nature reviews physics

https://doi.org/10.1038/s42254-023-00565-4

Review article

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Diffusion metamaterials

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Abstract

Diffusion and wave propagation are both fundamental transport mechanisms, but they have intrinsically different dynamics, governing equations, and applications. Over the past decade, studies have emerged that use the transformation principle and metamaterials to control diffusion. Such research has led to new discoveries and exciting applications for manipulating the transport of mass (for example, particles and plasmas) and energy (such as heat). This Review introduces the basic principles, materials advances and applications of metamaterials that modulate the diffusion of heat, particles and plasmas. The theory begins with the application of the transformation principle to the diffusion equations. This approach is then generalized to incorporate non-Hermiticity, topology, non-reciprocity and spatiotemporal modulation, thus going beyond the conventional scope of metamaterials. Finally, we analyse the primary challenges associated with the design and fabrication of diffusion metamaterials and suggest several future directions, such as research into topological diffusion and machine-learning-assisted materials design.

Sections

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Key points

- Metamaterials can control the diffusion of heat, particles and plasma to enable the cloaking, rotating or converging of the respective fields.
- The transformation principle can be used to control processes with governing equations that are form-invariant under coordinate transformations, enabling the control of diffusion processes by changing material parameters such as conductivity.
- The unique properties of diffusion metamaterials make them useful tools for studying new physical concepts such as non-Hermitian topology, non-reciprocity and spatiotemporal modulation.
- Some potential engineering applications of these devices include thermal camouflage, heat harvesting, particle separation and plasma-based wound healing.
- Future work in the field of diffusion metamaterials may include the development of devices to control topological diffusion or the use of machine learning to help with the design of new metamaterials.

Introduction

In 2006, several studies[1–3] inspired by curiosity about cloaking and invisibility proposed the theory of transformation optics and designed devices for optical cloaking. These studies led to a surge of research interest in metamaterials, transformation optics and related topics[4–8]. The underlying physics is general because transformation theories can be regarded as the equivalence principle, which states that the transformation of spatial coordinates is equivalent to the spatial modulation of the relevant material parameters. The spatial modulation of material parameters is usually complex and can be inaccessible when using natural materials but can be achieved with artificially structured materials, such as metamaterials. Driven by the value of the potential applications, advanced methods such as scattering cancellation[9–12] and numerical algorithms[13–16] (for example, the covariance matrix adaptation evolution strategy and particle swarm optimization) were developed for designing metamaterials and metadevices with various functions[17–21], including negative-refractive-index metamaterials[22–24]. Metamaterials thus became an interdisciplinary field[25–31] that attracts tremendous research interest in unconventional structural materials and their applications[32–38].

The fundamental science of metamaterials has also developed in other intriguing ways over the past decade. First, the concept of metamaterials was extended to various types of diffusion, including heat diffusion, which helped to drive research on metamaterials beyond wave-based systems. The proposal of transformation thermotics in 2008 (refs. [39,40]) and its experimental verification in 2012 (ref. [41]) were also notable developments in the field of thermal metamaterials[42–47]. Moreover, metamaterials were generalized to particle diffusion. Although the Fick equation does not perfectly meet the requirements of transformation theory, it was confirmed in 2013 that particle-diffusion cloaking can still be achieved under low-diffusivity conditions[18]. This work inspired the development of the diffusive-light cloak in 2014, which treated light propagation as photon diffusion[48] and considerably increased the attention on particle-diffusion metamaterials[49–54]. Plasma diffusion[55–59] can also be manipulated by transformed parameters[60], inspiring advanced plasma technologies based on transformation theory and metamaterials. Second, owing to the unique properties of diffusion metamaterials, it is natural (and exciting) to use them to study non-Hermitian effects and other intriguing concepts. For example, the dissipative nature of diffusion is closely tied to non-Hermitian physics. As a result, anti-parity–time symmetry was first demonstrated in diffusion systems. Additionally, heat transport in a spatiotemporal lattice has non-Hermitian topological band structures. Therefore, diffusion metamaterials have become a prosperous frontier for fundamental research.

In this Review, we provide a unified summary of diffusion metamaterials and the emerging physics in diffusion systems, mainly emphasizing materials, applications, and new concepts such as non-Hermiticity, topology, non-reciprocity and spatiotemporal modulation. Although the underlying physics is not limited to diffusion metamaterials, the nature of diffusion metamaterials makes them a unique platform for emergent fundamental phenomena and applications in cutting-edge technologies.

Thermal conduction

Theory

In 2007, it was shown that transformation theory (Box 1) and metamaterials can be used in the zero-frequency limit of electromagnetic waves, for example for electrostatic and magnetostatic fields[61,62]. Subsequently, the cloaking of electrostatic fields[63], magnetostatic fields[64] and direct current fields[65,66] was theoretically predicted and experimentally demonstrated. Research interests then turned to heat conduction, which is described by the Laplace equation at the steady state, similar to electrostatics.

Thermal cloaking inspired by transformation optics was then discussed[39,40], opening a pathway towards transformation thermotics and its extended theories. Before 2012, transformed thermal media were mainly applied to steady-state heat transfer, which limited the applications of transformation thermotics. Transformation thermotics was then extended from the steady state to the transient state by considering the time-dependent term in the conduction equation[67]. Based on this generalized theory, thermal cloaking and concentrating suitable for transient heat transport were designed. In general, heat conduction with a heat source is described by

$$\rho C\frac{\partial T}{\partial t}+\nabla\cdot(-\kappa\nabla T)=S, \qquad (1)$$

where t is time, ρ, C, T and κ are the density, heat capacity, temperature and thermal conductivity of the system, and S is the power density generated by the external heat source. Under a 2D coordinate transformation from (x,y) to (x',y'), this equation becomes

$$\rho' C'\frac{\partial T}{\partial t}+\nabla'\cdot(-\kappa'\nabla' T)=S', \qquad (2)$$

where $\rho' C'=\rho C/\det J$, $\kappa'=J\kappa J^{\mathrm{T}}/\det J$, and $S'=S/\det J$, where J is the Jacobian transformation matrix. Therefore, equation (1) is form-invariant.

Transformation thermotics faces various challenges: for example, materials with singular and anisotropic thermal conductivities are needed, which do not often exist naturally. Thus, a method that can experimentally achieve transformed parameters is urgently required. Thermal metamaterials, the thermal counterpart of electromagnetic metamaterials, make transformation theory applicable in practice.

As a type of macroscopic composite, the properties of thermal metamaterials can be predicted using effective medium theory. The Maxwell–Garnett theory and the Bruggeman formula are two basic theoretical tools. An early experiment on thermal metamaterials[41] was performed using an ABAB layered structure to achieve anisotropic thermal conductivity. This study experimentally demonstrated that the structure was capable of cloaking a steady heat flux. Subsequent experiments on thermal metamaterials extended this approach to transient cases[68].

Applications

Many proposals and designs targeting various applications have emerged to enrich and develop transformation thermotics. Intelligent and multifunctional thermal metamaterials have been proposed to explore new applications. Advanced methods using effective medium theories and numerical algorithms guided by the theory of transformation thermotics have achieved unprecedented phenomena and applications, such as intelligent thermal metamaterials that adapt their responses and functions to the environment. Additionally, smart thermal devices, such as chameleon-like concentrators[69] and rotators[70], have been investigated. Based on transformation-invariant thermal metamaterials[71] with highly anisotropic thermal conductivities, these chameleon-like devices work even when the properties of the surroundings change. Thermal-null media have also been presented[72–74]: these devices have highly anisotropic thermal conductivities that are homogeneous and independent of the geometry of the device. In other words, the conductivity does not have to be recalculated when the shape changes. The principle of extreme anisotropy is similar to that of chameleon-like devices. Nevertheless, these devices still face challenges because materials with extreme anisotropy do not exist naturally. Fortunately, artificial materials such as multilayer graphene or van der Waals thin films may provide alternatives[75–77].

Thermal metamaterials with multiple functions are also desirable. Such metamaterials are achieved by transforming multiple physical fields and the relevant material parameters. Simultaneously managing heat and electric currents is one of the key topics[78]. In 2010, a cloak for electric and heat currents was proposed[79], showing that it could be possible to construct bifunctional devices based on transformation theory. A bifunctional cloak was experimentally demonstrated in 2014 (ref. [80]). Coupled thermoelectric transport, where heat and electric currents influence each other, was also considered[81], leading to the development of a thermoelectric cloak that shields both electric and thermal fields. Other multifunctional devices such as thermo-hydrodynamic cloaks[82] and thermoelastic cloaks[83] have also been proposed.

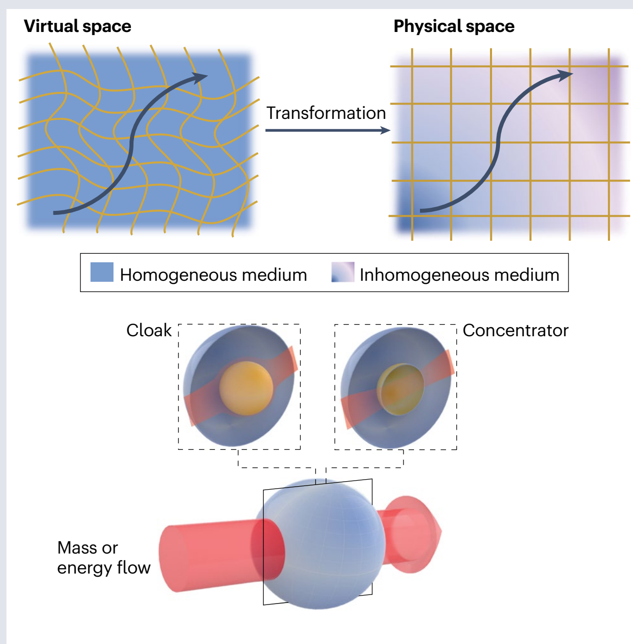
The scattering cancellation method, which was initially used for cloaking electromagnetic fields, has also been widely applied to the design of thermal metamaterials[84]. Here, for brevity, thermal invisibility and illusion are collectively treated as thermal camouflage. In 2014 one study demonstrated a 2D bilayer thermal cloak made of bulk isotropic materials that was designed by calculating the thermal conductivity of each layer by directly solving the heat-conduction equation[10]. Based on this bilayer structure, a camouflage device was reported that can transform the thermal scattering signature of a hidden object into another fake signature[42]. Although this device worked well for steady-state cases, the performance under transient conditions was unsatisfactory because the scattering cancellation method only considers the steady state. Later, an improved scattering cancellation method, which is suitable for transient cases, was proposed[85]. Therefore, the scattering cancellation method can solve the problem

Box 1

Transformation theory

The transformation principle originates from transformation optics but is not limited to optics. It plays a vital role in controlling physical processes with governing equations that are form-invariant under coordinate transformations, such as acoustics[237–239], elastodynamics[240,241], particle dynamics, thermotics[242] and plasma physics. Transformation theory aims to achieve the same effect as a spatial transformation by instead transforming material parameters (see top of the figure). In the figure, the background colours represent the properties of the material (such as the thermal conductivity), the grids represent space, and the arrows denote diffusion flow.

