

Heterogeneous preferences, decision-making capacity, and phase transitions in a complex adaptive system

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There has been a belief that with the directing power of the market, the efficient state of a resource-allocating system can eventually be reached even in a case where the resource is distributed in a biased way. To mimic the realistic huge system for the resource allocation, we designed and conducted a series of economic experiments. From the experiments we found that efficient allocation can be realized despite a lack of communications among the participants or any instructions to them. To explain the underlying mechanism, an extended minority game model called the market-directed resource allocation game (MDRAG) is constructed by introducing heterogeneous preferences into the strategy-building procedures. MDRAG can produce results in good agreement with the experiments. We investigated the influence of agents' decision-making capacity on the system behavior and the phase structure of the MDRAG model as well. A number of phase transitions are identified in the system. In the critical region, we found that the overall system will behave in an efficient, stable, and unpredictable mode in which the market's invisible hand can fully play its role.

biased distribution | invisible hand | economic experiments | minority game | resource allocation

Most of the social, economic, and biological systems involving a large number of interacting agents can be regarded as complex adaptive systems (CAS) (1), because they are characterized by a high degree of adaptive capacities to the changing environment. The interesting dynamics and phase behaviors of these systems have attracted much interest among physical scientists. A number of microscopic CAS models have been proposed (2–6), among which the minority game (MG) (7–9) becomes a representative model. Along with the progress in the research of econophysics (10), MG has been mostly applied to simulate one kind of CAS, namely the stock market (11, 12). Alternatively, MG can also be interpreted as a multiagent system competing for a limited resource (13, 14) that distributes equally in 2 rooms. However, agents in the real world often have to face a competition to the limited resource, which distributes in different places in a biased manner. Examples of such phenomena include companies competing among markets of different sizes (15), drivers selecting different traffic routes (16), people betting on horse racing with the odds of winning a prize, and making decisions on which night to go to which bar (17).

From a global point of view, the ideal evolution of a resource-allocating system would be the following: Although each agent would compete against others only with a self-serving purpose, the system as a whole could eventually reach a harmonic balanced state where the allocation of resource is efficient, stable, and arbitrage-free (which means that no one can benefit from the “misdistribution” of the resource). Note that during the process of evolution to this state, agents could neither have been told about the actual amount of the resources in a specific place nor could they have any direct and full communications, just as if there were an “invisible hand” directing them to cooperate with each other. Then, does this invisible hand always work? In practice, there is plenty of evidence that the invisible hand does

have very strong directing power in places such as financial markets, although sometimes it does fail to work. Such temporary ineffectiveness implies that there must be some basic conditions required for the invisible hand to exert its full power. Through an experimental study and a numerical study with a market-directed resource allocation game (MDRAG, which is an extended version of the MG model), we found that agents equipped with heterogeneous preferences as well as a decision-making capacity that matches with the environmental complexity are sufficient for the spontaneous realization of such a harmonic balanced state.

To illustrate the system behavior, we designed and conducted a series of economic experiments, in collaboration with university students. In the experiments, 89 students from different (mainly physics, mathematics, and economics) departments of Fudan University were recruited and randomly divided into 7 groups (Groups A–G, see Tables 1–4). The number of students in each group was just set for convenience and denoted by N in Tables 1–4. In the games played in the experiments, students were told that they had to make a choice among a number of rooms, in each round of a session, for sharing the different amounts of virtual money in different rooms. Students who got more than the global average, namely those belonging to the relative minority, would win the payoff. At the beginning of a session, participants were told the number of rooms (2 or 3) and in some cases, the different but fixed amount of virtual money in each room. In the following, M_i is used to denote the amount of virtual money in room i . A piece of global information about the payoff in the preceding round in all rooms is announced before a new round starts. In each round, the students must make their own choices without any kind of communication. The payoff per round for a student in room i is 2 points if $M_i/N_i > \sum M_i/N$, and -1 point otherwise. Here, N_i is the number of the students choosing room i . The total payoff of a student is the sum of payoffs of all rounds, which will be converted to money payoffs in Renminbi (RMB, or Chinese yuan) with a fixed exchange rate. Because the organizational and statistic procedures were done by a human, one session of 10 rounds took ≈ 20 min. More details can be found in the leaflet to the experiments in *Appendix*.

