Reply to "Comment on the use of the method of images for calculating electromagnetic responses of interacting spheres"

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In this Reply, we use the data offered in the Comment by Markel [Phys. Rev. E. **72**, 023401 (2005)] to investigate the electrorotation and dielectrophoretic spectra of dielectric particles such as biological cells discussed in our two papers [Phys. Rev. E. **65**, 021401 (2002); **69**, 051402 (2004)]. We find that the numerical results predicted by our method and Markel's method are in good agreement. Therefore, this further shows the validity of our approximation method for dielectric particles and that the Comment by Markel does not affect the significance of the two papers.

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Markel's Comment [1] is nothing but a presentation of a more accurate numerical calculation for two approaching homogeneous (only) spherical particles. It gives no new insight into the physical phenomena at hand. Below, we give a detailed account and also point out some inaccuracies in the Comment.

In our two papers [2,3] (which were commented by Markel [1]), we have discussed ac electrokinetics of two touching particles by using the multiple image method. However, in his Comment Markel sticks to extinction spectra only. It is worth noting that the focus of our two papers is on ac electrokinetics, and the good agreement (Figs. 1 and 2) between our method and the exact method further shows that our results and statements in the two papers are both correct and reasonable for ac electrokinetics of dielectric particles, e.g., biological cells. Therefore, the Comment does not affect the significance of the two papers.

For the extinction spectra, Markel focused on the small-frequency limit for conductors and expressed the dielectric constant as $4\pi i\sigma/\omega$, where σ is the static conductivity. He focused on $0.5\times10^{15}\sim1.0\times10^{15}$ Hz (optical frequencies), as derived from his Fig. 3. However, in our consideration [2,3], the real parts, ϵ_1 and ϵ_2, of $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ [here $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ denote the complex dielectric constant of the particles and the host medium, respectively, and $\tilde{\epsilon}=\epsilon+\sigma/(2\pi if)$] should be added, in order to discuss the ac electrokinetics of our interest for frequency $f<10^9$ Hz (beyond optical frequencies). In this case, the (complex) material parameter $s=1/(1-\tilde{\epsilon}_1/\tilde{\epsilon}_2)$ has always a negative real part since both $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ have positive real parts. Therefore, $s-s_n$ will never vanish. As long as the sum rule $\Sigma_n F_n$ is obeyed, our method should work well. That is shown in Figs. 1 and 2. Here s_n and F_n are, respectively, the poles and residues arising from the spectral representation.

Markel showed numerical results for F_n versus s_n for several $L/R=1.2,\ldots,1.01$. We can take one step forward and use Markel's numerical results to calculate the dielectrophoretic and electrorotation spectra [2,3] for these cases and to compare with our approximate method for the same values of L/R (Figs. 1 and 2). For our calculations of electrorotation spectra, we need the corresponding F_n and s_n for both longitudinal and transverse field cases. However, the values in Markel's Fig. 2 are only for the longitudinal field cases. In the absence of the (exact) transverse spectral representation (TSR) data, in Fig. 2, the TSR results were extracted from the method of multipole images by HYG. This is reasonable in the sense that the HYG TSR is more accurate than the HYG LSR (longitudinal spectral representation) when respectively compared to the corresponding exact data.

It is evident from the comparison (Figs. 1 and 2) that our approximate method yields good results. In fact, an excellent agreement can be obtained at frequencies higher than 10^6 Hz even at a very close encounter, $L/R=1.01$. There is, however, some deviation at lower frequencies, but usually the ac responses at lower frequencies can be influenced by numerous other factors. This demonstrates numerically that our method of multiple images has predicted physical results in our previous two papers [2,3] where ac electrokinetics have been investigated. Thus, the repetition of the calculations by the exact numerical method of Markel only gives some quantitative improvement, but not qualitative improvement or new insight.

It is quite well known that in a rotating electric field, the induced dipole moment will align two touching particles in the plane of the applied rotating field. Thus, random orientation is only a transient effect, and the angular averages $\langle\sin^2\theta\rangle=\langle\cos^2\theta\rangle=1/2$ are always in the steady state. In his Comment [1], Markel assumed that the electric field vector is three-dimensional.

Furthermore, in our paper [3], we have focused on *inhomogeneous* (graded) particles. However, Markel's formulas can apparently only be applied to *homogeneous* spherical particles, at least thus far.

