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Backstory

Physical principle used in reliability

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Summary

An effective approach based on the principle of maximum entropy is developed to analyze reliability in systems with dynamics of electric circuits and infectious diseases like coronavirus disease 2019 (COVID-19).

Human beings do not live forever due to aging processes, diseases, or accidents. Similarly, with the increase in mileage, vehicles will gradually wear out or damage. More generalized, both biological organisms and mechanical systems will lose some fundamental functions at different times with some randomness. Analyzing the expected duration of time until failure is called survival analysis or reliability theory, an interdisciplinary subject which is of high interest in many branches of scientific research. Reliability theory focuses on quantities including both the probability that a system can survive past a certain time (called survival function) and the probability that a surviving system might fail thereafter (called hazard rate function, which can be derived from the survival function). The estimation of survival function or hazard rate function is important for predicting the lifetime of systems while limited observation data often increases the difficulty of estimation, especially when people know little about the evolution mechanism of systems.

Recently, a work by Du, Ma, Wei, Guan and Sun at China Academy of Engineering Physics and Beijing Computational Science Research Center (Du et al., 2020) introduces an innovative method to estimate the hazard rate function from the perspective of physics. They adopted the principle of maximum entropy (MaxEnt) expounded by Jaynes in 1956 (Jaynes, 1956), which means the probability distribution matching a system best with stochastic nature should have the largest entropy. Entropy is a fundamental quantity in thermodynamics and statistical mechanics, and the concept has been extended to other disciplines such as Shannon entropy (Shannon, 1948) in information theory and Kolmogorov-Sinai entropy (Kolmogorov, 1958) in dynamical system theory since entropy can measure the information or complexity of a system. In particular, MaxEnt bridges statistical mechanics and information theory, revealing that the entropy used in the two fields are basically the same (Jaynes, 1956). MaxEnt is commonly applied to inferential problems in statistics. For example, the covariance matrix adaptation evolution strategy (Hansen and Ostermeier, 1996), a widely used random search algorithm in numerical optimization, is established on the basis of the MaxEnt.

The work by Du et al. (Du et al., 2020) uncovers that this MaxEnt method is similar to the least-action principle in Lagrangian mechanics, and estimating the most probable hazard rate is just like formulating the equation of motion (the Euler-Lagrange equation). Taking experimental data (which can be time-varying) and some prior knowledge as constraints, the MaxEnt means solving an optimization problem with Lagrange multipliers to make the objective function (entropy) achieve the largest value. This approach can induce many typical hazard rate curves and also can be applied to both single-function systems and multifunction systems. For example, the authors scrutinized a two-lamp circuit model (shown in Figure 1), which is a double-function system. The factor driving the lamps to failure here is heat generation of the two wires. It is obvious that the two lamps are correlated since Lamp 2 cannot work when Lamp 1 fails. So the system is irreducible. In this case, the hazard rate function is a matrix with off-diagonal terms representing the interaction between the two lamps. The estimated data echo well with the actual simulation data.

Later, in another work, Du and Sun used the developed method based on the MaxEnt to determine the parameters in susceptible-infective-exposed-removed model for epidemic disease dynamics of COVID-19 (Du and Sun, 2020). The hazard rate function here is related to death rate, incidence rate, and healing rate. Traditional approaches focus on overall data like the total number of cases to establish all kinds of

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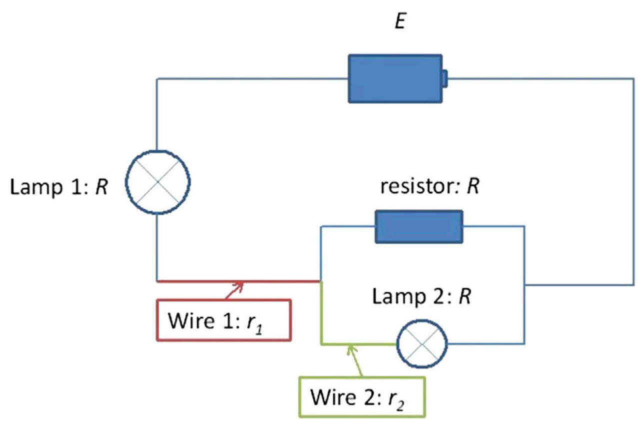

Figure 1. Schematic graph showing the two-lamp circuit model

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dynamical models and would have to phenomenologically handle a parameter-fitting problem of deterministic equations. However, along this way, a large amount of data is required to get explicit estimations. The approach developed in (Du and Sun, 2020) could connect the statistics of small amounts of cases to the phenomenological parameters in the modeling and utilizes the MaxEnt to solve the maximum likelihood probability distributions. It incorporates the elements of incomplete information into the model through statistical inference and does not depend on the choice of specific dynamical models. In addition, since people usually have only limited knowledge about emerging infectious diseases and can only get a small number of samples at an early stage, the MaxEnt approach is suitable for practical scenarios. To one's interest, this approach can help to reveal typical phenomena in epidemic transmission, such as the tending-to-zero infectious ratio in steady states when the healing period is much smaller than the duration of immunity.

By using the MaxEnt to link statistical inference and statistical mechanics and showing the analogy to solving Euler-Lagrange equations in analytical mechanics, these theoretical works offer a more physical interpretation to the traditional problem of reliability. It is very attractive that this effective approach works well in distinct subject areas of nature, from electric transport to disease transmission, being useful even without enough historical data. Outlooking its future, one direction would be to combine this microscopic perspective with macroscopic statistical data for epidemic disease dynamics (Du and Sun, 2020). It may also help researchers tackle problems within other aspects of reliability theory, such as tracking animal migration and risk analysis in finance and economics.

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