In a virtual space (x,y), the energy or mass flow can be twisted by warping the space. However, it is challenging and even impossible to bend a real space. According to the transformation principle, the same twisting of the energy or mass flow can be achieved by modulating the spatial distribution of the material parameters. The transformation from virtual space (x,y) to physical space (x', y') is described by the Jacobian transformation matrix $J = \partial x_i'/\partial x_j$. The effect of a spatial transformation on a uniform material parameter (A) in the virtual space can be effectively achieved by replacing A with the spatial distribution of the material parameter $A' = JAJ^{\mathrm{T}}/\det J$, where J^{T} is the transpose of J, and $\det J$ is the determinant of J. In some applications, the desired function (for example, cloaking, or concentration) can be achieved in physical space through the transformed spatial distribution of material parameters as determined by the coordinate transformation (see the bottom of the figure).



Glossary

Bulk–edge correspondence
A topological phenomenon that means that the topological properties of the bulk of the system determine the character of the edge modes.

Concentrator
A structure that creates a larger flow (for example, heat flow) inside a device without disturbing the external fields (for example, temperature distribution) in the surrounding area.

Cloak
A structure that provides a zero gradient (for example, temperature gradient) inside a device without disturbing the external fields (for example, temperature distribution) in the surrounding area.

Darcy's law
An equation to describe slow and viscous fluid flow. This equation is usually applied to describe fluids in porous media.

Diffusive Fizeau drag
The observation that wavelike temperature fields propagate at different speeds in opposite directions when an orthogonal advection is applied. It is analogous to the Fizeau drag observed in electromagnetic fields moving through flowing water.

Geometric phase
The phase difference obtained when a system completes one cycle of an adiabatic process, which is caused by the geometrical properties of the Hamiltonian parameter space.

Reynolds number
A dimensionless quantity that is used to classify the flow type as laminar (values less than 1) or turbulent (values more than 1).

Rotator
A structure that changes the transport direction of the flow (for example, heat flow) inside a device without disturbing the external fields (for example, temperature distribution) in the surrounding area.

Scattering cancellation
An approach that designs coatings that can be applied to objects to cancel the scattering, making the object invisible to observers.

of extreme parameters introduced by transformation theory and aid the design of new thermal metamaterials.

The other primary application of thermal metamaterials is heat management in fields ranging from electronics to biology. For example, with the rapid development of nanoelectronics, conventional thermal management approaches, such as through-silicon-via optimization and thermal pipes, face many challenges that could be addressed by thermal metamaterials[86,87]. Unlike conventional methods, thermal metamaterials can dissipate heat at will and avoid thermal crosstalk and local hot spots. Additionally, thermal memory and computing could be implemented by defining binary states of heat flux using preset alternating sequences of concentrating and cloaking states[88]. Thermal encoding has been proposed as a possible scheme for encoding optical or electronic information[89]. Personal thermal management is another exciting application. For example, a Janus layered textile was developed that works as a heating or cooling device[90]. The textile could increase (or decrease) the temperature of a skin simulator by 8.1 °C (or 6 °C) under sunlight.

Numerical optimization algorithms can also help the design and fabrication of thermal metamaterials. In 2014, topology optimization was used to design thermal-composite structures for heat flux manipulation, including shielding, focusing and rotating the heat flux[13]. The anisotropic thermal composite consists of elliptical inclusions embedded in the surrounding matrix. To simplify the structures, a covariance matrix adaption evolution strategy, a powerful stochastic method, was used to tackle nonlinear optimization problems and design a thermal cloak composed of natural materials[15]. Constructing such a thermal cloak involves two steps. First, the background temperature distribution must be preset without the obstacle present. Second, the undisturbed temperature field must be reproduced using a thermal cloak designed by topology optimization. Printable freeform thermal metamaterials have been proposed by directly creating functional cells[14] (Fig. 1a). The thermal conductivity tensor is calculated according to transformation thermotics and realized using local microstructures composed of die steel and polydimethylsiloxane. This approach solves challenges such as the limited shape adaptability of thermal metadevices and the need for prior knowledge of the background temperatures. Numerical methods have now become advanced and powerful for designing feasible thermal metamaterials with various applications such as invisible sensors[16,91,92], not to mention the emerging machine-learning techniques[93,94].

In addition to achieving abundant conventional functions, metamaterials can also be used to study counterintuitive physical phenomena. We refer to such phenomena as 'new physics', among which topology is a typical example. In the following sections, we focus on non-Hermitian topological phenomena such as thermal topology, in addition to introducing other new phenomena.

New physics

Nonlinear thermal diffusion, that is, temperature-dependent thermal diffusion, is an intriguing topic that involves unconventional phenomena and applications[95–104]. In thermal metamaterials, one can exploit a phase transition driven by temperature change to greatly modify the material parameters. A dual-functional thermal metamaterial based on nonlinear thermal diffusion was proposed[105], which automatically switched from a concentrator to a cloak when the environmental temperature changed. A later study proposed an energy-free thermostat composed of asymmetric phase-transition materials with thermally responsive thermal conductivities[106]. In such a device, the temperature is kept approximately constant in the central region. Based on these concepts, a theory of nonlinear thermal metamaterials was developed to describe nonlinear effects in thermal metamaterials and functional devices[107].

Another intriguing topic is topological phenomena in thermal metamaterials. The study of topological phenomena has revolutionized our understanding of phases of matter, leading to the discovery of quantum Hall effects and topological insulators. Using the analogy between quantum mechanics and classical wave dynamics, the study of topological phenomena has been extended to other types of systems including photonic, acoustic and mechanical systems[108–113]. However, extending topological phenomena to diffusion systems is challenging because of their incoherent nature and the lack of oscillatory dynamics. Nevertheless, with insight from non-Hermitian physics and non-Hermitian topology, the most fundamental topological phenomenon, that is, bulk–edge correspondence, has been established in diffusion metamaterials. In particular, the Su–Schrieffer–Heeger (SSH) model — a 1D lattice model with dimerized couplings — has been realized in thermal metamaterials[114–116]. These achievements are based on the mapping of the imaginary damping eigenvalues of the diffusion equation of a lattice to the real frequency eigenvalues of the wave equation on the same lattice. In this case, the real values form the band structure of

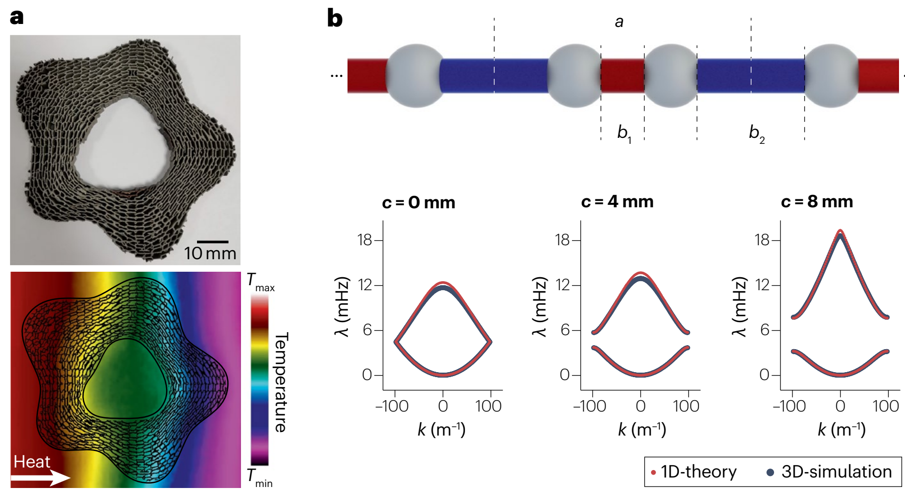


Fig. 1 | Thermal metamaterials for manipulating heat conduction. a, A 3D-printed cloak (top) and a colour map of its experimentally measured temperature field (bottom), where T_{max} and T_{min} are the maximum and minimum temperature, respectively. The arrow indicates the direction of heat flow. There is a minimal temperature gradient inside the object, and the temperature field outside the object is only slightly disturbed, indicating that the metamaterial is an effective cloak. **b**, Top: a model of a thermal lattice with periodically varying thermal conductivities between sites. Intercell and intracell heat conduction is represented by the blue and red rods, respectively. a is the unit length of the cell, and b_1 and b_2 are the lengths of the intracell and intercell rods, respectively. Bottom: 1D theoretical calculations and 3D simulations of the effect of changing the spacing between sites on the band structure of the system, where k and λ are the wave vector and frequency, respectively. Parameter $c = (b_2 - b_1)/2$ is a length used to characterize the spacing in the structure. When $c = 0$, the two bands are degenerate at $k = \pm\pi/a$, and when $|c| > 0$, a bandgap opens at $k = \pm\pi/a$. These results demonstrate the bulk–edge correspondence in this system. Part **a** adapted from ref. [14] under a Creative Commons licence CC BY 4.0. Part **b** adapted with permission from ref. [115], Wiley.

the wave dynamics of the lattice and the imaginary values form the damping band structure of the diffusion. As the damping modes can also have geometric phases, the topology of the damping band structure can be defined, and topological phenomena can be derived (Fig. 1b).

