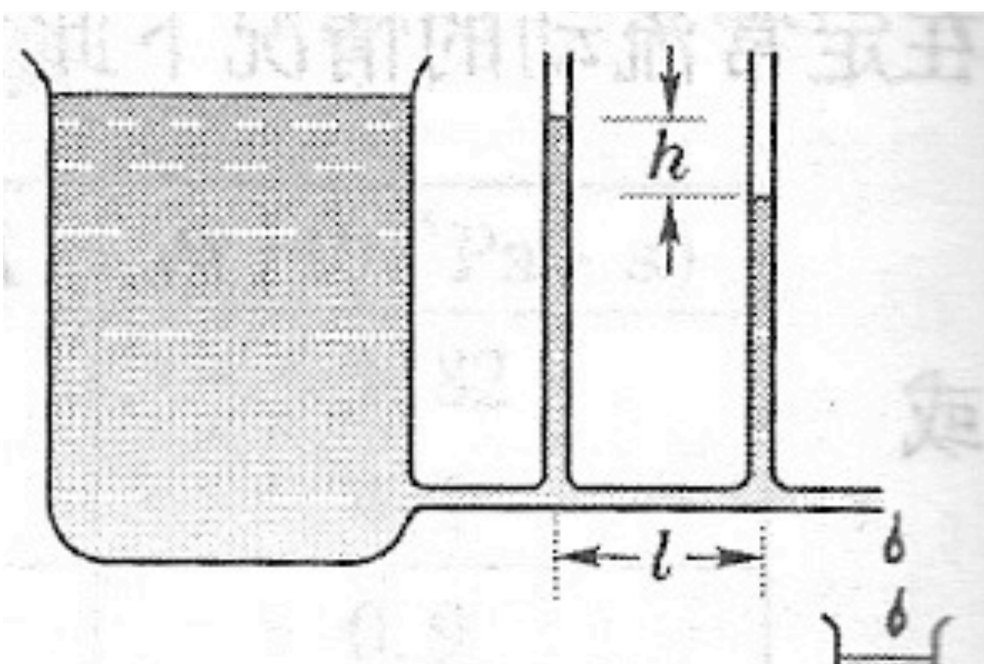


图 5-53 测量黏滞系数的装置

泊肃叶定律：

$$Q = \frac{\pi R^4}{8 l \eta}(p_1 - p_2)$$

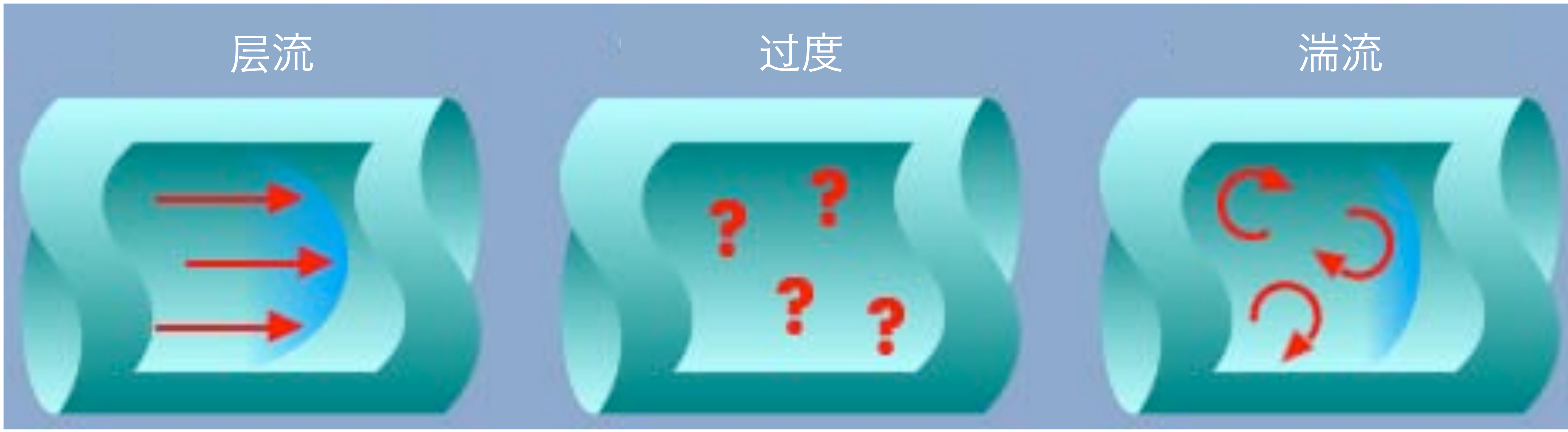
$$p_1 + \frac{1}{2}\rho v^2 = p_0 + \rho g h_1$$

