



A transformation theory for camouflaging arbitrary heat sources

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ABSTRACT

Camouflage devices have attracted intensive research interest for their significant applications. However, most camouflage devices are specifically designed according to target heat sources. Here, by applying the transformation thermotics approach, we develop a coordinate transformation, and design an unspecific camouflage device which can camouflage arbitrary heat sources into a circular one with an anisotropic shell. We verify the ability of our unspecific camouflage device with both steady and transient simulations. We also find the “apparent negative thermal conductivity” under certain conditions without violating the second law of thermodynamics. To ensure the feasibility, we further put forward the effective medium approximation for sample fabrication, and only two natural materials are required. Our results have relevance to the different applications of infrared misleading, uniform heating, and so on; they may also provide guidance to the research on other diffusive fields, such as magnetostatic and electrostatic fields.

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1. Introduction

The past few years have witnessed the prosperity and development of thermal camouflage, such as invisibility [1–16], illusion [17–21], etc. However, camouflage devices are rarely designed for heat sources. As a matter of fact, heat sources exist almost everywhere in nature, and the temperature profiles around heat sources play a crucial role in many aspects. For example, snakes hunt through detecting the temperature distributions of the preys (namely heat sources); people can move freely at night by using infrared detector to know the existence of heat sources; even there is a type of missile which attacks the targets through tracing heat sources. So camouflaging heat sources has significant research value and vast potential applications for human life, industrial production, and even military misleading.

Fortunately, recent research has made some breakthrough by considering the heat sources [21–26] and designed some specific camouflage devices. However, once these camouflage devices are fabricated, the functions of them are also determined. In other words, if we change the characteristics of the heat sources (or other conditions), these camouflage devices will not work again.

Such restriction limits the potential value of the camouflage devices.

So, an unspecific camouflage device is urgent to be studied; see Fig. 1. If a soldier [Fig. 1(a1)] or any other heat sources is detected by the infrared camera, he will be found due to the specific temperature profile [Fig. 1(a2)]. Now, if the soldier is wrapped up in an unspecific camouflage device [Fig. 1(b1)], he will be regarded as a panda [Fig. 1(b2)], for the temperature profile of a soldier with camouflage device exactly looks like a real panda [Fig. 1(c1, c2)].

Here, we successfully design such an unspecific camouflage device which can disguise arbitrary heat sources into a circular one with an anisotropic shell. Namely, if one detects the temperature profiles of the heat sources, they will always look like a circular one, and he/she will not get any useful information. We design the unspecific camouflage device through the transformation thermotics approach and verify its ability through both steady and transient simulations. We also give the design scheme with the effective medium approximation, and the results show that the camouflage device works indeed. Such unspecific camouflage device has potential applications in infrared misleading and uniform heating due to the character of circular heat sources. We expect this work may arouse the continuous research interest on the existing thermal metamaterials [27–33] through considering heat sources.

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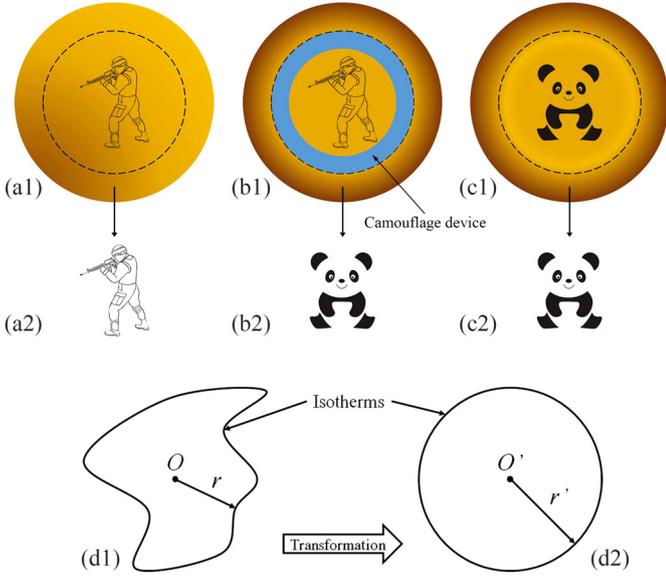


Fig. 1. Concept of camouflaging arbitrary heat sources. (a1–c1) shows the thermal field of a soldier, a soldier with camouflage device, and a panda, respectively. When the thermal fields are detected (a2–c2) by infrared camera, (b2) and (c2) are totally the same. So, a soldier is camouflaged into a panda. The transformation is intuitively presented in (d1, d2).

