

Long-living photonic dipole oscillations in photonic crystals

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We investigate the occurrence of photonic dipole oscillations (PDOs) in one-dimensional photonic crystals containing nonlinear composites. Because of the modulation of the graded pump electric fields, optical pulses undergo oscillations inside the curved band structure, analogous to the dipole oscillations in ultracold gases. The implementation of numerical simulations shows that our proposed scheme can produce stable and long-living PDOs. The physical origin is explored to give a clear physical picture. Finally, the tunability of PDOs is discussed. © 2009 Optical Society of America

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Recently, matter wave dynamics in cold atoms has sparked considerable interest for its major relevance from both fundamental and applied viewpoints [1,2]. These include the dipole oscillation (DO), i.e., an oscillatory motion of the wave packet in the periodic trapping potential superimposed on a harmonic potential [3]. The analogs in photonic structures provide visible models to interpret these phenomena [4,5] and bring new tools with widely tunable parameters for controlling light [6,7]. But until now, the lifetime of DOs in such contexts as cold atoms and photonic structures has been very short. For example, in cold atoms the DOs will stop after several periods because of the limited coherence and interaction among atoms [8]. Or in optical waveguides with transverse confinement, the oscillations will become unstable, for the pulse quickly spreads over the full structure [9]. For these reasons it is of interest to investigate long-living DOs.

The aim of this work is to demonstrate a class of *long-living* photonic dipole oscillations (PDOs) in chirped photonic crystals (PCs) provided by a parabolic curved band structure. To achieve them, we shall present a tractable method by applying a spatially linearly graded pump electric field to a one-dimensional PC with periodically alternating layers of a third-order Kerr-type nonlinear composite and a linear dielectric; see Fig. 1. Our idea relies on the fact that the permittivity of such nonlinear composites depends on pump electric fields. To exemplify this idea, we implement numerical simulations to study the propagation of light and discuss them in terms of semiclassical analysis. Our proposal can achieve stable and long-living PDOs and provide dynamical control of oscillations out of ultrafast response time.

The optical system under investigation is formed by periodically alternating layers of a Kerr dielectric and a linear dielectric as depicted in Fig. 1 [10]. For the use and justification of nonlinear media in the modulation of optical propagation, please see [10,11]. For model calculations, we investigate a Au:SiO₂ composite layer (the Kerr dielectric layer with effective nonlinear permittivity $\tilde{\epsilon}_1$) versus an air layer (the linear dielectric layer with permittivity $\epsilon_2 = \epsilon_0$).

Each Au:SiO₂ composite layer is composed of Au nanoparticles randomly embedded in a SiO₂ host with linear permittivity ϵ_h [12]. Owing to the local-field enhancement, the third-order nonlinearity of composites can be enhanced significantly [12,13].

Actually, the composite layer's dielectric response to the probe field \mathbf{E}_1 (which exhibits oscillations, see Fig. 1) is rather stable in the condition of weak nonlinearity [10]. In other words, since the pump field \mathbf{E}_0 is much larger than the probe field \mathbf{E}_1 , the probe EM waves are stable under the modulation of \mathbf{E}_0 . In the following, we assume $\mathbf{E}_1 \perp \mathbf{E}_0$ (see Fig. 1), which results in an effective nonlinear response to the probe field $\tilde{\epsilon}_1 = \epsilon_1 + \chi_1^{(3)} |\mathbf{E}_0|^2$. In the case of low concentrations of nanoparticles, it can be shown with some derivations [10] that the effective permittivity $\tilde{\epsilon}_1$ of the Au:SiO₂ composites is given by $(\tilde{\epsilon}_1 - \epsilon_h) / (\tilde{\epsilon}_1 + 2\epsilon_h) = p(\tilde{\epsilon}_p - \epsilon_h) / (\tilde{\epsilon}_p + 2\epsilon_h)$, where $\tilde{\epsilon}_p$ is the nonlinear permittivity of particles, written as $\tilde{\epsilon}_p = \epsilon_p + \chi_p^{(3)} \langle |\mathbf{E}_p|^2 \rangle$ by substituting the volume average of the enhanced local field $\langle |\mathbf{E}_p|^2 \rangle = [9|\epsilon_h|^2 / ((1-p)\epsilon_p + (2+p)\epsilon_h)^2] |\mathbf{E}_0|^2$,

