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An agent-based model of stock markets incorporating momentum investors

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\textbf{HIGHLIGHTS}

\begin{itemize}
  \item We propose a simple agent-based model of trading incorporating momentum investors.
  \item The model is able to reproduce some of the stylized facts observed in real markets.
  \item We show that real market data can be used to constrain the model parameters.
  \item The model parameters provide information on the momentum behavior in real markets.
\end{itemize}

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\textbf{ABSTRACT}

It has been widely accepted that there exist investors who adopt momentum strategies in real stock markets. Understanding the momentum behavior is of both academic and practical importance. For this purpose, we propose and study a simple agent-based model of trading incorporating momentum investors and random investors. The random investors trade randomly all the time. The momentum investors could be idle, buying or selling, and they decide on their action by implementing an action threshold that assesses the most recent price movement. The model is able to reproduce some of the stylized facts observed in real markets, including the fat-tails in returns, weak long-term correlation and scaling behavior in the kurtosis of returns. An analytic treatment of the model relates the model parameters to several quantities that can be extracted from real data sets. To illustrate how the model can be applied, we show that real market data can be used to constrain the model parameters, which in turn provide information on the behavior of momentum investors in different markets.

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1. Introduction

Momentum strategy in stock markets refers to buying stocks that have high returns over a recent period of time and selling those with poor returns. Traders who adopt a momentum strategy are referred to as momentum investors. Jegadeesh and Titman \cite{1} reported an average return of 1% per month using the momentum strategy in 1993. Their work attracted much attention on the effects of momentum strategy in stock markets \cite{2–5}. In fact, studies on the use of momentum strategy dated back earlier. Grinblatt and Titman \cite{6} found that a majority of mutual funds in the United States (US) tended to buy stocks that had scored positive returns over 3 months. Grinblatt, Titman and Wermers \cite{7} found that 77% of US mutual funds were momentum investors. Chen, Jegadeesh and Wermers also found that US funds, in aggregate, tended to buy stocks that outperformed those they sell \cite{8}. Thus, understanding the momentum strategy in mature and emerging markets has both academic and practical values.

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In the present work, we propose and study an agent-based model incorporating momentum investors. Agent-based modeling, in which a system is modeled as a collection of autonomous decision-making agents, has been proved to be a powerful method for simulating human systems [9–16]. Some of the existing agent-based models have incorporated momentum investors as one type of the trading agents [17,18]. However, these researchers mainly focused on simulating the real markets via agent-based modeling. Here, we focus on investigating the behavior of momentum investors, which may offer a hint on how to understand the momentum behavior on the basis of agent-based modeling.

In our model, we consider a population consisting of two types of agents: random investors and momentum investors, although there may be other types of investors, such as fundamentalists, noise traders, and contrarians in real markets [19]. The random investors trade all the time randomly. The momentum investors implement an action threshold to assess the most recent movement in the price and decide on their actions which could be idle, buying or selling. The time series of a stock price generated by the model shows some of the well-known stylized facts observed in real markets, including volatility clustering [20,21], fat-tail in the returns [22–24], weak long-term correlation [25,26] and scaling behavior in the kurtosis [27,28].

We construct a multi-agent model consisting of $N_r$ ($N_r \gg 1$) random investors who trade randomly and $N_m$ momentum investors who adopt a momentum strategy in trading. The agents join a virtual market repeatedly in which they trade only on one stock and they can only trade a single unit of stock at a time step when they decided to do so. At a time step, say $(t - 1)$, every random investor decides to buy or to sell randomly, and every momentum investor decides to buy, to sell, or to remain inactive. The difference between the number of agents buying and selling creates an excess demand $D(t - 1)$ that drives the stock price to move from $p(t - 1)$ to $p(t)$ via [27,34]:

$$
\ln p(t) = \ln p(t - 1) + D(t - 1).
$$

The momentum strategy of a momentum investor is implemented by referring to the excess demand and thus the price movement at the previous time step. Every momentum investor $i$ carries an action threshold $\lambda_i$ with $\lambda_i > 0$ [34]. If $D(t - 1)/\sigma \geq \lambda_i$, he/she will buy; if $D(t - 1)/\sigma \leq -\lambda_i$, he/she will sell. The agent will remain inactive if $|D(t - 1)/\sigma| < \lambda_i$, here, $\sigma$ is taken to be $\sqrt{N_r}$ $\equiv \sigma_r$ so that the excess demand is assessed in units of the spread solely caused by the random investors. Theoretically, the time series of excess demand in the model gives a Markov process, in which the excess demand at time $t + 1$ depends only on that at time $t$. Thus, for given $N_r$, $N_m$ and $\lambda_i$ ($i = 1, 2, \ldots, N_m$), a time series of the stock price can be generated from an initial price $P_0$. It is noted that a larger population of random agents would lead to a larger fluctuation in the price, as the price is generated by the difference between supply and demand. However, by comparing the excess demand with $\sigma$, which also increases with the number of random agents, the triggering of the momentum investors to enter or leave the market is expected to remain the same.