High-dimensional topological insulators can bring about a greater number of unexpected boundary states than 1D topological insulators. A zigzag or an armchair edge can be realized in honeycomb lattice thermal metamaterials[117]. A high-dimensional thermal-diffusion model based on the sphere–rod structure was used to reveal diffusive second-order topological insulators with zero-dimensional corner states and 1D edge states[118]. Additionally, the quadrupole topological phase of a non-Hermitian thermal system can be observed in a 2D diffusion system using quantized bulk quadrupole moments[119]. These results suggest that topological phases can be applied generally to diffusion systems.

Thermal convection

Theory

Besides conduction, convection is also a fundamental heat transport mechanism, which can be induced by moving fluids or solids. Because thermal convection is always accompanied by conduction, it is critical to explore whether these two mechanisms can be manipulated simultaneously. Transformation theory was first applied to fluid-flow control in the case when Darcy's law, $\mathbf{v} = -\eta/\mu\nabla P$, is valid[120], where $\mathbf{v}$, η, μ and P are the convective velocity, permeability, dynamic viscosity and pressure, respectively. Darcy's law requires that the Reynolds number is less than 1, meaning that the fluid flow is laminar. Transformation theory was then adapted to the thermal conduction–convection equation[52]. Although the thermal conduction–convection equation is form-invariant under coordinate transformations, practical methods for producing metamaterials to achieve the required velocity field remained elusive. Later, a transformation theory for thermal convection was proposed for porous materials filled with fluids[121]. In the case of porous materials filled with incompressible fluids, the conduction–convection equation and the continuity equation are

$$(\rho C)_e \frac{\partial T}{\partial t} + \nabla \cdot (-\kappa_e \nabla T + (\rho C)_f \mathbf{v} T) = 0 \quad (3a)$$

$$\nabla \cdot \mathbf{v} = 0 \quad (3b)$$

where $(\rho C)_e$ and κ_e are the effective product of the mass density and heat capacity, and the effective thermal conductivity, respectively. The average volume method equates the physical parameters of a mixture consisting of homogeneous components to the average of the physical parameters of each component weighted by the volume fraction. This approach is used to calculate the effective properties of the system: $(\rho C)_e = \phi(\rho C)_f + (1-\phi)(\rho C)_s$ and $\kappa_e = \phi\kappa_f + (1-\phi)\kappa_s$, where ϕ is the porosity, and the subscript s (or f) denotes solid (or fluid) components. Darcy's law, the conduction–convection equation and the continuity equation are all form-invariant under coordinate transformations, and the transformation rules are

$$(\rho C)'_s = \frac{(\rho C)'_e - \phi(\rho C)_f}{1-\phi} \quad (4a)$$

$$\kappa'_s = \frac{\kappa'_e - \phi\kappa_f}{1-\phi} \quad (4b)$$

$$\eta' = \frac{J\eta J^{\mathrm{T}}}{\det J} \quad (4c)$$

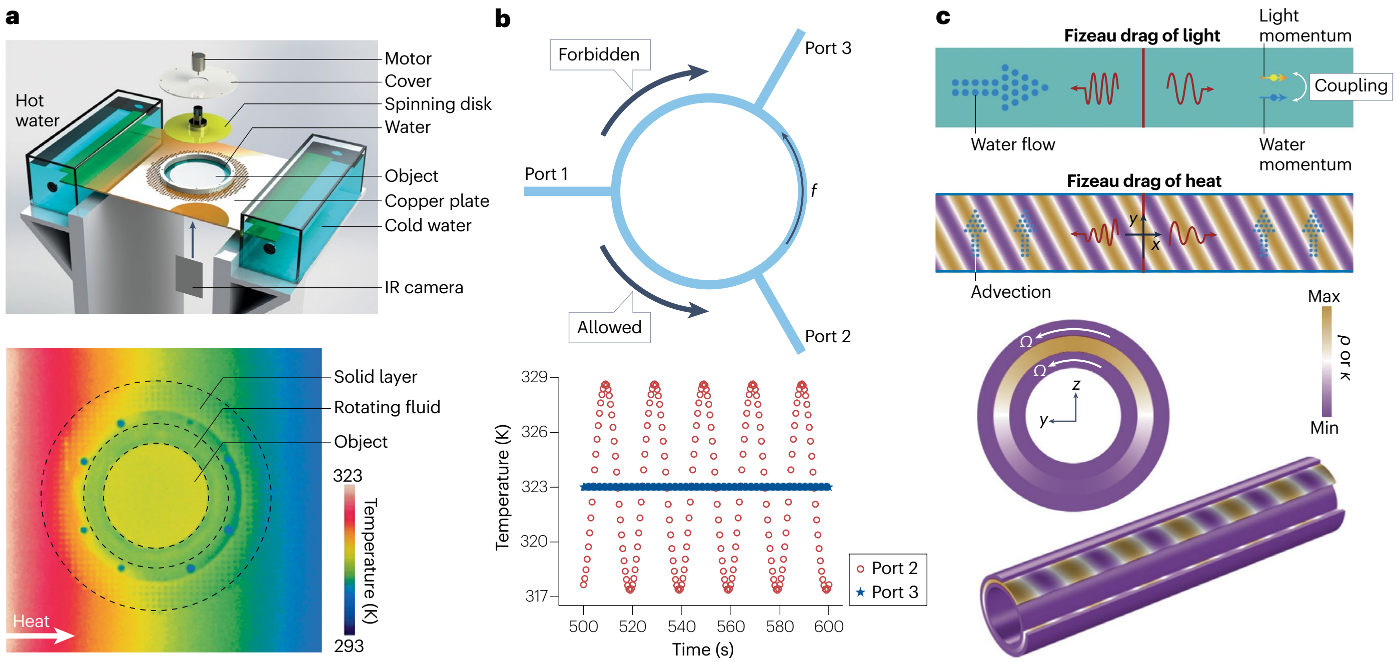


Fig. 2 | Thermal metamaterials for manipulating heat conduction and convection. a, Top: the experimental set-up used to create a thermal zero-index cloak by circulating water around the object from hot and cold water tanks to create a stable temperature field. Bottom: the temperature profile of the object and its surroundings measured with an infrared (IR) camera when the water is circulating around the object. The homogeneous temperature field in the object shows that there is no heat flow, which demonstrates the cloaking effect of the bilayer device. **b**, Top: a schematic of a water-filled angular-momentum-biased ring. By applying high pressure to port 1 and low pressure to ports 2 and 3, a volume force (f) is imposed on the ring in the anticlockwise direction. The arrows show the forbidden and allowed directions of travel for the thermal wave. Bottom: the average temperature measured at port 2 (red dots) and port 3 (blue stars) when $f = 2\ \mathrm{N\,m^{-3}}$, showing that the thermal wave at port 3 is isolated. **c**, Fizeau drag of light (top) and heat (middle). The red arrows indicate the propagation of the fields in the positive and negative x directions and contain information about the wavenumber and amplitude. Bottom: the results of simulations in 2D and 3D three-layer pipes to demonstrate the Fizeau drag of heat. The density (ρ) and thermal conductivity (κ) are spatially periodic, as shown by the colour bar. The layers rotate with angular velocity (Ω) to provide the surface advection to the central layer. Part **a** adapted from ref. [126], Springer Nature Limited. Part **b** adapted with permission from ref. [133], AIP. Part **c** adapted with permission from ref. [135], APS.

where $(\rho C)'_e = (\rho C)_e/\det J$ and $\kappa'_e = J\kappa_e J^{\mathrm{T}}/\det J$. In this way, solid metamaterials can be designed, instead of needing to change the parameters of a fluid, which are difficult to adjust.

Experimental demonstrations of transformed thermal convection are rare, owing to the difficulty in controlling fluid flow. Hydrodynamic metamaterials have been fabricated for controlling the Stokes flow, with promising experimental results[122]. The flow is controlled by constructing microstructures to achieve anisotropic viscosity. This work implies that transforming viscosity has the same effect as transforming permeability in porous media. Thus, combining the fluid method with the transformed thermal conductivity is a promising approach for fabricating thermal-convection metamaterials. Moreover, this methodology can be used in Hele–Shaw flows, which have flow depths that are much smaller than the other dimensions[123–125]. The situation is more complicated for fluid flows with large Reynolds numbers, a regime that has not yet been explored.

Applications

Many interesting physical properties and phenomena have been discovered in hybrid conduction–convection systems because the heat flow can be controlled through the conduction component or the convection component, giving these systems more freedom for manipulating heat transport than conduction systems. Such discoveries include zero-index thermal metamaterials, thermal wave control, thermal non-reciprocity and thermal topology.

Based on the similarity between zero refractive index in the Maxwell equations and infinite thermal conductivity in the Fourier law, a thermal analogue of zero-index metamaterials was used to construct a bilayer cloak with an integrated convective element[126] (Fig. 2a). The device behaves like a conventional thermal bilayer cloak by rotating the fluid in the inner layer. The integrated convective element creates an extremely high effective thermal conductivity in the rotating fluid layer. It has previously been challenging to design a thermal cloak to work in environments with very large thermal conductivities because the scattering cancellation method requires that the thermal conductivity of the outer layer is larger than that of the surrounding environment, and natural materials have a narrow range of conductivities. Therefore, this work solves this problem and provides a new route for manipulating thermal convection. Additionally, inspired by this work, adaptive metadevices with rotating solid layers were subsequently reported[127,128].