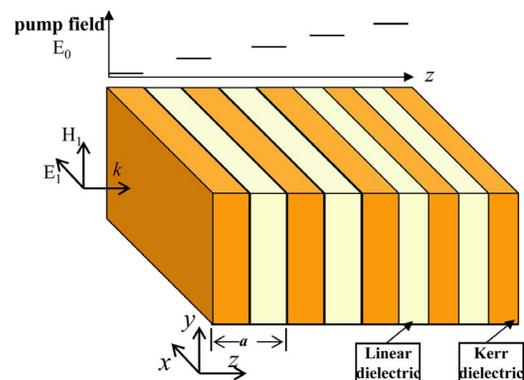


Fig. 1. (Color online) Schematic view of a one-dimensional PC with periodically alternating layers of a Kerr dielectric (e.g., Au:SiO₂ composite as used in this work) and a linear dielectric. A probe wave is incident along the z direction, with an electric component E_1 along the x direction. The pump electric field E_0 is directed along the y axis. For plotting Figs. 2–4, we set the thickness of each layer to be $0.5a$, where a is the lattice constant.

and p is the volume fraction of nanoparticles. Since the probe field in our use is much weaker in strength than the pump field, the dispersion of the probe field can be neglected. So we take $\varepsilon_p = -9.97\varepsilon_0$ and $\chi_p^{(3)} = 8 \times 10^{-8}$ esu, which correspond to a pump field produced by a laser source at 620 nm [13]. In addition, we set $p = 0.2$ and $\varepsilon_h = 2.25\varepsilon_0$. We should remark that the actual values for ε_p and $\chi_p^{(3)}$ do not change our results originating from the field-induced nonlinearity.

In order to obtain the desired PDOs, we adjust the photonic bands by applying a graded pump electric fields on the PCs (Fig. 1). That is, all the composite layers form a structure with graded responses due to the spatially varying pump field with a gradation profile $E_0^2(N) = (gN + b)^2$. Here, we use $E_0^2(N)$ to indicate the strength of a pump field acting on the N th Au:SiO₂ layer, with g and b being two tuning parameters. Figure 2 displays the second depth-dependent photonic band under the gradation $g = 13$ and $b = -390$, which is calculated via the transfer matrix method. The figure clearly shows that the z -dependent band diagram approximates the parabolic curve very closely (please see the dashed-dotted curve). In fact, many other parameters of nonlinearity have also been investigated, and similar conclusions can be obtained. This originates from the action of the graded pump fields, which causes a parabolic configuration of refractive index contrast between consecutive Au:SiO₂ and air layers. Also, one can see that by injecting probe waves with angular frequency between 2.23 and 2.84, one might possibly observe PDOs in the second band. The two-headed arrow indicates the region where the PDOs may take place for $\omega_0 = 2.78$, which is to be demonstrated in Fig. 3.

To study the propagation of electromagnetic waves in the modulated PCs, we implement the numerical simulations in a finite stack of layers. We look for its eigenmodes through solving the Helmholtz equation on a regular grid by approximating the differential operator with a finite-difference stencil. Then we decompose the initial packet using the basis of eigenmodes and find the solution in the whole structure

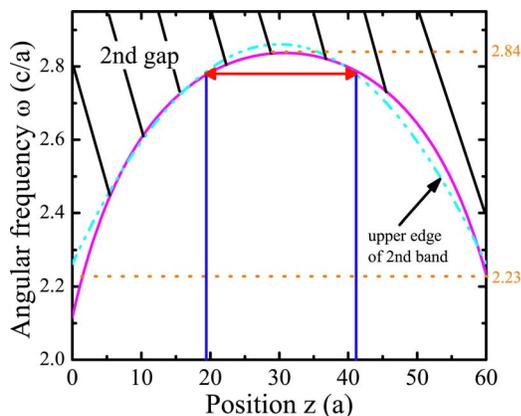


Fig. 2. (Color online) Depth-dependent second photonic band structure for the gradation profile with parameters $g = 13$ and $b = -390$. The dashed-dotted curve represents the fitting parabola $\omega = -6.64 \times 10^{-4}(-95.6 + z)(35.6 + z)$. PDOs may appear in the cross section between the two dotted lines.