Three kinds of games, GAME-I, GAME-II, and GAME-III, have been investigated. GAME-II differs from GAME-I in the global information being announced. In GAME-I, both the resource distribution M_i and the current population N_i in room i were announced, whereas only payoffs (2 or -1) in each room of the current round were conveyed to players in GAME-II. Note that the environmental complexity was increased in GAME-II, because to win the game, players would have to predict other players' decisions, and, in the meantime, infer the actual amount of virtual money in different rooms. In GAME-III, the global information is the same as that of GAME-II,

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Table 1. Results of GAME-I

| Session | Group | Round | M_1 | M_2 | M_3 | $\langle N_1 \rangle$ | $\langle N_2 \rangle$ | $\langle N_3 \rangle$ |
|---------|----------------|---------|-------|-------|-------|-----------------------|-----------------------|-----------------------|
| 1 | A ($N = 12$) | 1 to 10 | 3 | 2 | 1 | 5.3 | 4.6 | 2.1 |
| 2 | A ($N = 12$) | 1 to 10 | 3 | 2 | 1 | 5.5 | 3.8 | 2.7 |
| 3 | B ($N = 12$) | 1 to 10 | 3 | 2 | 1 | 5.5 | 4 | 2.5 |
| 4 | C ($N = 24$) | 1 to 20 | 3 | 2 | 1 | 12.2 | 7.4 | 4.4 |
| 5 | D ($N = 10$) | 1 to 10 | 5 | 3 | - | 6.1 | 3.9 | - |
| 6 | D ($N = 10$) | 1 to 10 | 3 | 1 | - | 7.4 | 2.6 | - |

except that an abrupt change of amount of virtual money is introduced during the play of the game without an announcement. (On the contrary, all of the participants have already been told that each M_i is fixed.) No further information was given to the participants.

Results of 6 sessions of GAME-I, 4 of GAME-II, and 1 of GAME-III are given in Tables 1–4. In Table 1, the results of GAME-I are listed, where the time average of the player number in room i is represented as $\langle N_i \rangle$. As the data show, a kind of cooperation seems to emerge in the game within 10 rounds. In particular, ratios of $\langle N_i \rangle$ converge to the ratios of M_i , implying that the system becomes efficient in delivering the resource even if it was distributed in a biased way. To the players, no room is better or worse in the long run; there is also no evidence that any of them could systematically beat the resource allocation “market.” One might naively think that the system could evolve to this state only because the participants knew the resource distribution before the play of the games and the population in each room during the play. However, results of GAME-II show that this explanation could not be correct. As shown in Table 2, although players who know neither the resource distribution nor the current populations in different rooms seem not to be able to adapt to the unknown environment during the first 10 or 15 rounds, eventually the relation $\langle N_1 \rangle / \langle N_2 \rangle \approx M_1 / M_2$ is achieved again in groups C and F. For instance, Table 3 shows the track through which group F gradually found the balanced state under the environmental complexity $M_1 / M_2 = 3$. Furthermore, the results of GAME-III support the conclusion of GAME-II, in which the system can reach this state even with an abrupt change of the unknown resource distribution during the play of the game, see the results of 21 to 45 rounds played by the G group in Table 4. It is surprising that players can “cooperate” even without direct communications or information about the resource distribution. We can define the source of a force that drives the players to get their quota evenly as the “invisible hand” of the resource-allocation market. In the sequel, however, we shall show that the effectiveness of this invisible hand relates to the heterogeneous preference and the adequate decision-making capacity of the participants of the game.