Conventional thermal waves triggered by thermal relaxation[129] are usually hard to study, owing to the short relaxation time and rapid dissipation. Hence, thermal metamaterials for controlling thermal waves were proposed based on a complex thermal conductivity[130]. The real and imaginary parts of the complex thermal conductivity are related to conduction and convection, respectively. The study proved that the complex conduction equation that dominates the conduction–convection process of thermal waves is form-invariant under a coordinate transformation. Accordingly, functional devices such as cloaking, concentrating and rotating devices were also designed using this theory[130]. The complex conduction equation is the conduction–convection equation with a mathematical substitution. Hence, the corresponding transformation rules should be the same as equations (4a–c). This type of thermal wave can be manipulated using conventional transformation theory, which led to a proposed thermal waveguide with graded parameters[131,132].

New physics

With the inclusion of convection in thermal metamaterials, some unprecedented properties emerge: for example, thermal diffusion non-reciprocity (that is, directional dependence of the transport properties of heat). Inspired by the Zeeman effect, an angular-momentum bias induced by the application of a volume force was used to generate non-reciprocal thermal waves in a three-port ring[133] (Fig. 2b, top). By modulating the volume force applied along the anticlockwise direction, the propagation behaviour of the thermal waves can be manipulated. When the optimal volume force is applied, the amplitude of the temperature oscillations at port 3 is reduced to zero, whereas these oscillations are still present at port 2 (Fig. 2b, bottom). The maximized thermal non-reciprocity indicates that the temperature propagation at port 3 is fully isolated. Another study proposed a structure in which multiple three-port ring systems are arranged in a graphene-like array to produce thermal edge states, meaning that thermal waves can only be transported in the edges of the array[134]. Diffusive Fizeau drag was also demonstrated using spatiotemporal modulation[135] (Fig. 2c). The vertical advection and periodic inhomogeneity of the density and thermal conductivity cause the temperature field to travel with different velocities in the positive and negative x directions. These studies pave the way to a new technique for controlling non-equilibrium heat transfer and are suitable for mass diffusion systems.

Topological phenomena emerge not only in thermal conduction systems but also in thermal conduction–convection systems[118,136–141]. Anti-parity–time symmetry[142–150] was also discovered in thermal-diffusion systems[151]. Such symmetry was demonstrated using two rings rotating with the same speed (v) but in opposite directions that are thermally connected through an intermediate layer[152] (Fig. 3). The wavelike temperature fields in this system allow the heat transfer problem to be reduced to an eigenvalue problem characterized by a 2 × 2 non-Hermitian matrix

$$\hat{H} = \begin{bmatrix} -\mathrm{i}\left(k^2D_\mathrm{t}+h\right)+kv & \mathrm{i}h \\ \mathrm{i}h & -\mathrm{i}\left(k^2D_\mathrm{t}+h\right)-kv \end{bmatrix} \tag{5}$$

where k is the wavenumber of the wavelike temperature fields, D_t is the thermal diffusivity of the rings, and h describes the heat exchange rate between the rings. The eigenvalue of $\hat{H}$ is

$$\omega_\pm = -\mathrm{i}\left(k^2D_\mathrm{t}+h\pm\sqrt{h^2-k^2v^2}\right) \tag{6}$$

where ω is the frequency of the wavelike temperature fields.

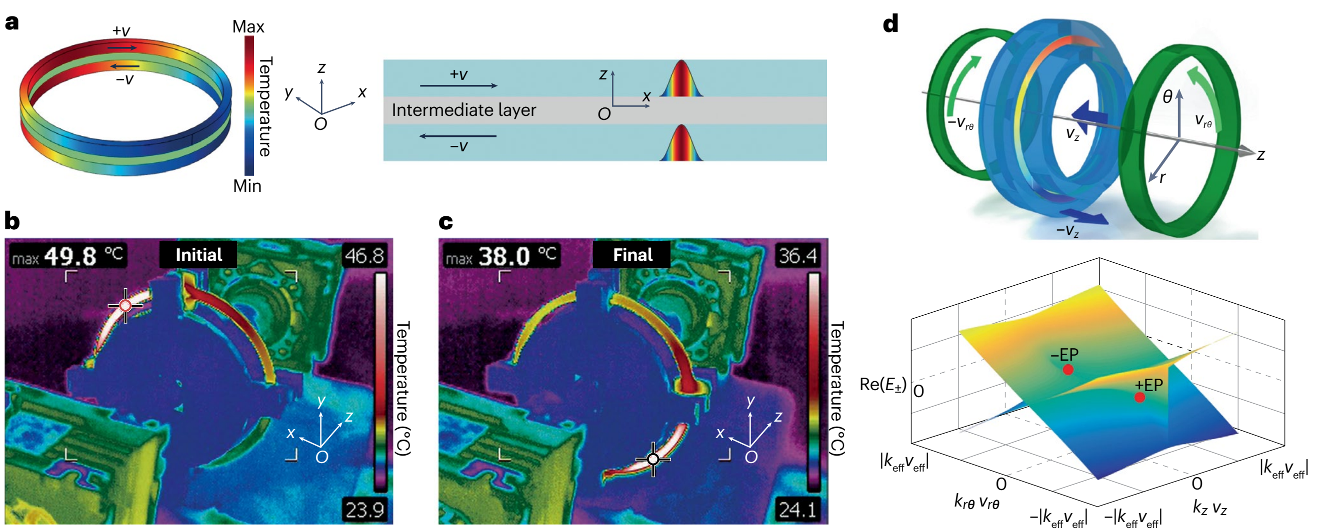


Fig. 3 | Topology and non-Hermitian physics in thermal metamaterials. **a**, 3D model (left) and simplified 2D model (right) for investigating the geometric phase of two counter-rotating rings (with velocity v) that are coupled through an intermediate layer. The Gaussian profiles show the locations of the maximum temperatures within the two rings. **b**,**c**, Initial (**b**) and final (**c**) states of the temperature of the counter-rotating rings when their rotation speed increases from zero to a value above the exceptional point and back to zero. The location of the maximum temperature (indicated by the markers) moves from one side of the ring to the other, demonstrating a geometric phase of π. **d**, Top: a schematic diagram of a four-ring system coupled with two orthogonal pairs of counter-moving advection components. The green rings rotate in opposite directions in the r–θ plane with a speed of $v_{r\theta}$ and a wavenumber of $k_{r\theta}$. The blue rings move in opposite directions along the z axis with a speed of v_z and a wavenumber of k_z. The multicoloured band shows the temperature profile of the central medium. Bottom: the real part of the complex spectrum of the eigenvalue of the thermal system ($E_\pm$) in the synthetic parameter space with the exceptional points (EPs) shown in red, where $k_\mathrm{eff}v_\mathrm{eff} = \sqrt{(k_{r\theta}v_{r\theta})^2+(k_zv_z)^2}$. Topological phase transitions can be achieved with this system by changing the number of EPs encircled in this parameter space. Parts **a**, **b** and **c** adapted with permission from ref. [152], Elsevier. Part **d** adapted with permission from ref. [155], APS.

As v increases from zero, ω changes from a purely imaginary number to a complex number with a real part. The exceptional point of $v = v_{EP} = \sqrt{h/k}$ plays a crucial role. When $v < v_{EP}$, ω is purely imaginary, indicating that the wavelike temperature field does not propagate and only decays with time. This case preserves the anti-parity–time symmetry. When $v = v_{EP}$, the two eigenstates ($\omega_\pm$) are degenerate; therefore, v_{EP} serves as an exceptional point. When $v > v_{EP}$, the real part of ω is non-zero, so the temperature fields start moving, corresponding to anti-parity–time symmetry breaking[153,154].

Interestingly, topological phenomena in thermal metamaterials were established using the non-Hermitian approach owing to the diffusive nature of the metamaterials. In fact, the first topological invariant of heat diffusion systems to be discovered was the geometric phase in a diffusion–convection system with a pair of coupled rotating rings[152]. When v evolves on a 'closed path' (meaning that the initial and final velocities are the same), the geometric phase can be calculated based on the eigenvectors. The phase is 0 when the path does not contain the exceptional point, and ±π when it does. Accordingly, if the geometric phase is ±π, the location of the maximum temperature point will rotate by π radians instead of returning to its original position. If the geometric phase is 0, the location of the maximum temperature will return to its initial position after the rotation speed has completed a closed evolution loop (Fig. 3a–c).

After that, the parameter space was extended to diffusion systems, revealing rich topological properties. Two pairs of rings coupled through a central medium were designed to demonstrate a configurable non-Hermitian topological phase transition in a 2D parametric space[155] (Fig. 3d, top). Viewing the system in cylindrical coordinates, $v_{r\theta}$ and v_z form an orthogonal parameter space, similar to k_x and k_y in momentum space. In such a parameter space, there is a pair of exceptional points (Fig. 3d, bottom). Closed paths in this parameter space that encircle different numbers of exceptional points are in different geometric phases. Thus, topological phase transitions can be achieved by changing the number of encircled exceptional points. This study shows that geometric phases are effective topological invariants. Additionally, eigenstate winding number and eigenvalue vorticity[138], which describe the exceptional point encircling behaviour of the complex eigenvector and eigenvalue respectively, can also be calculated and serve as topological invariants. For example, when two exceptional points are enclosed, a geometric phase of ±π, an integer eigenstate winding number and an integer eigenvalue vorticity are achieved. In this case, the system exhibits a dynamic-equilibrium temperature distribution that is topologically non-trivial. When only one exceptional point is encircled, the system has a geometric phase of ±π, a half-integer winding number eigenstate, and a half-integer vorticity eigenvalue. In this case, the temperature profile has a step-like π-phase transition that is also topologically non-trivial. Meanwhile, the SSH-like thermal conduction chain is also topological[156] and is the anti-Hermitian counterpart of the original SSH model[157], owing to the anti-Hermitian nature of heat conduction. These discoveries unveil intriguing and unprecedented physics at the interface of topological physics and non-Hermitian physics.