In what follows, we assume a common action threshold, i.e., $\lambda_i = \lambda$ for all momentum investors for simplicity. It is, sometimes, convenient to write $\lambda$ as:

$$
\lambda = \frac{\tilde{\lambda}}{\sqrt{N_r}}.
$$

with $\tilde{\lambda} > 0$ so that the criterion for activating a momentum investor becomes that of comparing the excess demand $D$ with $\tilde{\lambda}$. The number of momentum investors $N_m$ can also be expressed in units of $\sigma_r = \sqrt{N_r}$ as:

$$
N_m = \alpha \sqrt{N_r},
$$

with $\alpha > 0$. Giving $N_r$, $\alpha$ and $\lambda$ for the model is equivalent to specifying $N_r$, $N_m$ and $\tilde{\lambda}$. The plan of the paper is as follows. In Section 2, the model is defined in detail. In Section 3, we implement the model numerically and show that it gives some of the stylized facts in real markets. In Section 4, we provide an analytic treatment and illustrate how the model can be applied to real markets by using real market data sets to fix the model parameters. The work is summarized in Section 5.
Fig. 1. (a) A simulated time series of stock prices and its log-returns (inset), and (b) the probability density function of the log-returns (dots). The solid line (red curve) in (b) represents a normal distribution with the same mean and standard deviation. Parameters: $N_r = 100$, $\alpha = 3$ (i.e. $N_m = 30$), and $\lambda = 4$ (thus $\tilde{\lambda} = 40$). The simulation lasted for 60,000 time steps. The results are shown for returns observed by the time differences in $\Delta t = 20$ time steps. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3. Main features in numerical results

Fig. 1(a) shows a time series of a stock price obtained by simulations in a system with $N_r = 100$, $\alpha = 3$ (thus $N_m = 30$), and $\lambda = 4$ (thus $\tilde{\lambda} = 40$). The inset shows the log-return series obtained by using $\Delta t = 20$ time steps. The corresponding probability density function (PDF) of the log-returns is shown in Fig. 1(b), which shows fat-tails when compared with a normal distribution. Our agent-based model is thus capable of reproducing the fat-tail behavior, which has been frequently observed in empirical studies [23,24] on real market data. Here, we have to clarify that it is hard to know how many simulation steps should be taken to model the distribution of returns in real markets. However, we believe that observed fat-tail behavior in the distribution of returns should refer to $\Delta t \gg 1$ steps where the results are quite similar to real markets.

The model can also give other features observed in real market data. We analyzed the statistics of price returns using different values of the time delay $\Delta t$. For each value of $\Delta t$, a probability density function for the price return such as that in Fig. 1(b) can be constructed. The standard deviation and the kurtosis of the PDF can then be evaluated. Results are shown in Fig. 2 (see Fig. 2(a) and (c)) in a log–log scale. It is noted that no error bars are given in both Figs. 1 and 2, since we are not taking the average over several time series. The standard deviation increases with $\Delta t$ as a power law for large $\Delta t$ with an exponent of $\approx 0.53$, indicating a weak long-term correlation. For the kurtosis, the results show a non-monotonic dependence on $\Delta t$. For large $\Delta t$, the kurtosis decreases with $\Delta t$ as a power law. Interestingly, these behaviors are also observed in data of real markets [25–28]. As an example, we analyzed the time series of the Shanghai Stock Exchange Composite Index from January 2000 to December 2000 with data at 1 min intervals. The standard deviation and kurtosis of the returns are shown in Fig. 2(b) and (d), respectively. Here we focus on the variation trend of the statistics as $\Delta t$ increases, thus the absolute values of $\Delta t$ in real markets are not so important. The standard deviation also shows a weak long-term correlation with a power law of exponent $\approx 0.54$, similar to that in Fig. 2(a). It is noted that in both the real market and our model, a super-diffusive behavior occurs in the region of small $\Delta t$. The cross-over from super-diffusive to diffusive behavior has been found at the time interval of $\Delta t \approx 20$ steps and $\Delta t \approx 10$ min in Fig. 2(a) and (b), respectively. For the kurtosis, the Shanghai Index shows a monotonically decreasing behavior. For large $\Delta t$, there exists a range of nearly two decades with a power law decreasing behavior similar to that in Fig. 2(c). The same monotonic behavior was also reported by Cont et al. for high frequency data of S & P500 Index future prices and a power law behavior between the kurtosis and $\Delta t$ was revealed [28].
The non-monotonic behavior of kurtosis in our model can be understood as follows. Due to the behavior of the momentum investors, a large positive return tends to be followed by another large positive return. Therefore, for short time intervals $\Delta t$, this gives a fatter tail in the PDF of positive returns and yields an increasing kurtosis with $\Delta t$. However, for long time intervals $\Delta t$, the kurtosis decreases with $\Delta t$ as a result of the central limit theorem.