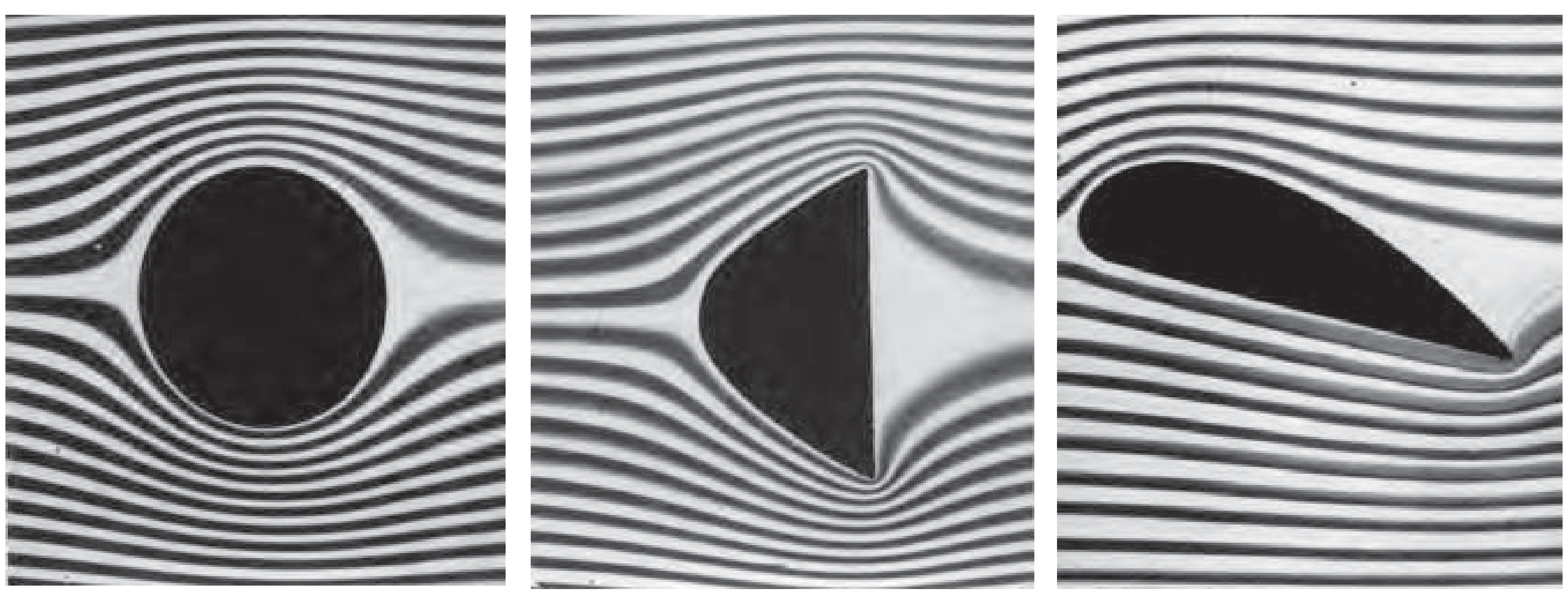
$$p_2 + \frac{1}{2}\rho v^2 = p_0 + \rho g h_2$$

$$p_1 - p_2 = \rho g (h_1 - h_2)$$

层流和湍流

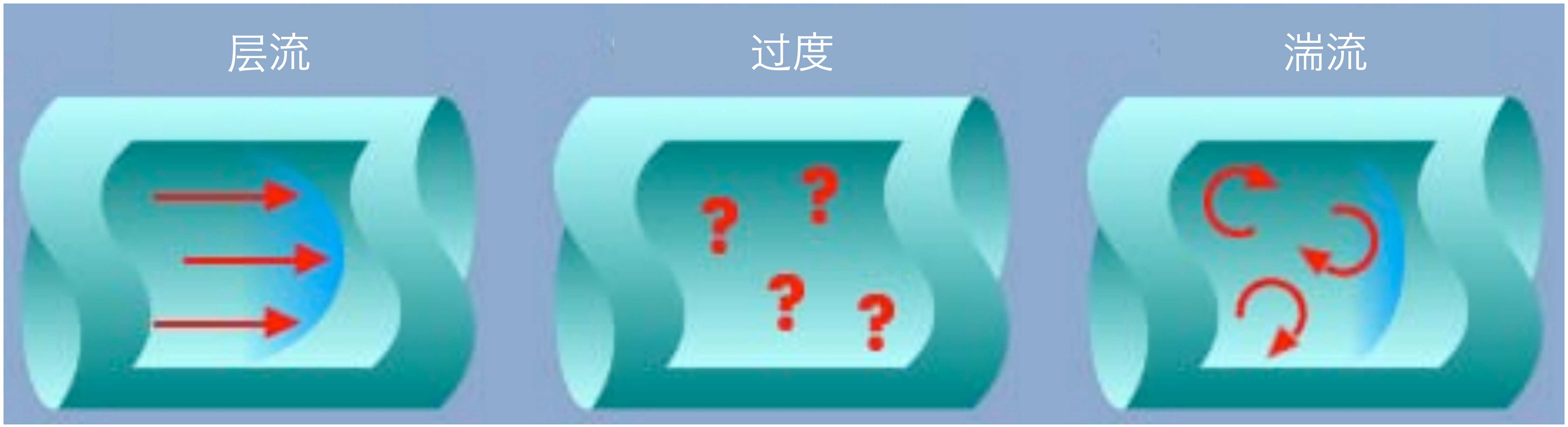
圆形管道中的流体　层流和湍流：http://v.youku.com/v_show/id_XMzgxNTg2MDA0.html