Model

To find out the mechanism behind this adaptive system of resource distribution, 2 multiagent models are used, and their results are compared with each other. The first model is the traditional MG, whereas the second one is an extended MG

Table 2. Results of GAME-II

| Session | Group | Round | M_1 | M_2 | $\langle N_1 \rangle$ | $\langle N_2 \rangle$ |
|---------|----------------|----------|-------|-------|-----------------------|-----------------------|
| 1 | D ($N = 10$) | 1 to 10 | 2 | 1 | 6.2 | 3.8 |
| 2 | E ($N = 10$) | 1 to 10 | 1 | 3 | 3.3 | 6.7 |
| 3 | F ($N = 11$) | 1 to 10 | 3 | 1 | 7.2 | 3.8 |
| 3 | F ($N = 11$) | 11 to 20 | 3 | 1 | 8.3 | 2.7 |
| 4 | C ($N = 24$) | 1 to 15 | 7 | 1 | 17.8 | 6.2 |
| 4 | C ($N = 24$) | 16 to 30 | 7 | 1 | 21.1 | 2.9 |

Table 3. Track of 11 players in Group F ($N = 11$) converging to $M_1 / M_2 = 3$

| Round | N_1 | N_2 | Round | N_1 | N_2 |
|-------|-------|-------|-------|-------|-------|
| 1 | 5 | 6 | 11 | 8 | 3 |
| 2 | 9 | 2 | 12 | 10 | 1 |
| 3 | 4 | 7 | 13 | 9 | 2 |
| 4 | 6 | 5 | 14 | 7 | 4 |
| 5 | 6 | 5 | 15 | 9 | 2 |
| 6 | 7 | 4 | 16 | 7 | 4 |
| 7 | 7 | 4 | 17 | 7 | 4 |
| 8 | 8 | 3 | 18 | 9 | 2 |
| 9 | 10 | 1 | 19 | 8 | 3 |
| 10 | 10 | 1 | 20 | 9 | 2 |

called a MDRAG. MG and MDRAG have a common framework: There are N agents who repeatedly join a resource-allocation market. The amounts of resource in 2 rooms are M_1 and M_2 . Before the game starts, each agent will choose S strategies to help him/her make a decision in each round of play. The strategy used in MG and MDRAG is typically a choice table that consists of 2 columns, as shown in Table 5. The left column is for the P possible economic situations, and the right side is for the corresponding room number, namely room 0 or room 1. Thus, if the current situation is known, an agent should immediately choose to enter the corresponding room. With a given P , there are totally 2^P different strategies. At each time step, based on a randomly given exogenous* economic state (18), each agent chooses between the 2 rooms with the help of the prediction of his/her best-scored strategy. After everyone has made a decision, agents in the same room will share the resource in it. Agents who earn more than the global average $(M_1 + M_2) / N$ become the winners, and the room that they entered is denoted as the winning room. To a strategy in the game, a unit of score would be added if it had given a prediction of the winning room, no matter whether it was actually used or not.

On the other hand, MDRAG differs from MG in the strategy-building procedures. In traditional MG, agents “randomly” choose S strategies from the strategy pool of 2^P size. Here, randomly means that each element of the right column of a strategy table is filled in with 0 or 1 equiprobably. By using this method, strategies of different preferences will have a binomial distribution. Here, the preference of a strategy is defined as the tendency or probability with which a specific room will be chosen when the strategy is activated. For a large P , the numbers of 0 and 1 in the right column are nearly equal. Hence, globally there would be no preference difference among agents who uniformly pick up these strategies. In MDRAG, however, we use another method to fill the strategy table to introduce heterogeneous preferences to the agents. First, K ($0 \leq K \leq P$), denoting the number of 0s in the right column, is randomly selected from the $P + 1$ integers. In other words, strategies with different preferences (different values of K) are chosen equiprobably from the strategy pool. Second, each element of the strategy’s right column should be filled in by 0 with the probability K/P and by 1 with the probability $(P - K)/P$. It is clear that a strategy with an all-zero right column can be picked with the probability $1/(P + 1)$ in MDRAG, whereas this could happen only with a probability of $1/2^P$ in the traditional MG and could practically never be chosen by any MG agents if $NS \ll 2^P$.