As shown above, our model is able to reproduce some of the well-known stylized facts. It should also be noted that, in its simplest form, the present model fails to reproduce some other stylized facts, such as power-law distribution of returns \([35-38]\), long-range correlations in volatility \([37,38]\) and the property of no memory. It is believed that, with some extensions of the present model, more stylized facts might be reproduced. However, we would like to emphasize that the main focus of the present model is to investigate the role of momentum investors. Thus, to reproduce all the stylized facts is outside the scope of the present model.

4. Analytic treatment and fixing parameters by data sets

In the model, for momentum investors with a common action threshold, they act collectively, i.e., either they are all inactive or all active with the same action. This simplification allows us to analyze our model further. Suppose that at time $t$, the momentum investors are all inactive. With only the random investors trading, the statistics of the excess demands $D$ is formally represented by

$$P(D) = \left(\frac{1}{2}\right)^{N_r} \binom{N_r}{D},$$

where $D = 2n - N_r$, with $n = 0, 1, \ldots, N_r - 1, N_r$, and thus $D = -N_r, -N_r + 2, \ldots, N_r - 2, N_r$. Here, $P(D)$ is the probability of having an excess demand $D$. In terms of $P(D)$, the switching probability $P_{inact\rightarrow act}$ that the momentum investors turn active at time $t + 1$ is

$$P_{inact\rightarrow act} = \sum_{D=-N_r}^{-\tilde{\lambda}} P(D) + \sum_{D=\tilde{\lambda}}^{N_r} P(D),$$

as the criterion $|D(t)| \geq \tilde{\lambda}\sqrt{N_r} = \tilde{\lambda}$ triggers the inactive momentum investors into action.
For many purposes, it is sufficient to use a continuum description and to characterize the PDF of the excess demand by a normal distribution with a vanishing mean and a variance $\sigma^2$. More specifically, the PDF is modeled by a normal distribution $f(x; \mu, \sigma^2)$ is in general given by

$$f(x; \mu, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \quad (6)$$

In terms of $f(x; 0, N_r)$, the switching probability $P_{\text{inact}\to\text{act}}$ that the momentum investors turn active at time $t + 1$ is then given by

$$P_{\text{inact}\to\text{act}} = \int_{-\infty}^{-\lambda} f(x; 0, N_r) \, dx + \int_{\lambda}^{+\infty} f(x; 0, N_r) \, dx$$

$$= \int_{-\infty}^{-\lambda} f(x; 0, 1) \, dx + \int_{\lambda}^{+\infty} f(x; 0, 1) \, dx,$$  \quad (7)

where $f(x; 0, 1)$ in the last line is the standard normal distribution and $\lambda = \tilde{\lambda}/\sqrt{N_r}$.

Following a similar argument, the switching probability that active momentum investors turn inactive is given in the continuum description by

$$P_{\text{act}\to\text{inact}} = \frac{1}{2} \int_{-\lambda}^{0} f(x; N_m, N_r) \, dx + \frac{1}{2} \int_{0}^{\lambda} f(x; -N_m, N_r) \, dx$$

$$= \frac{1}{2} \int_{-\lambda}^{0} f(x; 0, 1) \, dx + \frac{1}{2} \int_{0}^{\lambda} f(x; 0, 1) \, dx.$$  \quad (8)

The first and second terms result from the situations when all the active momentum investors bought and sold the stock in the previous time step, respectively. Note that $P_{\text{inact}\to\text{act}}$ and $P_{\text{act}\to\text{inact}}$ depend on the action threshold $\lambda$ and the number of momentum investors $\alpha$.