雷诺数

圆形管道中的流体



$R_e < 1000$ $1000 < R_e < 1500$ $1500 < R_e$

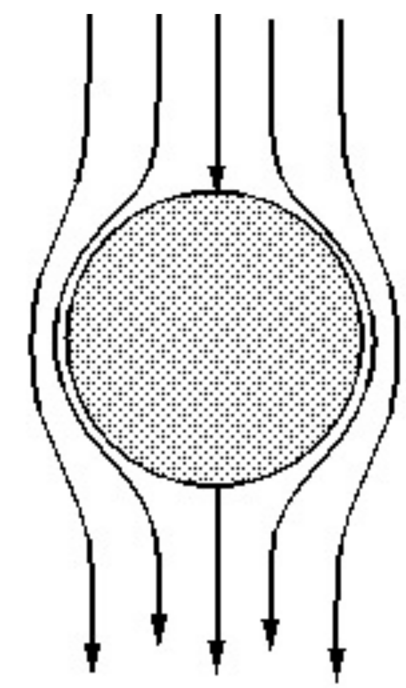
雷诺数： $R_e = \dfrac{\rho v r}{\eta}$

湍流（雷诺数11000）：http://v.youku.com/v_show/id_XMzkxOTk5MjU2.html

厨房中的层流和湍流：http://v.youku.com/v_show/id_XMzkxOTk3MTc2.html

湍流 - 声音 - 听诊（血液粘稠度）

雷诺数 - 小球



层流： $R_e < 0.1$

流体密度

颗粒相对流体速度

颗粒半径

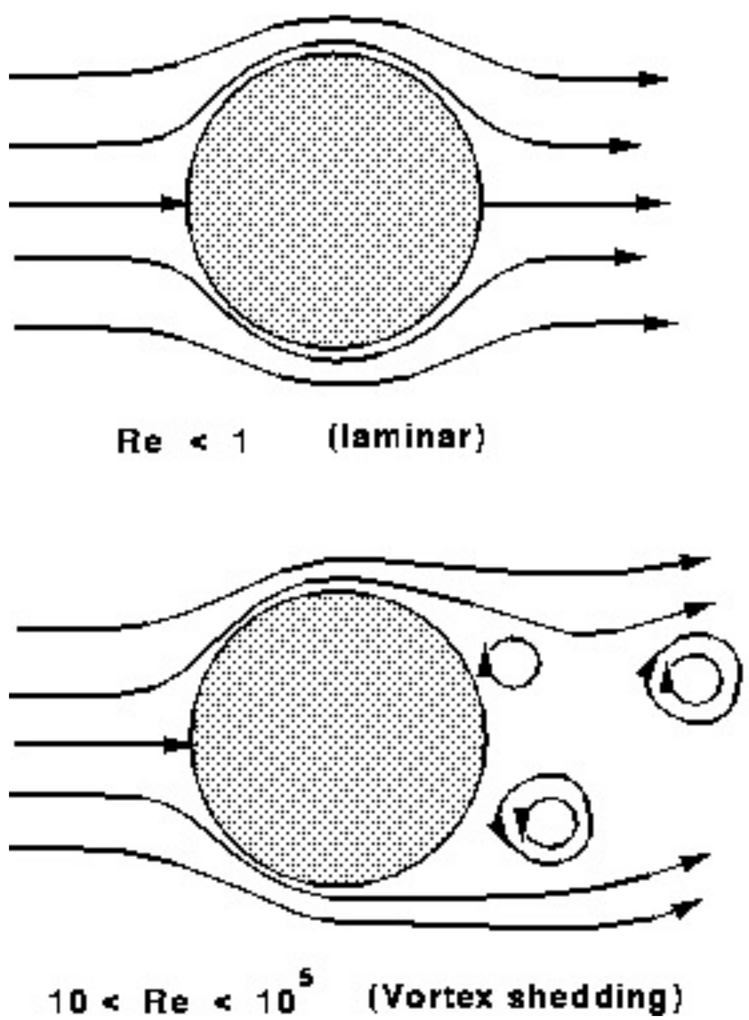