Thermal radiation

Theory

Thermal radiation — another essential form of heat transport — is ubiquitous and crucial in many fields, for example in the harvesting of solar radiation for energy applications. Metamaterials for manipulating thermal radiation have attracted tremendous research interest in the past decade. Transformation multithermotics theory was established to enable the simultaneous control of thermal radiation and conduction[158]. The Rosseland diffusion approximation, a perturbation of blackbody radiation that describes thermal radiation as the diffusion of photons, was developed to handle the thermal coupling between radiation and conduction. Based on this approximation, the dominant equation of multithermotics is

$$\rho C\frac{\partial T}{\partial t} + \nabla\cdot\left[-\kappa\nabla T + \left(-\frac{16}{3}\gamma^{-1}n^2\sigma T^3\nabla T\right)\right] = 0, \quad (7)$$

where the first term in the square brackets represents conductive flux, and the second represents radiative flux. ρ, C and κ have been mentioned previously, while γ, n and σ are the Rosseland mean attenuation coefficient, relative refractive index and Stefan–Boltzmann constant, respectively. The coordinate transformation of these parameters is described by

$$(\rho C)' = \frac{(\rho C)}{\det J} \quad (8a)$$

$$\gamma' = J^{-\mathrm{T}}\gamma J^{-1}\det J \quad (8b)$$

$$\kappa' = \frac{J\kappa J^{\mathrm{T}}}{\det J}, \quad (8c)$$

indicating that the transformed equation is form-invariant, which suggests that it could be possible to simultaneously manipulate thermal radiation and conduction. Using these coordinate transformations, prototype functional devices for multithermotics, such as cloaks, concentrators and expanders, have been proposed[158]. The temperature dependence of thermal radiation also makes it possible to extend the theory to nonlinear cases. In other words, applications using thermally responsive metamaterials could be possible.

In a more general scenario, thermal conduction, convection and radiation always appear together. However, owing to their distinct mechanisms, it is challenging to unify these three modes within the framework of transformation theory. A method that uses transformation omnithermotics to manipulate the three basic modes of heat transfer was deduced by adding a convective flux term to equation (7) (ref. [159]). In pure fluids, the heat transfer involving conduction, convection and radiation is governed by

$$\rho_f C_f\frac{\partial T}{\partial t} + \nabla\cdot\left[-\kappa_f\nabla T + \rho_f C_f \mathbf{v} T + \left(-\frac{16}{3}\gamma_f^{-1}n_f^2\sigma T^3\nabla T\right)\right] = 0, \quad (9)$$

in which ρ_f, C_f, κ_f, $\mathbf{v}$, γ_f and n_f are the density, heat capacity, thermal conductivity, velocity, Rosseland mean attenuation coefficient and relative refractive index of the fluid, respectively. The transformation rules for these parameters are

$$\rho_f' C_f' = \frac{\rho_f C_f}{\det J} \quad (10a)$$

$$\gamma_f' = J^{-\mathrm{T}}\gamma_f J^{-1}\det J \quad (10b)$$

$$\kappa_f' = \frac{J\kappa_f J^{\mathrm{T}}}{\det J} \quad (10c)$$

$$\mathbf{v}' = J\mathbf{v}, \quad (10d)$$

which show that the total heat flux can be modulated by adjusting $\rho_f C_f$, γ_f, κ_f and $\mathbf{v}$.

The parameters derived from transformation multithermotics are also anisotropic, which is a property that is rarely observed in natural materials. Such material parameters have been achieved using a layered structure design and effective media theory[158]. Alternatively, scattering cancellation theory can also achieve the same effect[160]. Functional devices with isotropic parameters could be directly obtained by solving equation (9).

Applications

Thermal metamaterials for manipulating radiation are primarily used in thermal vision and energy applications, such as thermal camouflage and radiative cooling. Various methods for overcoming the complexity of the transformed parameters have been reported.

The simplest application is camouflage (or thermal illusion) that produces heat signatures that are closely related to the actual scenario. In addition to hiding targets from external detectors, thermal illusion devices could exhibit fake signals to achieve a misleading effect. For example, a system was developed that cloaks one object by transforming its heat signature so that it is detected as a different object[42]. Another study used the regionalization transformation method to create thermal illusions and even achieve encrypted thermal printing, which hides information so that it is only visible when the correct heat source is applied[161]. Subsequently, the concept of 3D illusion thermotics was proposed to overcome the drawbacks of imperfect separation in 2D illusions[162,163]. Approaches to achieving camouflage for multiple physical fields have also been investigated. For example, a multispectral camouflage device for infrared, visible, laser and microwave bands has been proposed[164]. We anticipate that combining multiple disciplines and reducing the thickness of the devices will be future trends in the field of thermal camouflage.

Thermal-radiation patterns can also be controlled by thermal conduction, thereby simultaneously manipulating thermal radiation and conduction[165]. In 2018, a 2D structured thermal surface for radiative camouflage was developed based on transformation thermotics[166]. The camouflage surface was designed using a two-fold transformation and exhibited a small, cloaked region on the surface of the device in which an object can be hidden from thermal-radiation detection. The fabricated device can adjust the temperature profile of the surface regardless of the background thermal conductivity. The thermal camouflage surface was later extended to the 3D case[167]. The thermograph of the system demonstrates that the measured temperature profile of an object covered by the device is almost the same as the temperature profile of pure background (Fig. 4a). Another study used regionalization transformation theory to construct thermal metamaterial strokes that function as infrared signatures[161] (Fig. 4b). Heat at one location is used to create a heat signature of one basic stroke by adjusting the thermal conductivities of the rectangular regions. These individual strokes can then be combined to recreate the whole alphabet; therefore, this approach could be used for thermally encoding information so that it is only visible in an infrared image.

Different methods for regulating omnithermal fields have been explored to remove sophisticated parameters. For example, an omnithermal restructurable metasurface was designed to create infrared-light illusions with devices that look the same under visible light[168]. This study considered three basic modes of heat transfer for tuning the surface temperature and adopted the radiation-cavity effect to fabricate metasurfaces. This approach was used to create a device with tunable surface temperature and emissivity. The effective emissivity of a surface with a cavity (ε_b) is equivalent to that of a flat surface

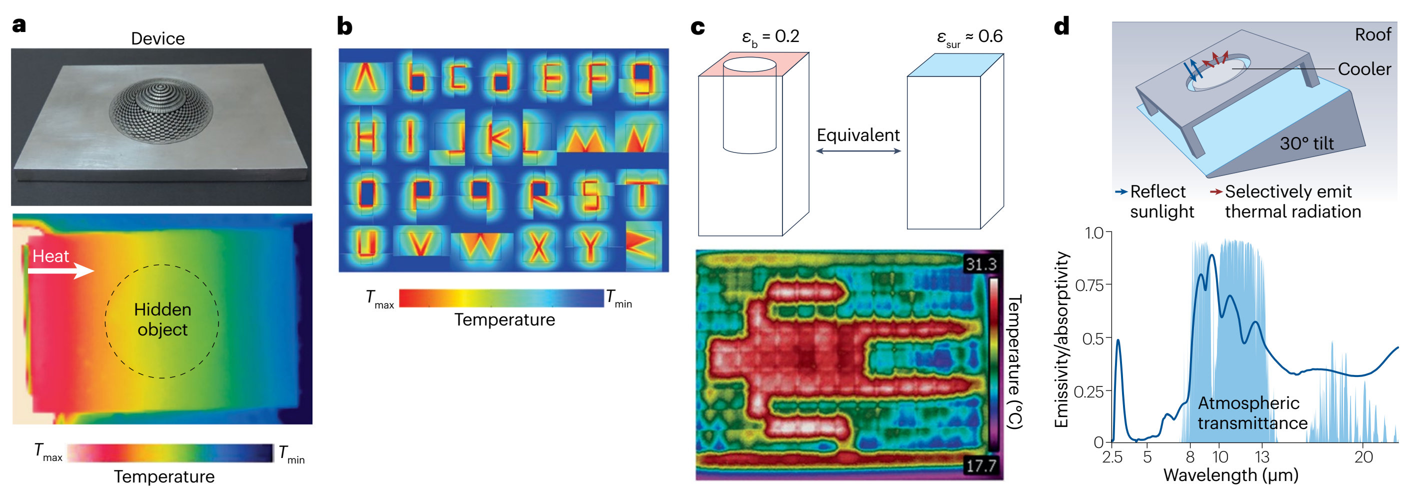


Fig. 4 | Metamaterials for manipulating heat conduction, convection and radiation. a, Top: an object covered by a fabricated 3D meta-helmet. Bottom: a thermograph of the object camouflaged by the helmet acquired with an infrared camera at normal incidence to the device, where T_{max} and T_{min} are the maximum and minimum temperature, respectively. The circle indicates the location of the device. **b**, Finite-element simulations of heat signatures of the entire alphabet composed of individual basic strokes made by adjusting the thermal conductivity of the regions. The colour bar shows the temperature of the regions. **c**, Top: schematic diagrams demonstrating the effective emissivity principle whereby the surface of a unit made from a material with emissivity ε_b containing a cavity (left) has the same surface emissivity ε_{sur} as a unit made from a material with different properties without a cavity (right). Bottom: thermograph showing the illusion of a human shape, made by constructing an array of components with and without cavities to control the emissivity of the surface. **d**, Top: schematic of a radiative cooler device. Bottom: the measured emissivity and absorptivity of the device over mid-infrared wavelengths, where the shaded region is the atmospheric transmittance window. Part **a** adapted with permission from ref. [167], Wiley. Part **b** adapted with permission from ref. [161], Wiley. Part **c** adapted with permission from ref. [168], APS. Part **d** adapted from ref. [171], Springer Nature Limited.

made from a material with a different emissivity (ε_{sur}); therefore, by combining unit components with or without cavities an image can be encoded into the heat signature of the surface to form the desired pattern. This technique was used to create an illusion of the outline of a human (Fig. 4c), demonstrating the validity of this theory.