2. Theory

The temperature profiles vary across heat sources, and hence one can derive the detailed information of them through detecting the temperature profiles. Here we expect to achieve the same temperature profile regardless of the different heat sources. Let us proceed as follows.

The governing equation of heat transfer is form-invariant under coordinate transformation. The thermal conductivity tensor κ' transformed from coordinate $X(x_1, x_2, x_3)$ to $X'(x'_1, x'_2, x'_3)$ is known to have the form as

$$\kappa' = \frac{J\kappa_0 J^t}{\det(J)}, \quad (1)$$

where κ_0 is the original thermal conductivity scalar, J^t is the transposed form of Jacobian transformation matrix J , and $\det(J)$ is the determinant of J .

We define a key parameter Λ to describe the extent of irregularity of one isotherm

$$\Lambda = \frac{r_{max} - r_{min}}{\langle r \rangle}, \quad (2)$$

where $\langle r \rangle = C/2\pi$ is the average radius of the isotherm, C is the perimeter of the isotherm, and r_{max} (or r_{min}) is the maximum (or minimum) radius of the isotherm. Clearly, Λ can reflect whether the shape of the isotherm looks like a circle. The larger Λ is, the more irregular the shape is. Especially, when $\Lambda = 0$, the isotherm is exactly a circle. For our purpose, it is required to ensure Λ to be small enough (~ 0) after transformation. We resort to the linear transformation in a two-dimensional polar coordinate system

$$\begin{cases} r' = \alpha r + \beta, \\ \theta' = \theta, \end{cases} \quad (3)$$

where α and β are two constants which remain to be discussed. The essence of the transformation is a space transformation, for the isotherms are spatially distributed. The transformation may be more understandable by considering the isotherms; see

Fig. 1(d1, d2). Then we check the value of Λ after transformation

$$\begin{aligned} \Lambda' &= \frac{r'_{max} - r'_{min}}{\langle r' \rangle} = \frac{(\alpha r_{max} + \beta) - (\alpha r_{min} + \beta)}{\langle \alpha r + \beta \rangle} \\ &= \frac{\alpha (r_{max} - r_{min})}{\langle \alpha r + \beta \rangle} = \frac{\alpha}{\alpha + \delta} \Lambda, \end{aligned} \quad (4)$$

where $\delta = \beta/\langle r \rangle$.

To camouflage heat sources, Λ' is expected to be zero. Since Λ may not be zero, $\alpha/(\alpha + \delta)$ should be zero. Therefore, as long as α is far smaller than δ , we can camouflage heat sources.

We calculate the Jacobian transformation matrix with Eq. (3)

$$J = \begin{pmatrix} \alpha & 0 \\ 0 & \frac{\alpha r + \delta(r)}{r} \end{pmatrix}, \quad (5)$$

and then we derive the thermal conductivity with Eq. (1)

$$\kappa' = \begin{pmatrix} \frac{\alpha r}{\alpha r + \delta(r)} & 0 \\ 0 & \frac{\alpha r + \delta(r)}{\alpha r} \end{pmatrix} \kappa_0 \approx \begin{pmatrix} \alpha/\delta & 0 \\ 0 & \delta/\alpha \end{pmatrix} \kappa_0, \quad (6)$$

where α is required to be far smaller than δ to ensure $\Lambda' \approx 0$, and we approximately regard $\langle r \rangle \approx r$.

Further, we divide Eq. (6) by α/δ to make the radial component to be κ_0 , which can further make the isotherm distribution in the camouflage device look like a circular one

$$\kappa'' \equiv \frac{\kappa'}{\alpha/\delta} = \begin{pmatrix} 1 & 0 \\ 0 & \delta^2/\alpha^2 \end{pmatrix} \kappa_0, \quad (7)$$

where $\delta^2/\alpha^2 \gg 1$. Without the operation from Eq. (6) to Eq. (7), the isotherms will concentrate in the camouflage device, and κ'' denotes the thermal conductivity tensor of the material required for constructing our camouflage device.