流体粘滞系数

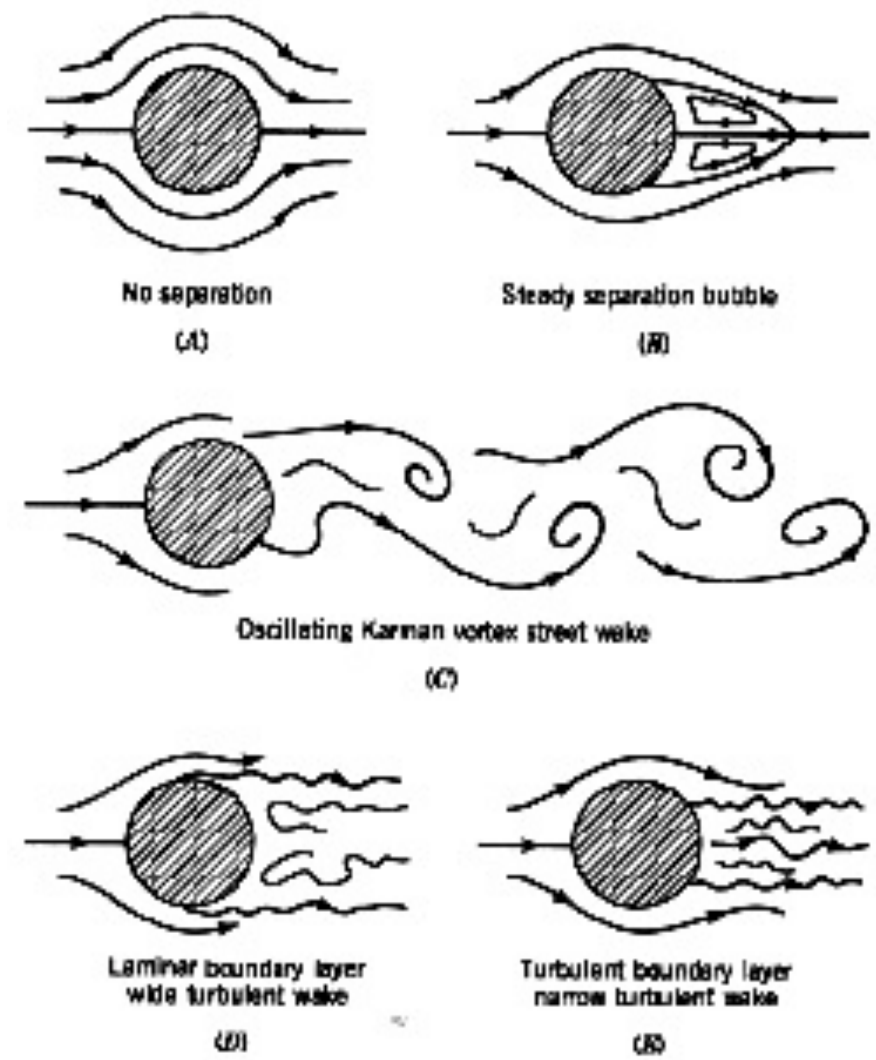
雷诺数： $R_e = \dfrac{\rho v r}{\eta}$

$$\rho_{\mathrm{air}} = 1.25\ \mathrm{kg/m^3}$$

$$\eta_{\mathrm{air}} = 1.8 \times 10^{-5} \mathrm{N\ s/m^2}$$

雷诺数





颗粒相对流体速度

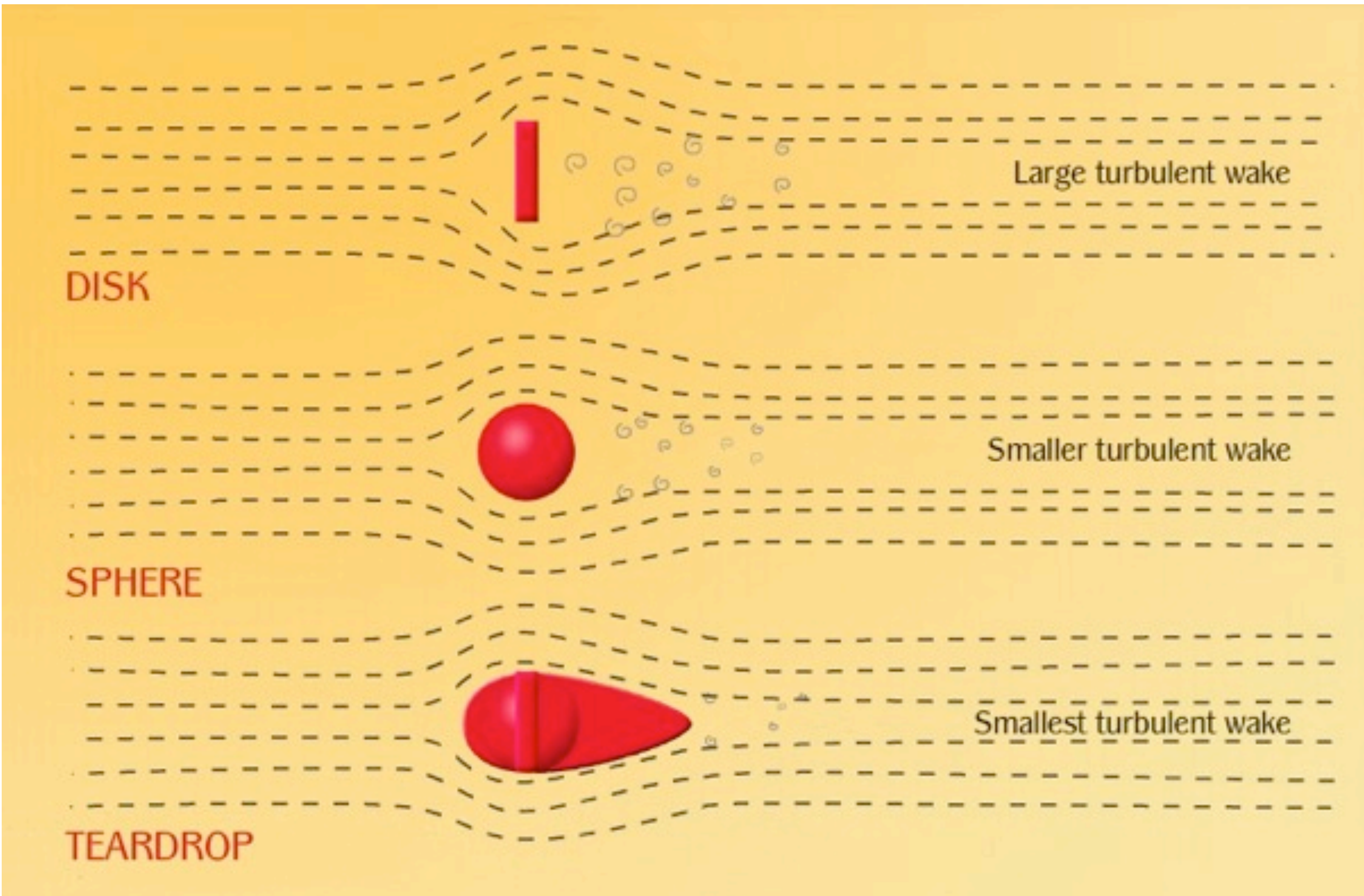
流体密度

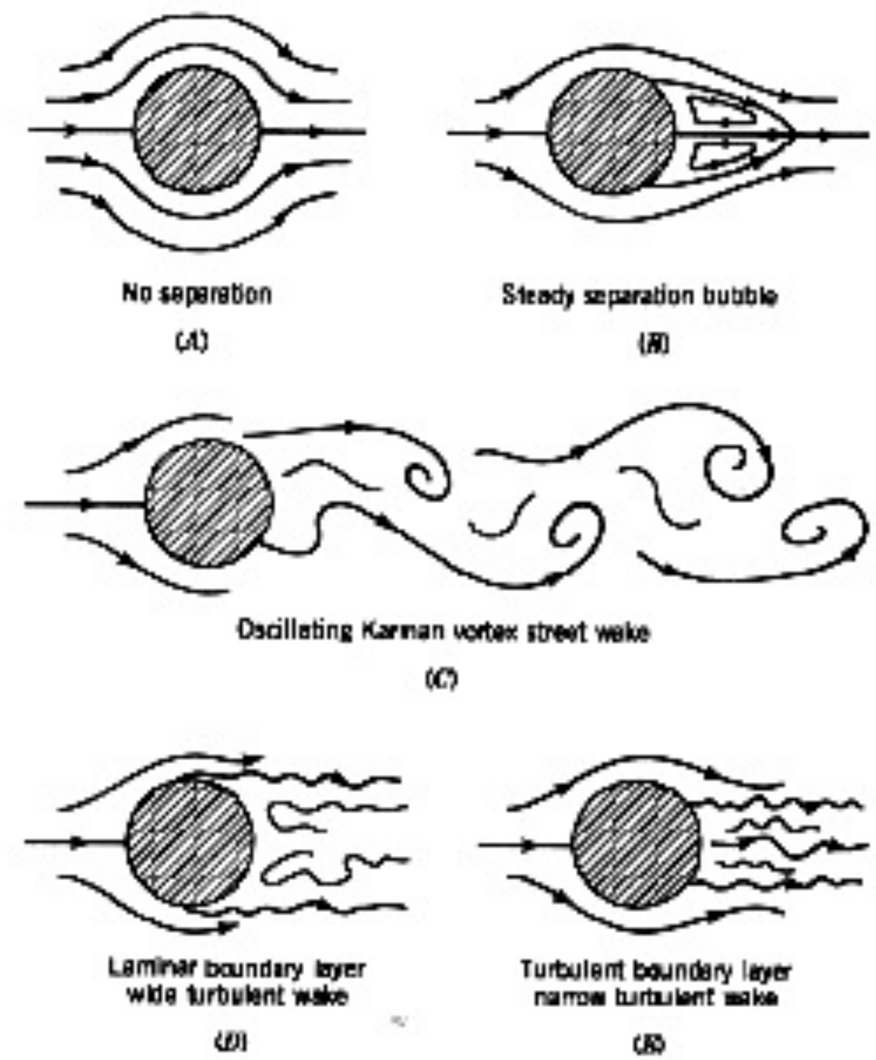
雷诺数： $R_e = \dfrac{\rho v r}{\eta}$

颗粒半径

流体粘滞系数