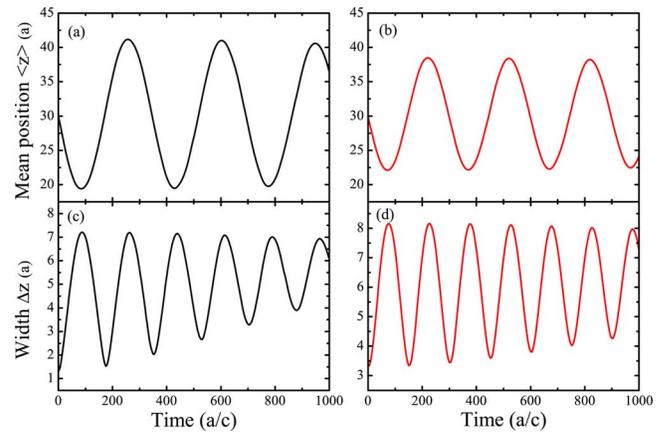


Fig. 3. (Color online) Time evolution of the wave packet with $\omega_0 = 2.78c/a$ under $E_0^2(N) = (gN + b)^2$ with (a), (c) $g = 13$ and $b = -390$ and (b), (d) $g = 15$ and $b = -390$. The DOs clearly develop in different ranges with different periods.

[14]. In our model, we employ the discretization scheme $(1/h^2)(E_{n+1} - 2E_n + E_{n-1}) + \varepsilon_n E_n = 0$ to produce the matrix, where $h = 0.1a$ is the grid step. For the initial packet we assume the Gaussian distribution $W_{(z,k)} = A \exp[-(z - z_0)^2/2\sigma_z^2 - (k - k_0)^2/2\sigma_k^2]$ centered at $(z_0 = 30a, k_0 = 0.18\pi/a)$ with $\sigma_z = 1.2a$ (here $\sigma_z\sigma_k = 1$). The time series of subsequent displacement of the pulse can be found in Fig. 3(a). Evidently, the pulse oscillates in the some spatial range, whose mean position echoes the estimation indicated by the arrow in Fig. 2. Meanwhile, the width of packet $\Delta z(t)$ remains bound in time (about 10% of the length of the stack) without spreading to the whole system and presents a oscillating behavior [Fig. 3(c)]. Interestingly, its period is just half of the corresponding period of the PDOs, and the reason will be found below. To access the lifetime of the PDOs, we fit the mean position with the formula of damped oscillations $\langle z \rangle \propto \exp(-\beta t) \cos(\omega t)$ with frequency ω and damping β . The fitting procedure produces the exponent $\beta = 0.0002$, indicating that the characteristic lifetime of the PDOs is $\tau = 1/\beta = 5000a/c$. During this time the pulse has been oscillating for 15 periods, which is much larger than previously reported values [8,9]. The damping is associated with the imperfect parabolic band diagram, yielding the incomplete recovery of the initial excitation.

Why can the oscillations of packets in such system remain long-lived and avoid diverging? To understand the mechanism, let us go to the semiclassical frame, in which the physics becomes particularly clear. Considering a material with space-dependent dispersion relation $\omega(\mathbf{z}, \mathbf{k})$, where \mathbf{z} and \mathbf{k} represent the position and the wave vector of probe waves. According to [15], the variational derivative of the Hamiltonian gives the equations of motion: $\dot{\mathbf{z}} = \partial H(\mathbf{z}, \mathbf{k}) / \partial \mathbf{k}$ and $\dot{\mathbf{k}} = -\partial H(\mathbf{z}, \mathbf{k}) / \partial \mathbf{z} = \mathbf{F}$. This means that the space dependence of energy bands gives an equivalent force \mathbf{F} acting on the wave packets. Generally speaking, the key point of oscillations is that waves are reflected on the bandgaps and experience equivalent force inside the bands. So the existence of

depth-dependent band structures allows for all kinds of dynamical effects of pulses such as PDOs.