To be mentioned, the designed camouflage device can be applied for camouflaging arbitrary heat sources into a circular one with an anisotropic shell. Without loss of generality, we consider camouflaging two linear heat sources represented by constant temperatures.

3. Simulation results

We perform finite-element simulations based on the commercial software COMSOL Multiphysics (<http://www.comsol.com/>) to verify the above theory. The stable simulation results are presented in Fig. 2.

There are two heat sources with different temperature in the middle of Fig. 2(a1, b1). We can detect the temperature profile [Fig. 2(a1)] to obtain the information of the heat sources [Fig. 2(a2)]. Fig. 2(b1) shows the temperature profile with our designed camouflage device, and the shape of isotherms outside the black dashed line is a set of concentric circles which exactly looks like the distribution in Fig. 2(c1). Without loss of generality, we plot the concrete temperature data [Fig. 2(d)] on the brown dashed line in Fig. 2(a1–c1) for intuitive comparison. Therefore, no one can uncover the detailed information about the heat sources in Fig. 2(b1). The only information we can obtain is that there is a circular heat source in the center [Fig. 2(b2)], and we can know nothing about the rest. So the heat sources are camouflaged as expected. To be mentioned, red line agrees well with blue line not only outside the device [denoted by “out” in Fig. 2(d)], but also in the device region [denoted by “device” in Fig. 2(d)]. The operation from Eq. (6) to Eq. (7) contributes to the effect.

We also find the “apparent negative thermal conductivity” in such a system presented in Fig. 2(b1). We know that the direction of heat flow is along the negative gradient of temperature, which