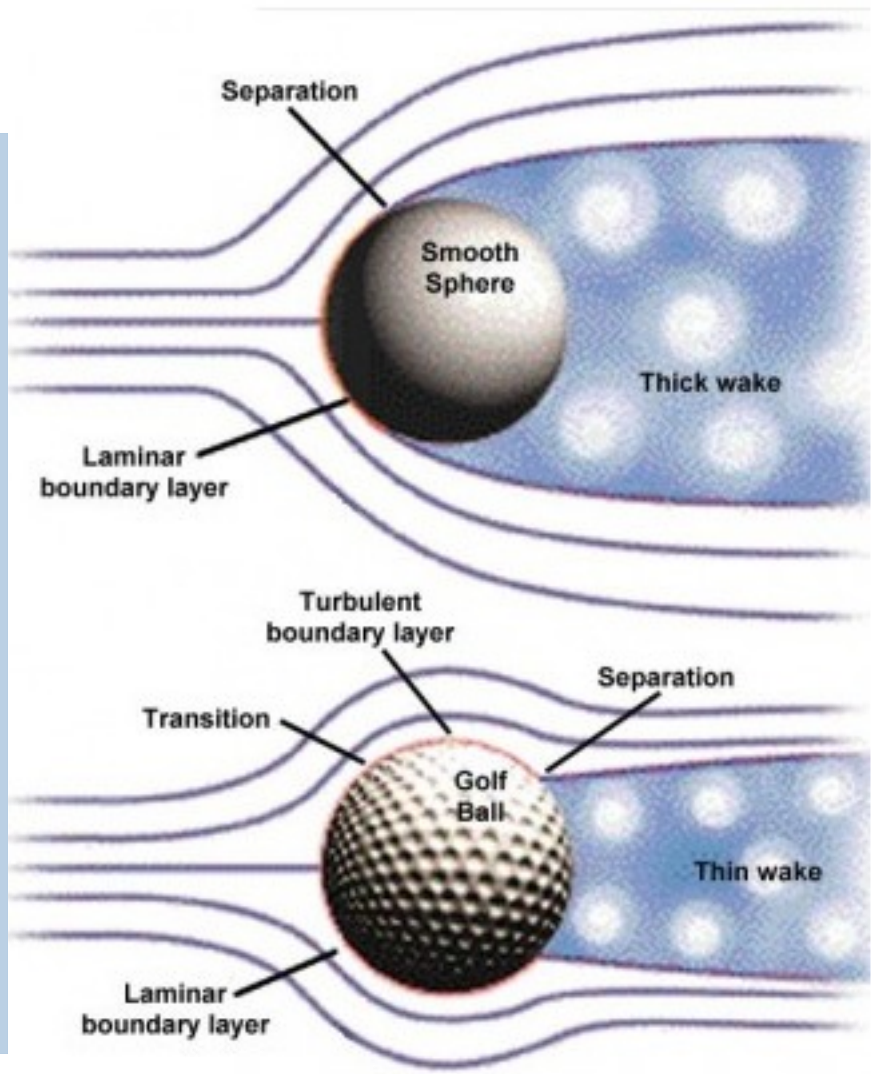
压差阻力

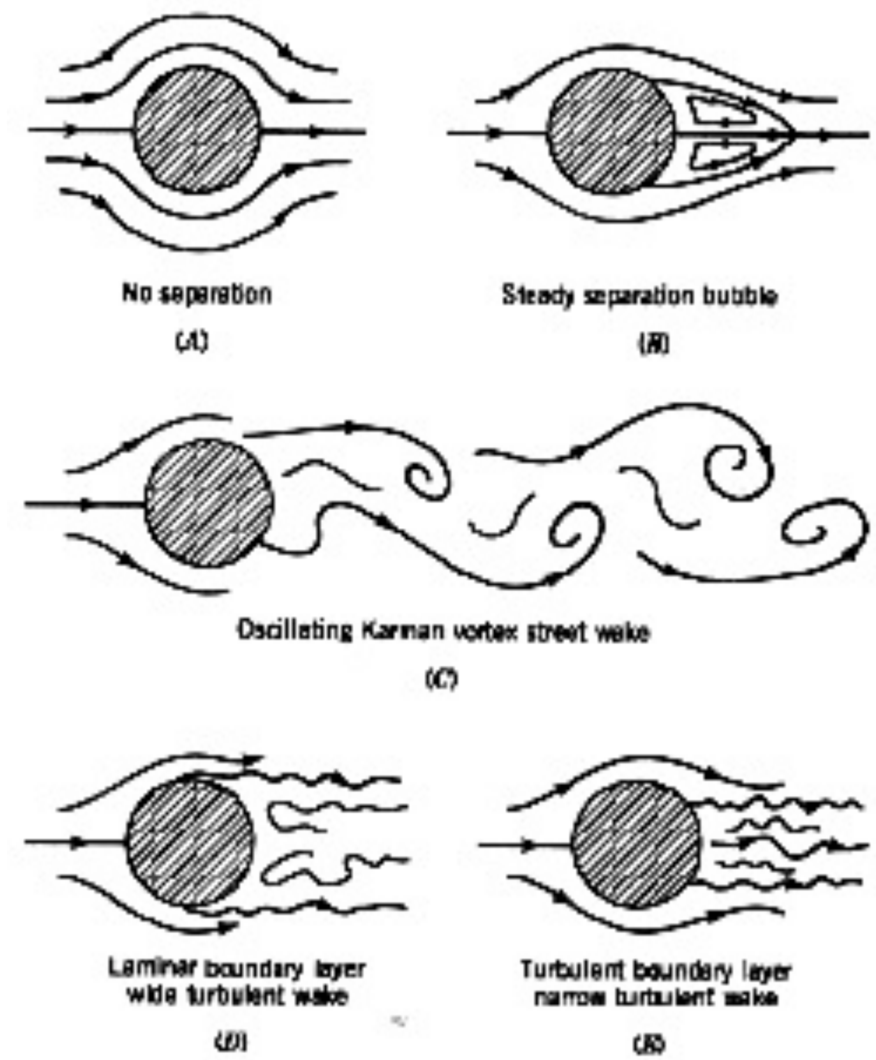




压差阻力：http://howthingsfly.si.edu/aerodynamics/pressure-drag

压差阻力


Separation
Smooth Sphere
Thick wake
Laminar boundary layer
Turbulent boundary layer
Separation
Transition
Golf Ball
Thin wake
Laminar boundary layer


No separation
(A)
Steady separation bubble
(B)
Oscillating Karman vortex street wake