In the Comment [1], Markel claimed that in Ref. [3] we presented the multipole image method "as exact and used

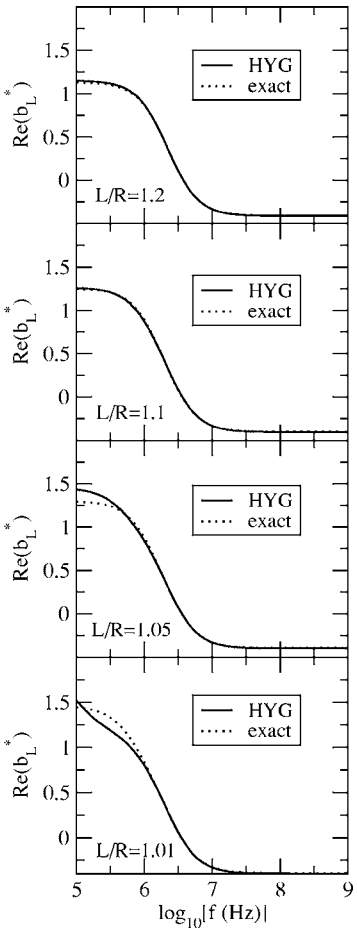

FIG. 1. Comparison between the method of multiple images (HYG) and the exact method (exact). We investigate the dielectrophoretic spectrum given by $\mathrm{Re}(b_L^*)$ for different L/R. Here $\mathrm{Re}(\cdots)$ means the real part of $\cdots$, and b_L^* the effective dipole factor for a pair of touching spherical particles in longitudinal fields. Parameters: $\sigma_1=2.548\times10^{-2}$ S/m, $\epsilon_1=14.858\epsilon_0$, $\sigma_2=2.8\times10^{-4}$ S/m, and $\epsilon_2=80\epsilon_0$.

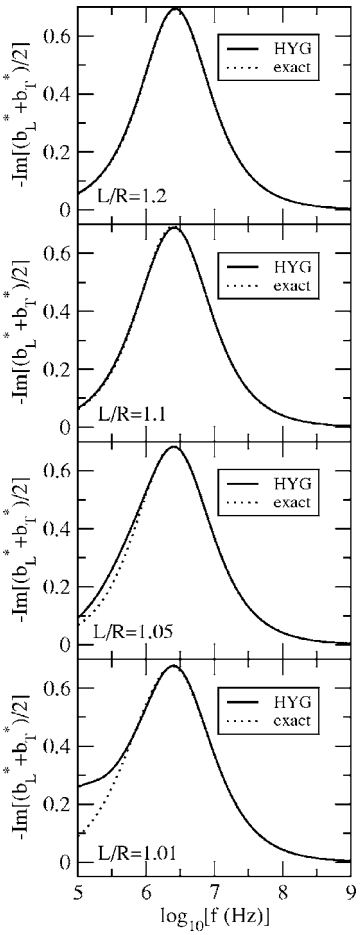

FIG. 2. Same as Fig. 1. But we investigate the electrorotation spectrum given by $-\mathrm{Im}[(b_L^*+b_T^*)/2]$. Here $\mathrm{Im}(\cdots)$ means the imaginary part of $\cdots$, and b_T^* the HYG effective dipole factor for a pair of touching spherical particles in transverse fields.

without restriction." Our method is certainly an approximate theory. The paper [3] actually cited our previous one [2] where the theory was already said to be approximate.

In our papers [2,3], we realized that there is possibly an L/R value below which numerical calculations become divergent. However, at that time, we did not know the actual value explicity. That is not a surprise as our method is only approximate.

Finally, Markel addressed that "More general analytical results can be obtained with the use of the theory of hypercomplex variables (a generalization of the conformal mapping)." This was referenced to Ref. 12 in Markel's comment (which is Ref. [4] in this Reply). Before actually solving the problem, we are not convinced which method is indeed valid for obtaining more accurate or general results, the method based on bispherical coordinates [5] or the theory of hypercomplex variables [4]. Thus far, the results predicted by the former are still lacking for two dielectric particles. In addition, its validity has been debated [6,7].

In summary, the Comment by Markel only provided a numerical approach to an already well-studied problem. In this Reply we have clarified some confusions raised by the Comment and pointed out some inaccuracies in it.

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