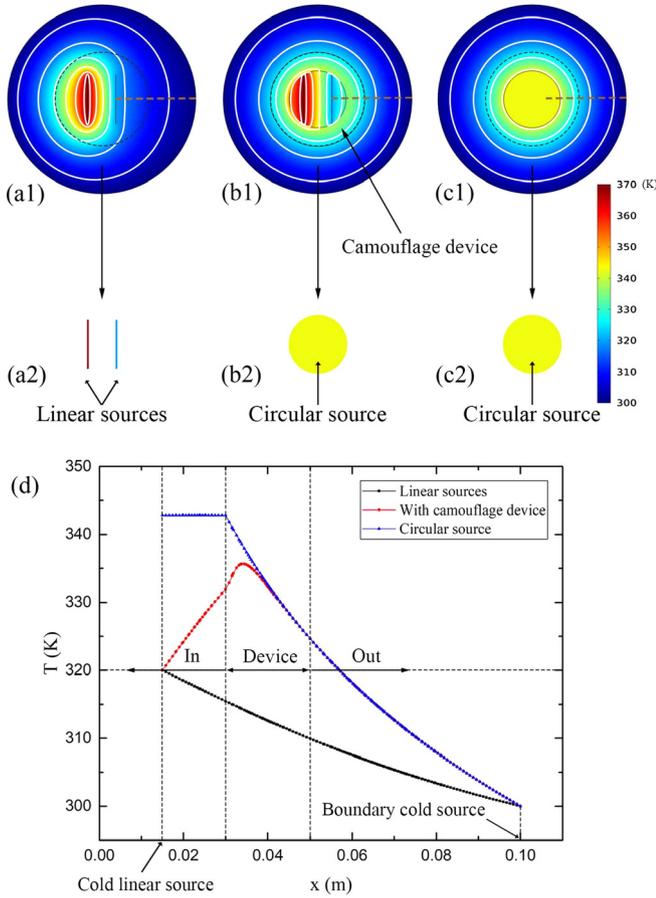


Fig. 2. Steady two-dimensional simulation results of the camouflage device. In (a1–c1), the simulation size is a circle with radius 10 cm. Except the camouflage device (namely an anisotropic shell), the thermal conductivities are all 1 W/(m·K). The boundary cold source is 300 K. In (a1), two linear heat sources are in the middle of the thermal field. One is in the position (–1.5, 0) with temperature 370 K and length 5.2 cm, and the other is in the position (1.5, 0) with temperature 320 K and length 5.2 cm. (a2) is basically the same as (a1). The only difference is that the designed camouflage device is applied outside the two linear sources. The inner radius of the camouflage device is 0.3 cm, and the outer one is 0.5 cm. The thermal conductivity is designed according to Eq. (7), where $\kappa_0 = 1$ W/(m·K) and $\delta^2/\alpha^2 = 200$. In (c1), there is a circular heat source with radius 1.5 cm and temperature 343 K in the middle of thermal field. When thermal fields [outside the black dashed lines in (a1–c1)] of the heat sources are detected by infrared camera, the results correspond to (a2–c2), respectively. We also measure the concrete temperature on the brown dashed lines in (a1–c1) to verify the ability of the camouflage device, and the result is shown in (d). (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

means temperature decreases from high temperature to low temperature. However, in our system, the temperature rises first then decreases, which behaves like negative thermal conductivity. The data (red line) is clearly shown in Fig. 2(d). Of course, the bizarre phenomenon does not violate the second law of thermodynamics, and can be explained. We know that materials with negative thermal conductivity do not exist in nature, but such negative thermal conductivity can be equivalent to an extra heat source [34]. In our system, the left linear heat source with temperature 370 K just acts as an extra heat source, compared with the right linear heat source with temperature 320 K, so there is a temperature rise inside the camouflage device. Thus, the phenomenon is called “apparent negative thermal conductivity”, because negative thermal conductivity actually does not exist in our system. To be mentioned, the two linear sources are both called heat sources in this work because the true cold source is on the outer boundary with temperature 300 K.

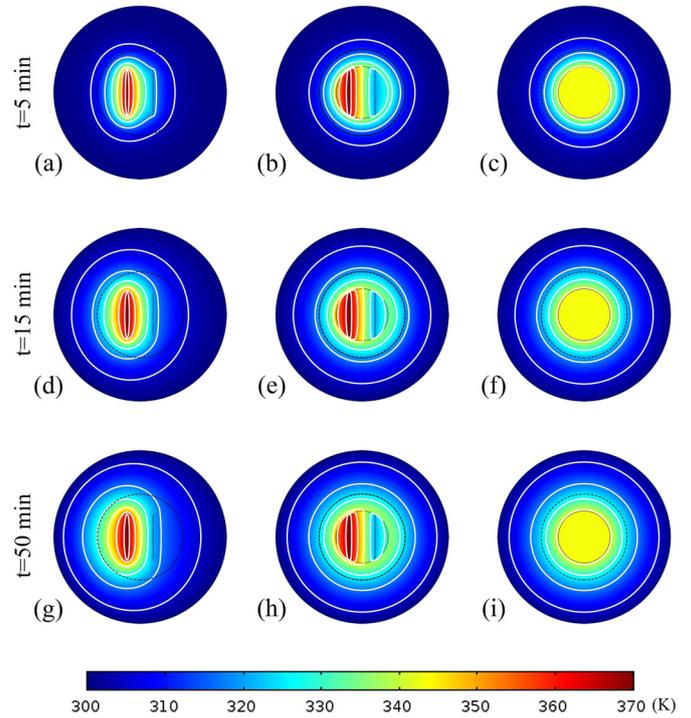


Fig. 3. Transient two-dimensional simulation results of the camouflage device. The material parameters and boundary conditions of the three columns are the same as those in Fig. 2(a1–c1), respectively. The density and heat capacity of all the regions are 1000 kg/m³ and 1000 J/(kg·K), respectively. The initial temperature is 300 K. It takes about 50 min to reach the steady state, and the results are given for $t = 5, 15, 50$ min.

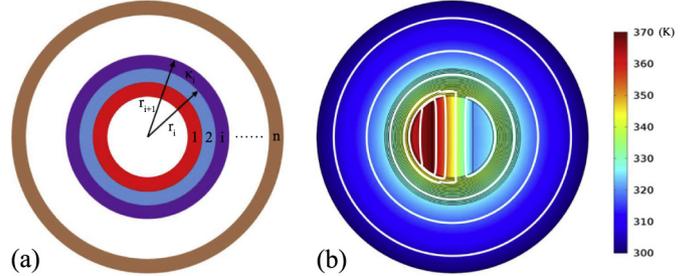


Fig. 4. Simulation result with effective medium approximation. (a) shows the schematic diagram. (b) is the steady simulation result with 20 layers composed of two natural materials with thermal conductivities 0.5 W/(m·K) and 400 W/(m·K), which approximately make $\kappa_0 = 1$ W/(m·K) and $\delta^2/\alpha^2 = 200$ in Eq. (7). The boundary conditions are the same as that in Fig. 2(b1).