(C)
Laminar boundary layer wide turbulent wake
(D)
Turbulent boundary layer narrow turbulent wake
(E)

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Trinity Atomic Bomb (1945)

Trinity bomb test: http://v.youku.com/v_show/id_XMzkyMDA0NTIw.html

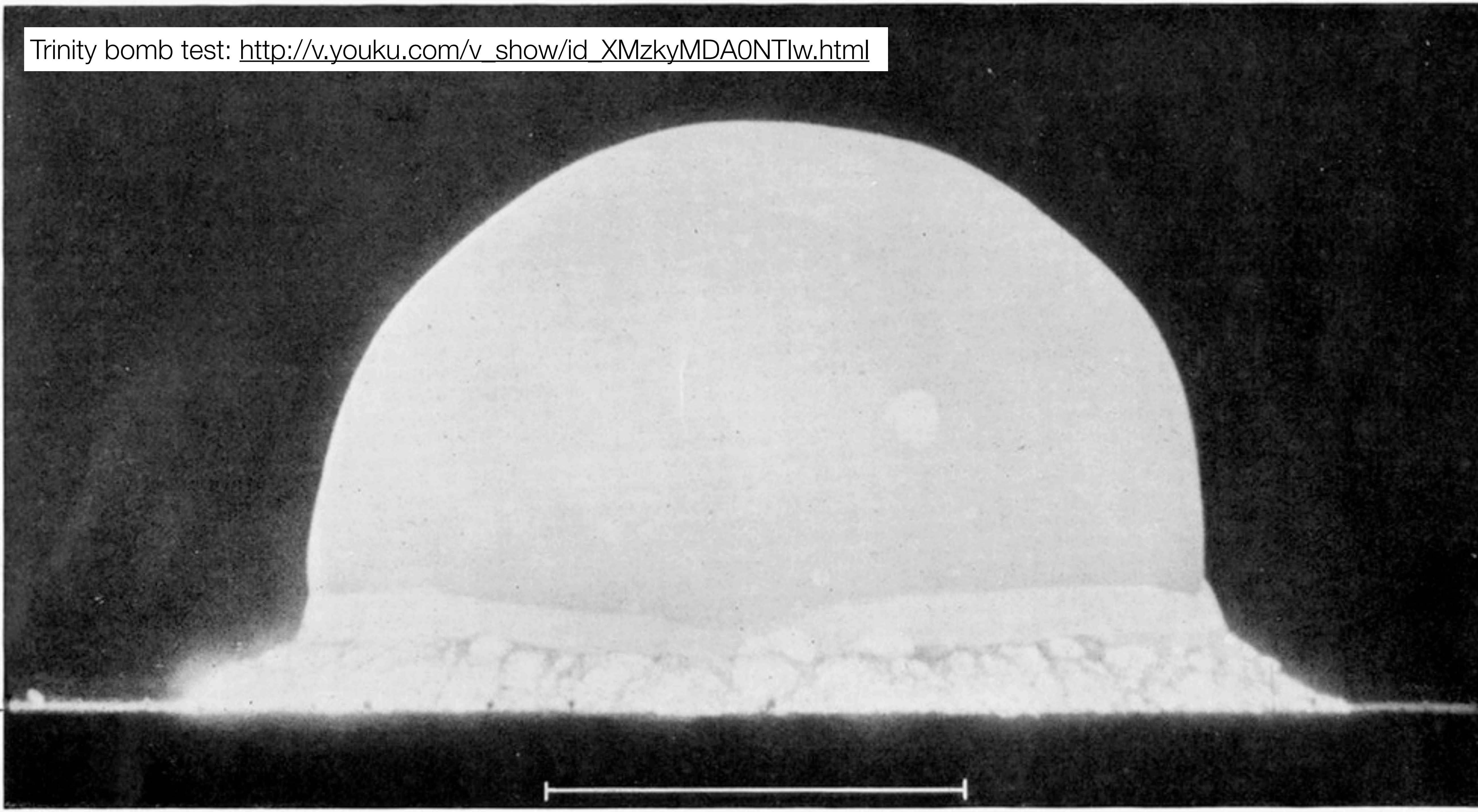

FIGURE 7. The ball of fire at $t = 15$ msec., showing the sharpness of its edge.