In our case, the time average of the equivalent force is zero within every period, as shown in Fig. 4(a). In other words, the driving force is restoring force, which drags the pulse to oscillate in the real space. In fact, our vanishing time-average force is due to the alternations of the direction and magnitude of the force. And it is the alternations that make the direction of the force parallel or antiparallel to the movement of pulse, causing the period of the width to be half that of the PDOs (see Fig. 3). Figure 4(b) shows that the first derivative of the second band diagram, i.e., the restoring force, is almost linearly related to the displacement of waves. That is, the properties of such oscillations are analogous to the harmonic oscillator. Consequently, all the components of the wave packet will oscillate with the same period, which can be explained by the harmonic oscillator model: the oscillation frequency is proportional to the square root of the second derivative of the band diagram, and it remains nearly constant in our configuration. So the set of plane waves of the packet travels together as a stable entity, and thus the broadening of width is suppressed and the wave packet is maintained. By now, we are convinced that the long-living PDOs occur indeed.

The existence of Kerr nonlinearity offers a handle for PDOs tunability by varying the band diagram. Figure 3 shows the oscillations for a fixed incidence $\omega_0=2.78$ under different gradients. Obviously as the pump electric field's gradient becomes larger, both the amplitude and oscillation period decrease because of the resulting steeper curvature of the photonic band.

Herein are some comments. Our formalism provides a dynamical control of PDOs because the non-

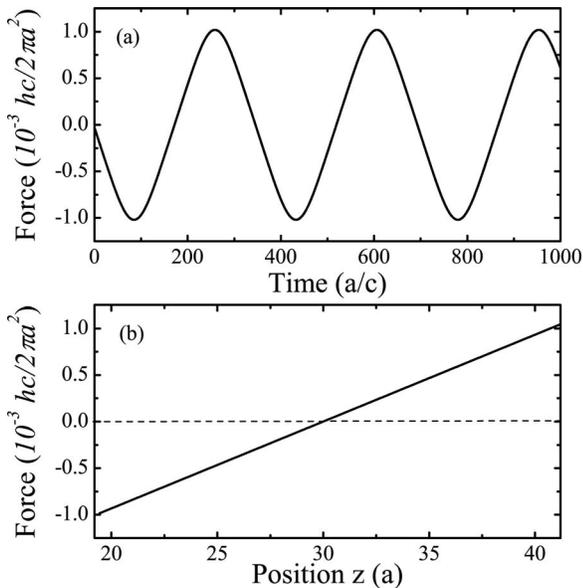


Fig. 4. (a) Time-dependent driving force acting on an incident pulse. (b) Linear relationship between the force and the position. Parameters are again taken to be the same as those used for Fig. 3(a). The time average of the force is zero within each period.

linear response obtained from the composites is ultrafast (approximately picoseconds [12]). Once the pump fields are applied suddenly, a pulse propagating in the structure will be trapped, and the oscillations subsequently commence. After the graded field is restored to the uniform, the waves carrying an oscillating modulation will be decoupled from the structure. Another advantage of our designed structure is the adoption of low pump field intensities, originating from the large nonlinear susceptibility $\chi^{(3)}$ of composites (up to 10^{-6} esu [12]). For example, in order to achieve terahertz modulation of IR wavelength $1.4 \mu\text{m}$ for the gradation $g=13$, $b=-390$, and lattice constant $a=600$ nm, the maximum pump field intensity is $|E|^2=40.3$ mW/cm², leading to PDO frequency 1.45 THz. And the intensity of the probe beam is often 1 or 2 orders of magnitude lower than that of the pump. They are all experimentally practical. We should also comment on the effect of the loss from the composites. Since the wavelength of pump field is off the surface plasmon resonance wavelength of composites (530 nm) [12], the effect of loss in composite layers is very weak. An estimate of the attenuation length is of the order of magnitude of 10^2 cm, which is long enough to sustain the PDOs in the structure.

To sum up, we have shown that the long-living PDOs with tunability can be realized in PCs with Kerr dielectrics by properly applying graded pump fields.

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