To make descriptions easier to understand, explanations of the model parameters are provided. The ratio M_1 / M_2 represents the

*The alternative is the use of endogenous binary history of the game results as the economic situations. We have confirmed that there would be no change in the simulation results.

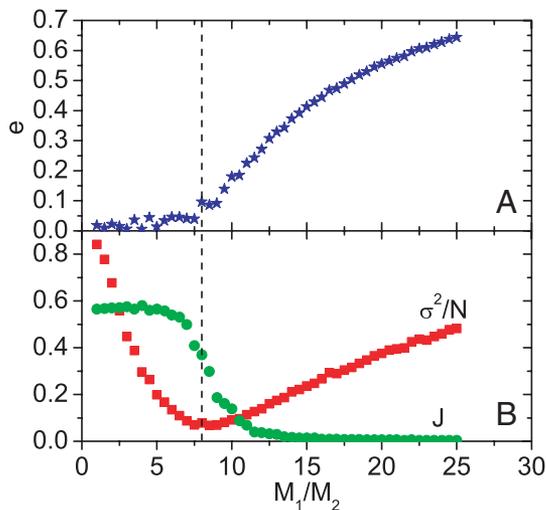


Fig. 2. The “ M_1/M_2 phase transition” in MDRAG, for $N = 100$, $P = 64$, and $S = 8$. Simulations are run 300 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). The dashed line denotes $M_1/M_2 = 8$. (A) e as a function of M_1/M_2 . (B) σ^2/N and J versus M_1/M_2 , respectively.

that describe system behaviors in 3 aspects, namely, efficiency, stability, and predictability. First the efficiency of resource allocation can be described as $e = |\langle N_1 \rangle / \langle N_2 \rangle - M_1/M_2| / (M_1/M_2)$. Note that $0 \leq e < 1$ and a smaller e means a higher efficiency in the allocation of the resource. The stability of a system can be described by $\sigma^2/N \equiv (1/2N) \sum_{i=1}^2 \langle (N_i - \bar{N}_i)^2 \rangle$, which denotes the population fluctuation away from the optimal state.[†] Here, $\bar{N}_i = M_i N / \sum M_i$, and $\langle A \rangle$ is the average of time series A_t . The predictability is related to $H \equiv (1/2NP) \sum_{\mu=1}^P \sum_{i=1}^2 \langle (N_i - \bar{N}_i | \mu)^2 \rangle$, in which $\langle A | \mu \rangle$ is the conditional average of A_t , given that $\mu_t = \mu$, one of the P possible economic situations. If $\sigma^2/N \neq H$, it means that agents may take different actions at different times for the same economic situation (namely, the market behavior is unpredictable). For clarity, we describe the predictability of system by defining $J = 1 - HN/\sigma^2$. It is obvious that $0 \leq J < 1$ and a smaller J means a higher predictability.

The variation of system behavior along with the change of environmental complexity M_1/M_2 is shown in Fig. 2. As shown in Fig. 2A, the system changes from an efficient state into an inefficient state at some critical value $(M_1/M_2)_c \approx S$. For other values of P , the system behavior stays the same as long as P is larger than M_1/M_2 . In Fig. 2B, around the same critical value of M_1/M_2 , σ^2/N changes from a decreasing function to an increasing function, giving the smallest fluctuation in the population distribution at the critical point. Meanwhile, the order parameter J also falls into zero at $(M_1/M_2)_c$, suggesting that a phase transition, named the “ M_1/M_2 phase transition,” occurs at this critical point. To be more illustrative, when the environmental complexity is much smaller than the critical value, the system could reside in an efficient, unpredictable, but relatively unstable state. Getting closer to the phase transition point, the stability of system will be improved until the most stable state is reached. Then, after crossing the critical point, the decision-making capability of the whole system has been exhausted, and it will fall into an inefficient, predictable, and unstable state. At the vicinity of the critical point, as if participants of the game worried about being eliminated from the competition, the market inspires all of its