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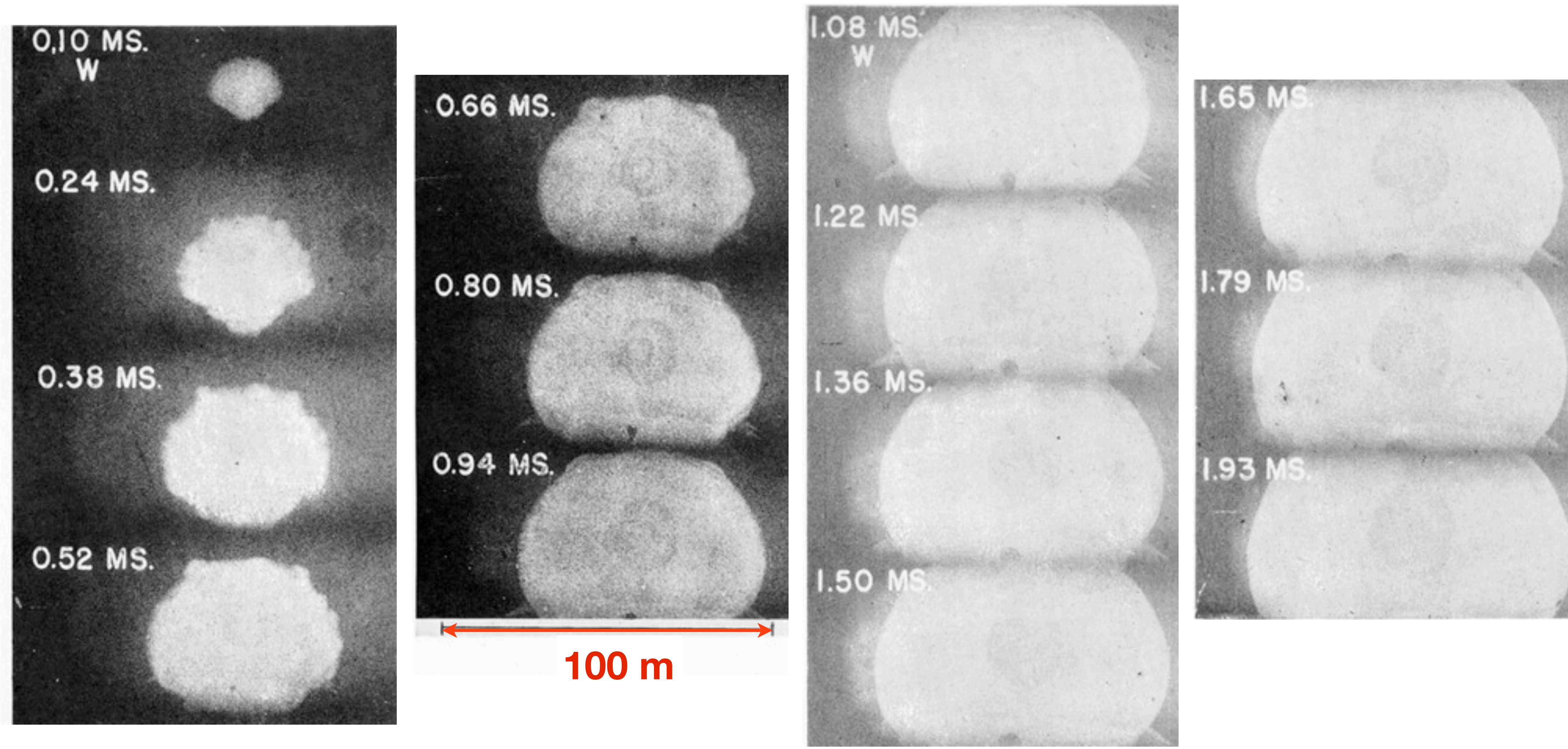
Trinity Atomic Bomb (1945)

1947年解密的原子弹鲍照火球随时间的系列照片，爆炸当量保密



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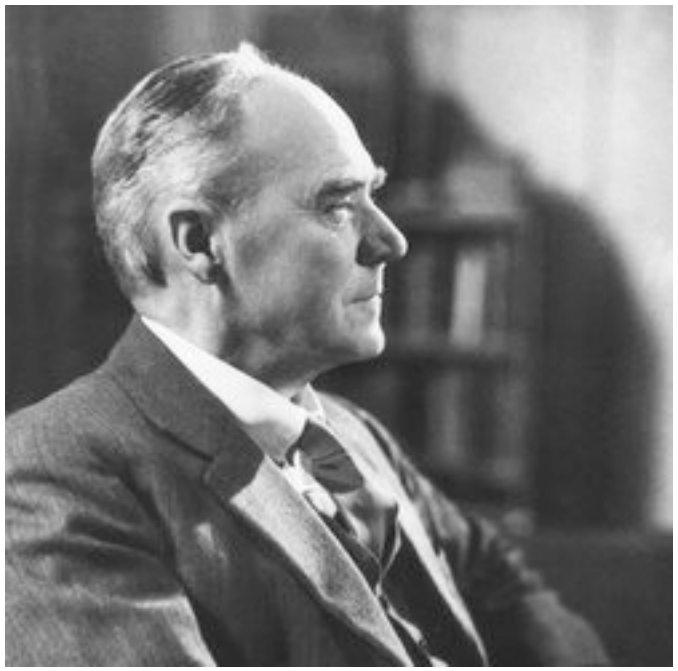
Trinity Atomic Bomb (1945)