We further verify the transient performance of our camouflage device in Fig. 3. The temperature profile at $t = 5, 15, 50$ min are simulated. The diffusion of the isotherms with camouflage device [Fig. 3(b, e, h)] is also a set of concentric circles, which shows the same character as that of a circular heat source [Fig. 3(c, f, i)]. Therefore, the detailed information of the heat sources [Fig. 3(a, d, g)] is camouflaged. So the camouflage device also performs well for transient situations.

4. A design scheme based on effective medium approximation

For practical applications, here we propose the effective medium approximation for an n -layer composite in the two-dimensional polar coordinate system; see Fig. 4(a).

The radial temperature distribution in the i -th layer is $T_i = A_i \ln r + B_i$, where A_i and B_i are two constants. The temperature difference in this layer is $\Delta T_i = A_i \ln \frac{r_{i+1}}{r_i}$. For the total heat flux

J is conserved, $J = -2\pi A_i \kappa_i$ is a constant, where κ_i is the thermal conductivity of the i -th layer. Considering all the n layers, the total temperature difference is $\Delta T = \sum_i \Delta T_i = \sum_i \frac{-J}{2\pi \kappa_i} \ln \frac{r_{i+1}}{r_i} = -\frac{J}{2\pi \kappa_{rr}} \ln \frac{r_{n+1}}{r_1}$, where κ_{rr} is the radial effective thermal conductivity. So we have

$$\kappa_{rr} = \frac{1}{\sum_i \frac{1}{\kappa_i} \ln \frac{r_{i+1}}{r_i}} \ln \frac{r_{n+1}}{r_1}. \quad (8)$$

Then we exert a tangential thermal gradient to consider the tangential thermal conductivity. The heat flux in the i -th layer is $J_i = \frac{-\kappa_i |\nabla T|}{2\pi} \ln \frac{r_{i+1}}{r_i}$. Therefore, the total heat flux is $J = \sum_i J_i = \sum_i \frac{-\kappa_i |\nabla T|}{2\pi} \ln \frac{r_{i+1}}{r_i} = \frac{-\kappa_{\theta\theta} |\nabla T|}{2\pi} \ln \frac{r_{n+1}}{r_1}$, where $\kappa_{\theta\theta}$ is the tangential effective thermal conductivity. So we obtain

$$\kappa_{\theta\theta} = \frac{\sum_i \kappa_i \ln \frac{r_{i+1}}{r_i}}{\ln \frac{r_{n+1}}{r_1}}. \quad (9)$$

Considering Eqs. (8) and (9), we can use only two materials with thermal conductivity 0.5 W/(m·K) and 400 W/(m·K) to realize the anisotropic thermal conductivity of the camouflage device as displayed in Fig. 2(b1). The simulation result is presented in Fig. 4(b) which agrees well with Fig. 2(b1). The designed composite can be used for uniformly heating with arbitrary heat sources, owing to the circular isotherms outside the composite. It can also be used for protecting the inside heat sources from being detected by infrared camera.

5. Conclusion

We have developed a coordinate transformation based on transformation thermotics approach, and then designed an unspecific camouflage device which can disguise arbitrary heat sources into a circular one with an anisotropic shell. Without loss of generality, we discuss the camouflage of two linear heat sources. The ability of our camouflage device is verified by both steady and transient simulations. We have also found “apparent negative thermal conductivity” under certain conditions without violating the second law of thermodynamics. To enhance the feasibility of fabrication, we have further put forward the effective medium approximation to give the design scheme with two uniform and isotropic materials. For the character of a circular heat source, our camouflage device has potential applications in infrared misleading, uniform heating, etc. This work may arouse further research interest on heat sources, and similar research can also be extended to other diffusive fields, such as magnetostatic and electrostatic fields.

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