The consideration of omnithermal fields is vital for energy applications. For instance, daytime radiative cooling, a passive process that does not require any energy input, has great potential for renewable energy research and for meeting the demand for air conditioning[169]. The idea of promoting radiative cooling by constructing a white paint from TiO_2 was proposed as early as 1978 (ref. [170]). However, this work did not draw much attention at the time. In 2014, a photonic approach was used to fabricate a radiative cooler[171]. The main principle is to design a device that reflects solar radiation with selective thermal emission in the mid-infrared range. A multilayered structure of HfO_2 and SiO_2 was used to obtain this particular property (Fig. 4d). The device was designed with a well-sealed air pocket to minimize the heating of external surfaces and air adjacent to the radiative cooler. The measured emissivity of the cooler confirmed that the device selectively emits radiation at wavelengths within the atmospheric transparency window (8–13 μm), allowing the infrared radiation to be transmitted to outer space. Another study fabricated hybrid metamaterials for daytime radiative cooling by embedding SiO_2 microspheres randomly within a polymeric matrix[172]. The fast and simple fabrication process of this device could enable it to be produced industrially.

Alternatively, thermal diodes designed using nonlinear theory[106] or spatiotemporal modulation[173–181] are of great value for harvesting heat energy because they are natural tools for ensuring that there is only a single path of heat flow[182]. A thermal trapping mechanism based on graded heat-conduction metadevices was proposed, inspired by black holes, which trap light within their event horizons. This mechanism has potential applications in waste heat recovery and provides insights into the combination of transformation thermotics and cosmology[183]. Additionally, nanoscale heat transport is essential to manipulate heat energy in energy-harvesting applications[184].

New physics

The freedom offered by the multiple components of the heat flow of omnithermal fields (that is, thermal conduction, convection and radiation, inclusively) is encouraging. It could lead to unprecedented thermal control and introduce new physics. Unlike thermal conduction, radiative heat flow is quadratic with temperature according to the Stefan–Boltzmann law, leading to strong nonlinearity[159]. This feature may bring new ideas to the study of thermal topology. Loading nonlinearities into classical topological models could enable many new functions[185–199], such as self-induced transitions from trivial to non-trivial topological states[200,201]. In a topological thermal-radiation system, nonlinearity can be achieved by transferring the higher-order dependence to the radiation coefficient. Topological thermal-radiation systems have the potential to achieve functions that linear thermal systems cannot, for example topological phase transitions. Moreover, thermal radiation with strong nonlinearity could reveal unprecedented topological phases in thermal higher-order topological insulators.

In addition to far-field thermal radiation, near-field thermal radiation with new properties has also been studied. Owing to the enhanced near-field coupling of evanescent waves on the surface, near-field radiation can overcome the far-field limit imposed by the Stefan–Boltzmann law, which affects various thermal technologies. One study developed metasurfaces composed of 2D periodic arrays of holes to enhance near-field radiative heat transfer[202]. This work relies on the ability to control the bandwidth of surface-plasmon polaritons. Another study performed a quantitative measurement of near-field heat transfer between two hyperbolic metamaterials[203]. This measurement validated the effective medium theory in near-field radiative metamaterials under a classical gap-period condition, providing quantitative experimental evidence for the enhanced radiation.

Transformation particle diffusion

Theory

As a ubiquitous phenomenon, particle diffusion described by the Fick equations has been investigated in various situations. Without external particle sources or sinks, Fick's second law is

$$\frac{\partial c}{\partial t} = \nabla \cdot (D\nabla c - \mathbf{v}c) \qquad (11)$$

where c, D and $\mathbf{v}$ are the particle concentration, diffusivity and advection velocity, respectively. Following a coordinate transformation, this equation becomes

$$\frac{1}{\det J}\frac{\partial c}{\partial t} = \nabla' \cdot (D''\nabla' c - \mathbf{v}''c) \qquad (12)$$

with $D'' = JDJ^{\mathrm{T}}/\det J$ and $\mathbf{v}'' = J\mathbf{v}/\det J$. In contrast to equation (11), an additional metric term emerges on the left side of equation (12), causing the breakdown of transformation invariance. Consequently, cloaking or concentrating transient particle diffusion is not perfect.

Despite this change in the form of Fick's second law following the coordinate transformation, a particle-diffusion cloak made using the low-diffusivity approximation was computationally demonstrated[18] (Fig. 5a). This pioneering study inspired tremendous work on transformation particle diffusion.

Moreover, Fick's law can be used to describe light propagation in disordered media. In 2014, a water-based invisibility cloak was produced by using diffusive-light propagation[48]. This cloak was designed using the scattering cancellation method and made of polydimethylsiloxane doped with melamine-resin microparticles (Fig. 5b). When tested in transient illumination conditions, the experimental results demonstrated that the cloak worked in static and quasistatic regimes[54]. In systems that contain many randomly distributed scattering centres, such as clouds, fog, milk and frosted glass, light propagation behaves like photon diffusion rather than a ballistic beam. This diffusive-light cloaking approach works for a broad bandwidth; therefore, it overcomes one of the limitations of conventional electromagnetic cloaking[204].

Alternatively, an optimized theory of transformation mass transfer was proposed to control transient particle diffusion and overcome the breakdown of transformation invariance[205]. This study considered Fick's second law in the transient state and transformed the equation as

$$\frac{\partial c}{\partial t} = \nabla' \cdot (D'\nabla' c - \mathbf{v}'c) \qquad (13)$$

with $D' = JDJ^{\mathrm{T}}$ and $\mathbf{v}' = J\mathbf{v}$. In this way, equation (13) approximately keeps the same form as equation (11) when the diffusivity and velocity are small. This approach was used to design theoretical devices for separating, cloaking, concentrating and rotating chemical waves (Fig. 5c). Computer simulations of these devices demonstrated that the devices could be used to manipulate chemical waves and confirmed the

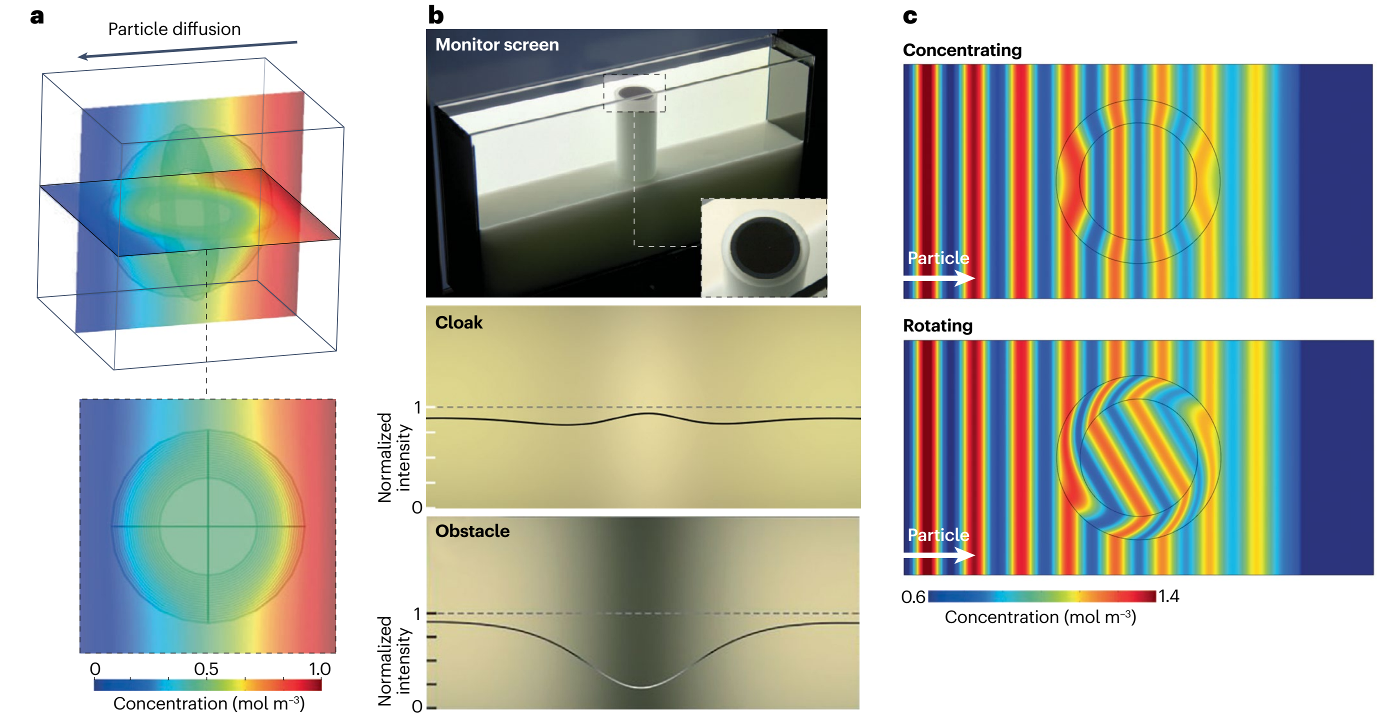


Fig. 5 | Transformation diffusion and metamaterials. **a**, Top: 3D view of a simulated particle-diffusion cloak, where the colour bar shows the particle concentration and the particles are diffusing from right to left. Bottom: 2D slice from the top plot. **b**, Top: the experimental set-up used to create a light-diffusion cloak that conceals the cylindrical obstacle (inset). The tank is filled with a mixture of deionized water and white paint. Middle and bottom: photographs of the light coming from the tank with (middle) and without (bottom) the cloak around the obstacle. The solid black curves show the normalized light intensity at the location indicated by the dashed line. **c**, Simulated chemical-wave concentration profiles after passing through concentrating (top) and rotating (bottom) devices, which were designed using the optimized theory of transformation mass transfer. The arrow shows the direction of propagation of the chemical wave. Part **a** adapted from ref. [18], CC BY 4.0. Part **b** adapted with permission from ref. [48], AAAS. Part **c** adapted with permission from ref. [205], Wiley.

excellent performance of particle-diffusion metamaterials based on this theory.