[†]Large fluctuations in populations can cause a higher dissipation in the system. Hence, an efficient and stable state means an optimal state with a low waste in the resource allocation.

guiding potential and leads the system to the ideal state for the resource allocation, a state that is both efficient and stable and where no unfair arbitrage chance can exist.

It is important to know that MDRAG and MG have totally different phase structures, which could be analyzed by comparing the $S - P$ contours of the descriptive parameters for the 2 models, see Fig. 3. From the analysis, we could also know how the decision-making capacity influence the overall performance of the resource allocation system, in case the environmental complexity is fixed ($M_1/M_2 = 4$). Features of the contour maps (Fig. 3) are summarized as the following (different M_1/M_2 s do not change the conclusions):

(i) Compared with the traditional MG as a whole, MDRAG has a much wider range of parameters for the availability of the efficient, stable, and unpredictable states. In particular, there is almost no eligible region in Fig. 3A if we take the criterion of efficiency as $e < 0.08$. Also, the predictable region ($J < 0.02$) in Fig. 3F is much smaller than that of the MG’s results in Fig. 3C. These facts indicate that MDRAG has a much better performance than MG as a resource-allocating system.

(ii) Patterns of the contour maps suggest that MG and MDRAG have totally different dependency on parameters. Fig. 3A–C indicates that P and S are not independent in the traditional MG model, which confirms the previous findings (19). On the other hand, in MDRAG, there is always a region where the system behavior is almost controlled by the parameter S . In Fig. 3D, for large enough P , the system can reach the efficient state if S exceeds a critical value S_p , where S_p will converge to the limit value M_1/M_2 with increasing P . For $S < S_p$, the system can never reach the efficient state no matter how P changes. For a very large P and $S < M_1/M_2$, it can be proved that the probability for agents to enter the richer room is $S/(S + 1)$, so that the system stays in the inefficient states ($\langle N_1 \rangle / \langle N_2 \rangle < M_1/M_2$).

(iii) Observing Fig. 3D and F, one may find both an “ S phase transition” and a “ P phase transition.” As mentioned above, for large enough P , the increase of S can abruptly bring the system from the inefficient/predictable phase to the efficient/unpredictable phase, and it is named S phase transition. On the other hand, in the narrow region where $S < M_1/M_2$, the increase of P can also produce of a drastic change from the unpredictable phase to the predictable phase, and it is named P phase transition. The existence of the S phase transition can be explained by the fact that the number of available choices in decision making is a key factor for agents to find the right choice from strategies with an adequate heterogeneity of preferences. But for $S \gg P$, it will also cause a slight decrease of the efficiency because of the conflicts of the different predictions from the equally good strategies with the same preference. This explains why MDRAG, $S = 48$ performs worse than MDRAG, $S = 8$, when $P = 16$ in Fig. 1. The P phase transition reflects that for some incompetent ability of choice deliberation, a critical value of the cognition ability can enhance the decision-making capacity to match the environmental complexity.

(iv) It is also noteworthy that the parameter $\alpha = 2\mu/N$, which is the main control parameter in the MG model (8, 9), no longer controls on the behavior of the MDRAG system. Varying N while keeping M_1/M_2 as a constant, the basic feature, especially the critical position of the contour maps, will remain unchanged.

In aspects of the competition for resources, the feed of global information, and the inductive optimization of strategies, both MG and MDRAG may be regarded as eligible models for the economic experiments. However, MG fails to reproduce the experimental results in most cases. By simply accommodating a broader preference distribution of the strategies, MDRAG fits the experimental results without any coordinating capability of the agents. This enables us to comment on the possible mech-

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