The fractions of time $T_{\text{act}}$ and $T_{\text{inact}}$ that the momentum investors are in the active and inactive modes can be found by $P_{\text{inact}\to\text{act}}$ and $P_{\text{act}\to\text{inact}}$ as

$$T_{\text{act}} = \frac{P_{\text{inact}\to\text{act}}}{P_{\text{inact}\to\text{act}} + P_{\text{act}\to\text{inact}}},$$

$$T_{\text{inact}} = \frac{P_{\text{act}\to\text{inact}}}{P_{\text{inact}\to\text{act}} + P_{\text{act}\to\text{inact}}},$$ \quad (9)

analogous to the analysis of a two-level system at dynamical equilibrium.

Fig. 3(a) shows the dependence of $T_{\text{act}}$ on the number of momentum investors $\alpha$ and the action threshold $\lambda$. As expected, a low threshold tends to activate the momentum investors. It is especially the case when many momentum investors trade in a background of a minority of random traders where the behavior of the random traders is sufficient to lead to price movements that keep the momentum investors active.

The kurtosis is another quantity that can be expressed in terms of model parameters. The distribution of price returns for $\Delta t = 1$ can be regarded as a superposition of distributions of excess demands in the presence and absence of the momentum investors, weighted by $T_{\text{act}}$ and $T_{\text{inact}}$. The kurtosis of the return distribution can then be calculated to give

$$\kappa = \frac{(\alpha^4 + 6\alpha^2 + 3) T_{\text{act}} + 3 T_{\text{inact}}}{(\alpha^2 + 1) T_{\text{act}} + T_{\text{inact}}^2},$$ \quad (10)

which depends on the number of momentum investors $\alpha$ and the action threshold $\lambda$. Fig. 3(b) shows the kurtosis as a function of $\alpha$ and $\lambda$. Recall that the normal distribution has a kurtosis of 3. Comparing with Fig. 3(a), the region of $\kappa > 3$ corresponds to the case when the momentum investors only trade occasionally. In addition, a specific value of the kurtosis with $\kappa > 3$ defines a rather restrictive portion of the $\alpha$-$\lambda$ space. This feature is useful in determining a specific set of values $(\alpha, \lambda)$ for the two parameters in our model for different markets.

Another quantity that can be obtained from both our model and real market data is the probability $P_{\text{opp}}$ that two consecutive price returns are of opposite signs, i.e., when a positive return is followed by a negative return or a negative return is followed by a positive return. This quantity is related to the excess demands in two consecutive time steps, either $(+;-)$ or $(-;++)$ where the signs of the demands indicate the stock price went up (down) and then down (up). To obtain $P_{\text{opp}}$, it is more convenient to consider $1 - P_{\text{opp}}$ instead, corresponding to the probability that two time steps have $(++;)$ or $(-;-)$ for the excess demands. In the presence of momentum investors, it is necessary to consider their status, i.e., active or inactive, in two consecutive time steps. For example, the probability of having $(+;-)$ can be broken down into four cases labeled by $(\text{status}(t-1), +; \text{status}(t), +)$; where $\text{status}(t) = \text{active or inactive}$ labels the status of the momentum investors.
Fig. 3. Contours showing constant (a) $T_{\text{act}}$, (b) kurtosis $\kappa$, and (c) probability $P_{\text{opp}}$ in the $\alpha$-$\lambda$ space. The color bars show the values of the corresponding quantity at the contours. (d) Based on the monthly returns of the S & P 500 index between 1950 and 2011, $\kappa$ is found to be $\kappa = 5.45$ and it gives a constant-$\kappa$ contour, while $P_{\text{opp}}$ is found to be $P_{\text{opp}} = 0.46$ and it gives a constant-$P_{\text{opp}}$ contour. These contours are shown in (d) and they intersect at a point corresponding to $(\alpha, \lambda)_{S & P} = (3.7, 2.5)$.

Similarly, there are four cases for $(-,-)$ excess demands. Considering all these eight cases, we obtain an expression for $1 - P_{\text{opp}}$ as

$$1 - P_{\text{opp}} = T_{\text{inact}} (1 - P_{\text{inact}}) \cdot \frac{1}{2} + T_{\text{inact}} P_{\text{inact}} \cdot r + T_{\text{act}} P_{\text{act}} \cdot \frac{1}{2} + T_{\text{act}} (1 - P_{\text{act}}) \cdot r,$$

where

$$r = \int_{-\alpha}^{\infty} f(x; 0, 1) dx = \int_{0}^{\infty} f(x; \alpha, 1) dx$$

is a number close to unity. Fig. 3(c) shows $P_{\text{opp}}$ on the $\alpha$-$\lambda$ space. Again, a specific value of $P_{\text{opp}}$ defines a restrictive portion of the $\alpha$-$\lambda$ space. It should be noted that Fig. 3a, b, and c are all theoretical results, which are described by the accurately derived formulas. Thus, it is believed that the model simulation will give the same results when the number of simulation rounds is quite huge.