Applications

Achieving cloaking with particle diffusion is particularly valuable for drug delivery, for example using concentric liposomes as a particle cloak to protect water-soluble drugs during transportation[18]. A series of techniques have been proposed for concentrating and cloaking particle diffusion[206,207]. As different types of particles have different diffusivities within the same media, particle separation can be achieved by cloaking one type of particle while concentrating the other. Hence, a bilayer metadevice[208] was proposed for separating O_2 and N_2. This device achieves cloaking (or concentrating) of O_2 (or N_2) in the central region, offering a potential application for separating mixed gases (Fig. 6a). This work uses two natural materials to design a bifunctional metadevice based on the scattering cancellation method, which simplifies the fabrication process. Additionally, a metamaterial membrane was proposed to separate two particle types into different locations[206,209,210] (Fig. 6b). Simulations demonstrated that the two different particles diffused to different positions after flowing through the membrane, suggesting that this metamaterial membrane is a good candidate for separating mixed particles.

Particle-diffusion metamaterials are also valuable for other applications: for example, one application used such a metamaterial to produce an electrical cloak[211]. The nearly flat electrical potential observed behind the device indicates the near-perfect cloaking effect (Fig. 6c). In reality, diffusion metamaterials are challenging to produce experimentally. Numerical algorithms are often used to design optimized devices that are experimentally feasible[49], particularly devices for general applications such as those involving diffusion and chemical reactions[50].

New physics

One of the most exciting developments in particle-diffusion metamaterials is from the fundamental aspect: the synergy with non-Hermitian physics and topology. One study reported that geometric phases could appear in particle-diffusion metamaterials when looping around an exceptional point[212]. The set-up (Fig. 6d) involves two rotating rings coupled through an intermediate layer with the rotation velocity (u) controlling the dynamics of the particle concentrations, similar to the approach described previously that was used to explore geometric phase in thermal-diffusion systems. A non-Hermitian Hamiltonian can be used to describe the particle-diffusion dynamics in such a system. The eigenvalues of the non-Hermitian Hamiltonian (that is, ω) are complex numbers with real and imaginary parts that describe the propagation velocity and temporal decay rate of the particle concentration, respectively. The relationship between ω and u contains exceptional points (Fig. 6d). It was found that when the velocity path encircles an exceptional point, a phase difference of π can be accumulated.

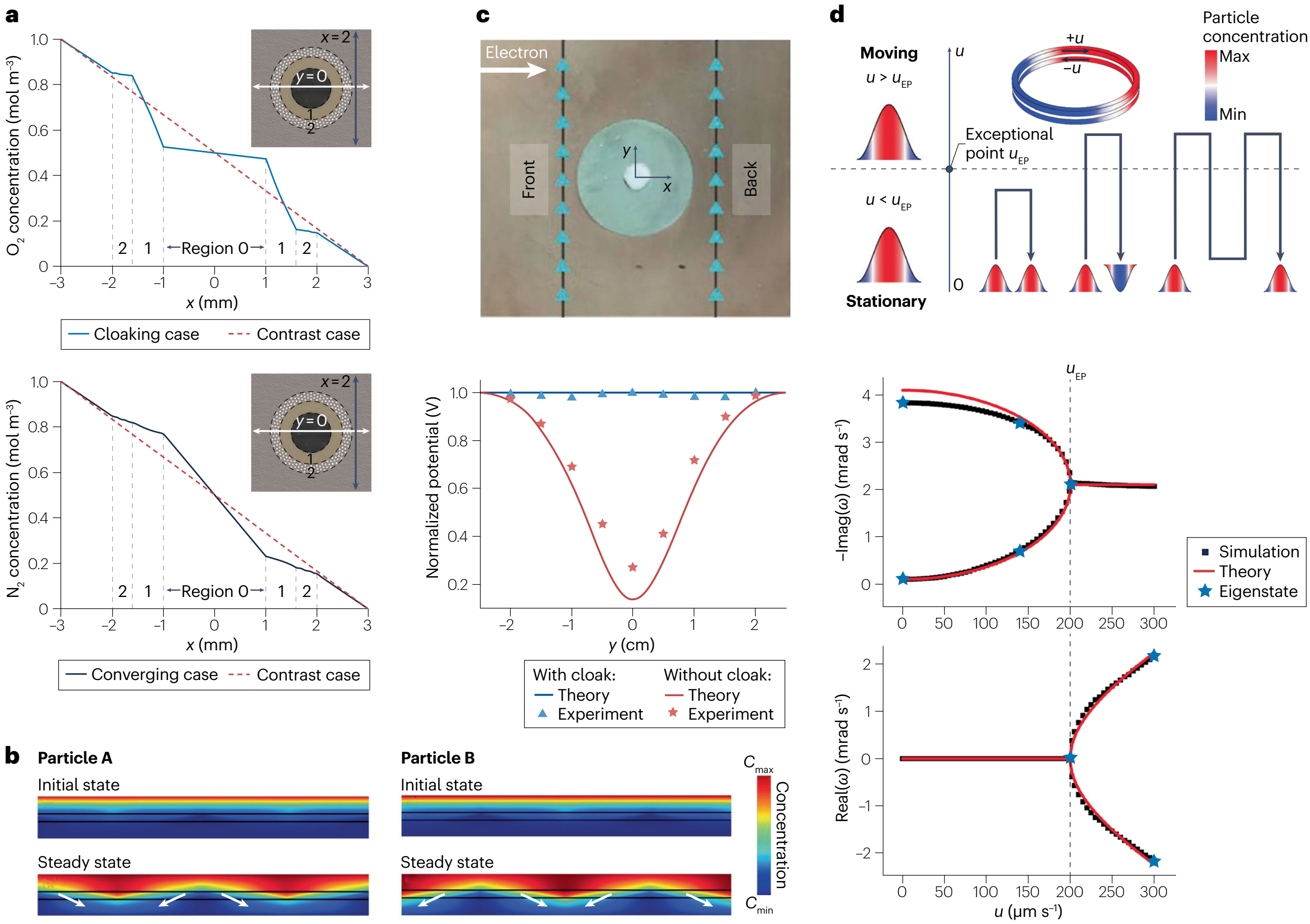


Fig. 6 | Manipulating particle diffusion with metamaterials. a, A bilayer metadevice that cloaks O_2 (top) and concentrates N_2 (bottom) in the centre of the device. The concentration of O_2 is almost constant across region 0, showing that nearly no O_2 flows into the region, and the concentration gradient of N_2 in region 0 is bigger than that of the contrast case, demonstrating the converging effect of the device. The concentrations were measured along the white $y = 0$ line shown in the inset. The vertical lines indicate the locations of regions 0, 1 and 2 shown in the insets. Insets show schematics of the samples. **b**, Simulations of the performance of a metamaterial membrane to separate different particle types by transient diffusion. The colour maps show the concentration distribution of the two particle types at the start and end of the process, where C_{max} and C_{min} are the maximum and minimum concentration, respectively. The solid black lines indicate the location of the membrane, and the white arrows show the diffusion direction. **c**, Top: photograph of a fabricated alumina plate on a silver plate, which functions as an electrical cloak. The arrow shows the direction of electron flow, and the black lines show the locations at which measurements were performed to test the cloaking. Bottom: the measured (triangles and stars) and simulated (lines) voltages along the back line, with and without the cloak. **d**, A system comprising two rings that rotate in opposite directions with velocity u, separated by a permeable layer (top inset), is used to study geometric phase in particle diffusion. Top: the change in the concentration profiles in the rings (shown by the Gaussian profiles) when u maps a cyclic path in velocity space crossing the velocity exceptional point (u_{EP}) different numbers of times. When $u > u_{EP}$, the concentration profiles in the rings begin to move. Depending on the number of times u crosses u_{EP}, the particle concentration returns to the initial state with or without a π phase shift. Bottom: the negative imaginary (imag) and real parts of the complex frequency ω as a function of u. The eigenstates are shown by blue stars. When $u < u_{EP}$, the system is in an anti-parity–time symmetric region. When $u > u_{EP}$, the real part of the eigenvalue appears, owing to the breaking of anti-parity–time symmetry. Part **a** adapted with permission from ref. [208], Elsevier. Part **b** adapted with permission from ref. [206], IOP. Part **c** adapted with permission from ref. [211], Wiley. Part **d** adapted with permission from ref. [212], APS.

The discovery of the geometric phase in particle diffusion unveils the possibility that new topological phenomena could be present in diffusion metamaterials. This understanding naturally leads to the development of topological diffusion metamaterials. A 1D particle-diffusion lattice with alternating high and low diffusivities can be used to mimic the SSH model. In such a set-up, bulk–edge correspondence can appear in particle diffusion, and new edge states could be uncovered, as has already been confirmed in heat-conduction metamaterials[117].

Transformation plasma transport

Theory

Plasma, the fourth state of matter, is a gaseous mixture of unbound ions, electrons and reactive radicals, and is highly conductive. Unlike general diffusion systems, plasma transport is affected by particle collisions and by the local electromagnetic field generated by local charge concentrations, and thus exhibits collective behaviours. As a result, plasma transport is also a unique diffusion system. Plasma can be artificially produced by charging gases with direct or alternating currents, radio-frequency waves or microwave sources[213]. The rich components of plasma give it unique properties such as high chemical activity. Therefore, studying plasma is both necessary and valuable.

Despite many theoretical and experimental studies, manipulating plasma transport still faces critical challenges. For example, it is difficult to obtain an analytical solution of the motion tracks or the dynamic details of the ionized species, because the strong coupling between charged particles and electromagnetic fields makes plasma transport a nonlinear process[214–216]. Therefore, researchers often use numerical simulation methods (such as the particle-in-cell or Monte Carlo collision models) to analyse plasma transport[217,218].