Sir Geoffrey Ingram Taylor
(1886 - 1975)

$$\Pi_1 = \overbrace{R}^{\text{火球半径}}\left(\frac{\overbrace{\rho}^{\text{空气密度}}}{\underbrace{E}_{\text{爆炸能量}}\underbrace{t^2}_{\text{爆炸时间}}}\right)^{1/5} \quad \text{and}\Pi_2 = \overbrace{p}^{\text{空气压强}}\left(\frac{t^6}{E^2\rho^3}\right)^{1/5}$$

$$R \approx \overbrace{k}^{\sim 1}\left(\frac{Et^2}{\rho}\right)^{1/5} \Rightarrow$$

$$\log R \propto \log t^{2/5} = \frac{2}{5}\log t$$

$$E \sim \rho\frac{R^5}{t^2}$$

Taylor, G. The Formation of a Blast Wave by a Very Intense Explosion. II. The Atomic Explosion of 1945. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* **201**, 175 -186 (1950).

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Trinity Atomic Bomb (1945)

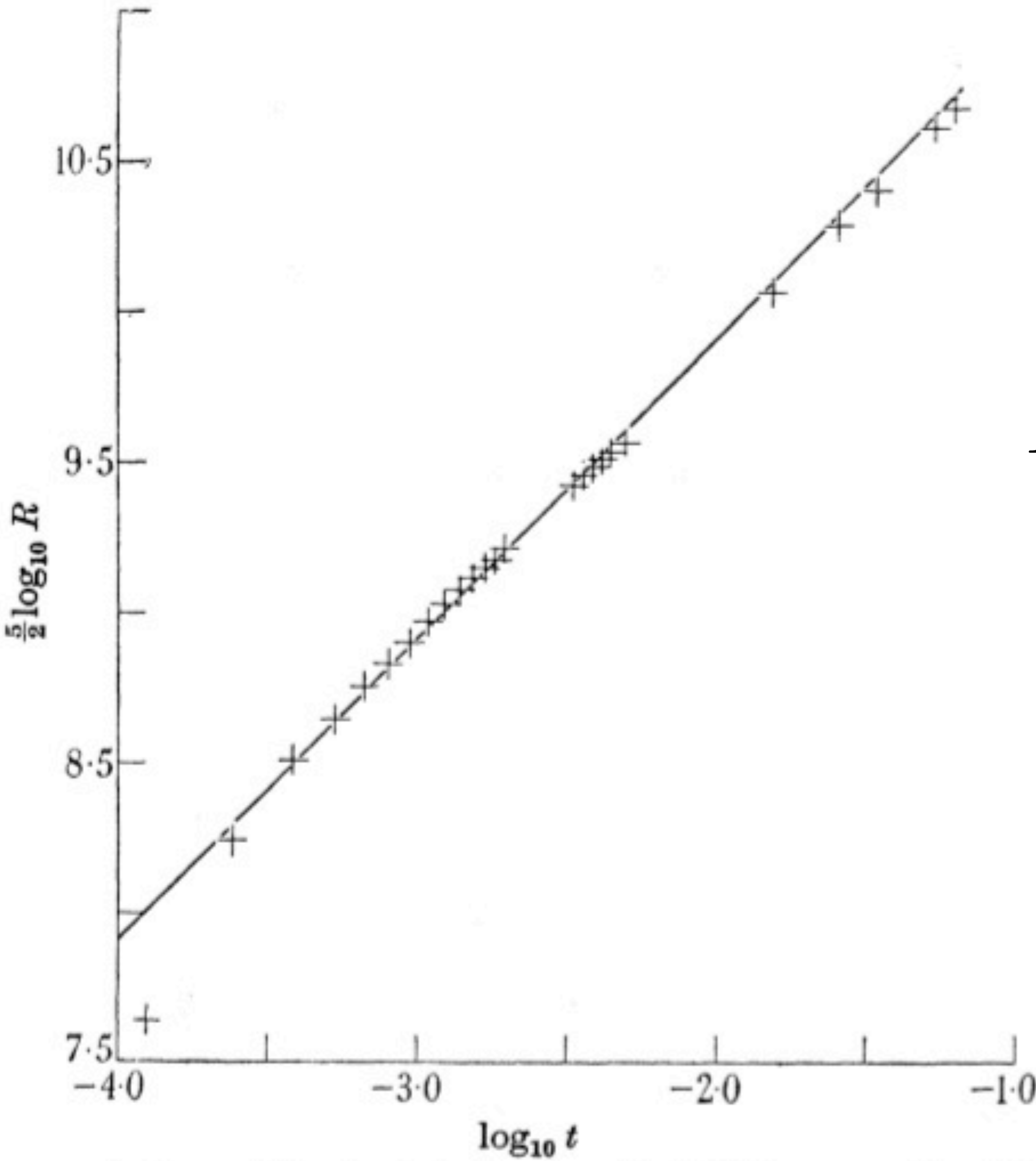

FIGURE 1. Logarithmic plot showing that $R^{\frac{5}{2}}$ is proportional to t.

$$\frac{R^5}{t^2} \approx 7 \times 10^{13}\ m^5/s^2$$

$$\rho_{\text{air}} = 1.25\ \text{kg/m}^3$$

$$E \sim 10^{14}\ \text{J} = 21\ \text{kton TNT}$$

$$1\ \text{ton TNT} = 4.18 \times 10^9\text{J}$$

$$\log R \propto \log t^{2/5} = \frac{2}{5} \log t$$

$$E \sim \rho \frac{R^5}{t^2}$$