To illustrate how to apply our model to real markets, we refer to the data of monthly returns of the S & P 500 index between 1950 and 2011 in the US stock market. The S & P 500 represents the performance of 500 selected stocks traded in the New York Stock Exchange and NASDAQ and thus is representative of the US market. From the data, we found that the kurtosis $\kappa_{S & P} = 5.45$ and the probability $P_{\text{opp(S & P)}} = 0.46$. It is noted that the returns of stock indices have a convergence to Gaussian as the time horizon increases. However, the convergence is so slow that one still sees fat tails at the monthly horizon. The result of $\kappa_{S & P} = 5.45$ is considered to be consistent with the previous Refs. [39,40].

Making use of Fig. 3(b), $\kappa = 5.45$ defines a contour in the $\alpha$-$\lambda$ space as shown in Fig. 3(d). Similarly, $P_{\text{opp}} = 0.46$ defines a curve in $\alpha$-$\lambda$ space as shown in Fig. 3(d) by referring to Fig. 3(c). The contour and the curve intersect at a point with $(\alpha, \lambda)_{S & P} = (3.7, 2.5)$. It is an encouraging sign to see that the two curves actually intersect, as it indicates that $\kappa$ and $P_{\text{opp}}$ of a market define the parameters $\alpha$ and $\lambda$ in the model uniquely.

We have carried out similar analysis on the Shanghai Stock Exchange Composite Index that has a monthly kurtosis $\kappa_{SSE} = 14.4$ and $P_{\text{opp(SSE)}} = 0.44$. These values fix the parameters $(\alpha, \lambda)_{SSE} = (4.2, 2.7)$ in our model for the Shanghai market. Similarly, analyzing the Hong Kong Hang Seng Index and the Nikkei 225 Index gives $(\alpha, \lambda)_{HSI} = (3.8, 2.6)$ for the Hong Kong market and $(\alpha, \lambda)_{NII} = (3.8, 2.9)$ for the Japanese market, respectively.

It is interesting to note that the parameters $(\alpha, \lambda)$ for different markets are by-and-large similar. The values for S & P 500, Hang Seng Index and Nikkei 225 Index are very close to each other. The similar values of $\lambda$ ($\approx 2.5-2.9$) indicate that
the momentum investors react to strong and significant volatility and the action thresholds are roughly 3\( \sigma \). The value of \( \alpha (\alpha_{SSE} = 4.2) \) in the Shanghai market is higher than that in the other three mature markets, suggesting that there is a better opportunity for momentum strategies in the Shanghai market (an emerging market). This is reasonable within the viewpoint of momentum behavior being related to market inefficiency. Barberis et al. [41] and Daniel et al. [42] developed models based on human cognitive bias that would lead investors to either under-react to information or adopt momentum strategies. As an emerging market, the Shanghai market is less mature and less efficient than the other three mature markets, thus providing more opportunities for momentum investors.

Also, it should be noted that we investigated the momentum behavior on a monthly time scale by using the monthly returns of S&P500 Index, Hang Seng Index, Nikkei 225 Index, and Shanghai Stock Exchange Composite Index. In real markets, momentum investors usually follow a stock and look at it over a long time horizon. We considered a monthly time scale because empirical studies showed momentum behavior on a time scale from one month to several months [6–8]. As an approximation, the monthly time scale is chosen. Thus, we used the data of monthly returns in real markets to fix the model parameters, in order to investigate the momentum behavior on the monthly time scale.

### 5. Summary

In summary, we have proposed and studied an agent-based model of trading incorporating momentum investors, which provides an alternative approach for studying the impacts of momentum investors on market behavior. The model gives simulated time series of stock prices that carries some of the well-known stylized facts. Furthermore, we established the relationships between the model parameters and several quantities that can be extracted from real data sets. We also illustrated how real data sets could be used to constrain the model parameters, which in turn provided information on the behavior of momentum investors in different markets.

Here, we only considered the case of a single action threshold for all the momentum investors. The model can be readily extended to include the effects of a distribution in the action thresholds among the momentum investors. This would introduce an additional parameter into the model that characterizes the spread in the action thresholds. It is, however, expected that a spread of action thresholds around a sharp mean threshold would only lead to quantitative changes in the results, rather than qualitative differences. The work here represents a step towards the understanding of momentum trading within the framework on agent-based modeling. Other interesting extensions of our model include investigating the competition between traders using different time horizons, different action thresholds, or different types of trading strategies including strategy evolution.

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