Transformation theory was introduced to plasma physics by using a diffusion–migration model to describe plasma transport[60]. This study designed three conceptual devices to control plasma diffusion based on transformation theory. First, several assumptions were made: for example, the effects of convection and magnetic fields were ignored, and the temperature of the charged particles was considered to be constant. The transport of positive ions in plasma can then be described by a diffusion–migration equation. Using the Einstein relation to define mobility in terms of diffusivity, the diffusion–migration equation can be written as

$$\frac{\partial n_i}{\partial t} - \nabla \cdot (D \nabla n_i) + \nabla \cdot \left[\left(\frac{D\mathbf{E}}{T_i} \right) n_i \right] = 0 \quad (14)$$

where n_i, D, $\mathbf{E}$ and T_i are the density, diffusivity, electric field and reduced temperature, respectively ($T_i = Tk_B/q$, where T is the temperature, k_B is the Boltzmann constant and q is the unit charge). Equation (14) can then be transformed into an approximate form under a coordinate transformation,

$$\frac{\partial n_i}{\partial t} - \nabla' \cdot (D' \nabla' n_i) + \nabla' \cdot \left[\left(\frac{D'\mathbf{E}'}{T_i} \right) n_i \right] = 0 \quad (15)$$

with $D' = JDJ^{\mathrm{T}}$ and $\mathbf{E}' = J^{-\mathrm{T}}\mathbf{E}$. Equation (15) suggests that plasma flows could be guided by modulating D and $\mathbf{E}$. To test this theory, three devices were designed for cloaking, concentrating and rotating plasma flows[60]. By assuming that the plasma density fluctuates in a plane-wave form, the transformed parameters can be calculated according to the transformation rules.

Using a harmonic profile of plasma density, boundary conditions were applied to excite a wavelike density oscillation in the plasma. Simulations of the plasma cloak, concentrator and rotator show that the devices perform well for manipulating plasma transport (Fig. 7a). The decrease in the concentration amplitude of the plasma as it travels through the medium is caused by the dissipative nature of diffusion. The dispersion relation for this system can be deduced by substituting a plane-wave solution into equation (14).

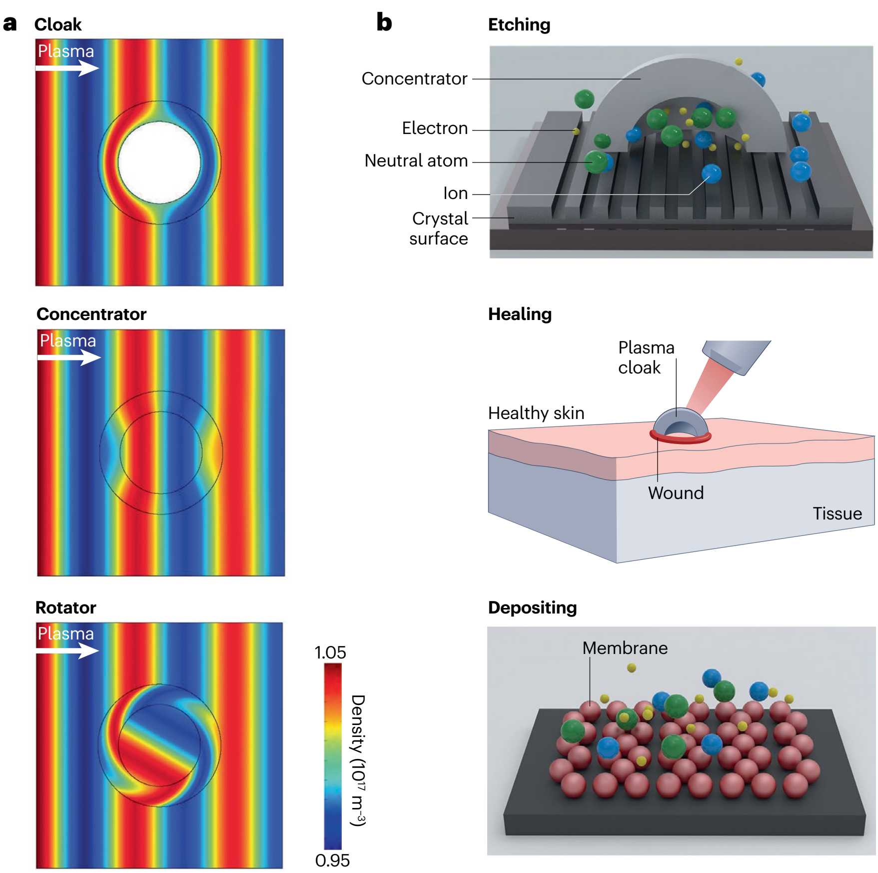


Fig. 7 | Plasma transport and metamaterials. a, Simulated plasma density profiles of cloaking (left), concentrating (middle) and rotating (right) devices. The arrows indicate the direction of plasma flow. The parameters for these simulations were chosen to produce appropriate decay rates and wave numbers to make these results clear and intuitive. **b**, Plasma applications for etching, cloaking healthy skin when healing wounds, and deposition. Part **a** adapted with permission from ref. [60], CPS.

Applications

Plasma technology is crucial for practical fields, such as micro- and nano-electronics, chemistry, biomedicine, aerospace and materials science[59] (Fig. 7b). Plasma is full of high-energy particle species that can uniquely modify the surfaces of materials. Thus, plasma-based technologies can be used for sputter-deposit processing, etching integrated circuits, and welding. The highly reactive radicals in plasma can induce chemical reactions at the plasma–matter interface that are usually difficult to achieve under normal conditions. Such techniques have also been used for catalyst preparation and greenhouse gas conversion[59]. Additionally, many diseases, such as infected wounds and cancers, can be treated with cold plasma[58]. In the aerospace industry, plasmas are used to break high-carbon chains of molecules in the fuel into low-carbon chains, which improves the ignition and combustion performance of aerospace engines[55]. The new approaches to manipulating plasmas introduced above could help to further improve plasma technologies[60]. The larger the plasma flow, the more beneficial it is to the etching or combustion process. Plasma concentrators can increase the plasma flow in some areas (Fig. 7b) to improve plasma etching. Similarly, concentrators can also increase the efficiency of catalysts and plasma-assisted engines. Plasma cloaks can be used to protect healthy tissue exposed to plasmas when curing infected wounds. Appropriate coordinate transformations can be used to separate or guide plasmas for directional deposition.

The transformed diffusivity and electric field are generally challenging to achieve. However, it is possible to produce the same effect through other methods. For example, a bilayer diffusive cloak could be fabricated using two homogeneous materials based on the scattering cancellation method[208]. Moreover, the static electric cloak could inspire the manipulation of electric fields[219]; that is, a metamaterial-based method for controlling plasma transport may trigger a new trend in plasma physics. Note that the term 'plasma metamaterials' has already been used to describe plasma-based metamaterials for manipulating electromagnetic waves[220–224]. In detail, plasma metamaterials refers to devices that use a medium composed of plasma with a periodic structure, which can control the transmission of electromagnetic waves by changing the dielectric constant and susceptibility of the plasma. Hence, plasma metamaterials could be regarded as a kind of electromagnetic metamaterial. However, the work highlighted in this Review emphasizes that plasma transport characteristics can be regulated by modifying the diffusion rate and electric field[60] to control the transmission of plasma particles. New mechanisms for developing a more detailed and accurate model of plasma transport should be explored despite the inevitable challenges: for example, generalizing the simplified model to include the influence of magnetic fields and gas-phase reactions in plasma, or adding an advection term to provide additional freedom for regulating plasmas. In short, further studies of transformation plasma physics are required.

Summary and outlook

Transformation optics has been extended to many fields throughout the past decade. Establishing the transformation principle in diffusion systems can lead to the discovery of unconventional diffusion properties and phenomena as well as enabling basic control of diffusion. In this Review, we systematically report the development of transformation theory and metamaterials in diffusion systems, especially for manipulating particle dynamics, heat diffusion and plasma physics. By presenting the basic principles, typical examples and exciting progress in these fields, we provide an overview of the development of various diffusion metamaterials that could give access to new phenomena and applications. Diffusion metamaterials are a growing field, and they solve the problem of the limited working bandwidth of wave systems, owing to the frequency independence of diffusion properties. This property alone could make diffusion metamaterials appealing for many practical applications.

However, some challenges remain to be solved. For wave and diffusion systems, the complexity and singularity of the transformed parameters are inherent problems of using transformation theory. Extremely high thermal conductivity cannot be found in natural or artificially fabricated materials, which inevitably affects the performance of designed devices. Additionally, most metadevices have only a single function and are often limited to specific operating conditions. Fortunately, opportunities arising from nonlinearity and multiphysical effects in diffusion systems allow the tuning of many parameters such as electrical conductivity and permeability. Studies of transformation theory on a microscale are still scarce. It is possible that studies of phonon diffusion could be used to fill this gap[225,226]. Additionally, the interfacial thermal resistance between different media, neglected in most research[227], is a problem worthy of further exploration. For mass diffusion, the main challenge lies in the imperfect manipulation of transient flows.

Many functional devices proposed in thermal systems, such as invisible sensors[228,229], can be transplanted into particle-diffusion systems. A promising future for diffusion metamaterials can be foreseen. Topological wave metamaterials could inspire the development of topological thermal metamaterials[230,231]. Diffusion systems have unique advantages and are convenient platforms for hosting various phenomena in non-Hermitian physics[108] (such as anti-parity–time symmetry), nonlinear physics (such as temperature-dependent thermal transport) and multiphysical fields (such as the thermoelectric effect). Combining these phenomena with topological effects in diffusion systems may lead to discoveries beyond wave systems. In addition, machine-learning techniques have been valuable for studying topological effects in wave systems[232–234]; such techniques are also expected to be helpful for studying topological diffusion metamaterials. These studies would not only fill a conceptual gap in topological physics but also provide new avenues for controlling diffusion. Beyond that, diffusion metamaterials could be used for many applications, such as materials separation, heat control, crystal growth control[235] and radiative cooling[236]. Thus, from both fundamental and application aspects, there are many intriguing topics to be explored in these metamaterials.

Published online: 23 March 2023

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Acknowledgements
We acknowledge financial support from the National Natural Science Foundation of China under grants no. 11725521, 12035004 and 12125504 and from the Science and Technology Commission of Shanghai Municipality under grant no. 20JC1414700.

Author contributions
Z.Z. researched data for the article. Z.Z., L.X., J.-H.J. and J.H. contributed substantially to discussion of the content. All authors wrote the article. J.-H.J. and J.H. reviewed and/or edited the manuscript before submission.

Competing interests
The authors declare no competing interests.

Additional information
Peer review information *Nature Reviews Physics* thanks the anonymous reviewers for their contribution to the peer review of this work.

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