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Partial information, market efficiency, and anomalous continuous phase transition

Guang Yang, Wenzhi Zheng and Jiping Huang

Department of Physics and State Key Laboratory of Surface Physics, Fudan University, Shanghai, People's Republic of China
E-mail: 11110190006@fudan.edu.cn, teasic137@gmail.com and jphuang@fudan.edu.cn

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Abstract. It is a common belief in economics and social science that if there is more information available for agents to gather in a human system, the system can become more efficient. The belief can be easily understood according to the well-known efficient market hypothesis. In this work, we attempt to challenge this belief by investigating a complex adaptive system, which is modeled by a market-directed resource-allocation game with a directed random network. We conduct a series of controlled human experiments in the laboratory to show the reliability of the model design. As a result, we find that even under a small information concentration, the system can still almost reach the optimal (balanced) state. Furthermore, the ensemble average of the system's fluctuation level goes through a continuous phase transition. This behavior means that in the second phase if too much information is shared among agents, the system's stability will be harmed instead, which differs from the belief mentioned above. Also, at the transition point, the ensemble fluctuations of the fluctuation level remain at a low value. This phenomenon is in contrast to the textbook knowledge about continuous phase transitions in traditional physical systems, namely, fluctuations will rise abnormally around a transition point since the correlation length becomes infinite. Thus, this work is of potential value to a variety of fields, such as physics, economics, complexity science, and artificial intelligence.

Keywords: applications to game theory and mathematical economics, critical phenomena of socio-economic systems, interacting agent models, stochastic processes

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Contents

1. Introduction

Complex adaptive systems formed by a large number of interacting agents (e.g., individuals, companies, etc) have attracted much attention in many disciplines [1]. Different from interacting entities such as particles, spins, etc., as studied in physical systems, agents in complex adaptive systems usually have a great ability to observe, learn and adapt in order to survive in fast-changing environments [2]. Due to the intelligence involved, interactions among agents can become much more intricate, which makes the bottom-up approach a natural choice for the study of this kind of system. The bottom-up approach includes both agent-based models (in which artificial agents with heterogeneous intelligence are designed accordingly [3]–[10]) and controlled laboratory experiments (in which human participants or even other living creatures are recruited to play under various conditions [11]–[16]). It is well known in statistical physics that there exist many phase transition phenomena, e.g., the melting of ice (classified as a first-order phase transition) and the ferromagnetic transition (classified as a second-order phase transition; both second-order and higher-order phase transitions are also called continuous phase transitions) [17]. In complex adaptive systems, phase transition phenomena can be seen as well [12], [18]–[21].

In economics and social science, it is a common belief that if more information is distributed to agents, the associated system will be more efficient (here 'more efficient' means that the system lies more easily in the optimal state and that the system's fluctuation level is also lower). For example, the well-known efficient market hypothesis implies that market efficiency will be higher if more information is available to investors, i.e., efficiency upgrading from the weak form to the strong form [22]. In this paper, we recheck the information effect in a complex adaptive system related to resource-allocation problems [6, 12], [23]–[26]. Research on the allocation of resources is of particular importance. For instance, some believe governments should get involved to guarantee that resources are distributed to places where they are needed most, while others think the

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'invisible hand' phenomenon will emerge naturally through agents' interactions and can ensure the efficiency of resource allocation [27, 28]. Both agent-based simulations and controlled human experiments were adopted in [12] to show the existence of the invisible hand. In another different resource-allocation system, i.e., artificial financial markets, the invisible hand phenomenon was studied as well [29, 30]. Note that in [12], all participants could get access to global information to evaluate their strategies. However, in many real problems, global information is either difficult to collect or confidential. For example, when a company decides whether or not to step into an emerging market, it is hard to know all the other competitive companies' reactions and it would also take a long time to determine the real return on investment. Therefore, the company can only draw up its own strategies under the currently obtained partial information. This leads to a question whether the invisible hand can still function well when market members obtain only partial information.

To model this question, agents in our system are connected via a directed random network and everyone evaluates his/her own performance through partial information gathered from his/her first-order neighborhood. It is obvious that a higher connection rate can make the system more information-concentrated. Our agent-based model is designed based on the market-directed resource-allocation game [12] (the market-directed resource-allocation game [12] and the minority game [6] were both inspired by the well-known El Farol bar problem [31]). We also conduct a series of controlled human experiments to show the reliability of the model design. We find that the system can almost reach the optimal (balanced) state even under a small information concentration. Furthermore, when the information concentration increases, the ensemble average of the fluctuation level goes through a continuous phase transition, which means that in the second phase, agents getting too much information will actually harm the system's stability (a higher fluctuation level means a lower stability of the system). This is contrary to the belief mentioned above (namely, market efficiency increases when more information is shared). Also, at the transition point, the ensemble fluctuations of the fluctuation level remain at a low value. We also show that when the system becomes infinitely large, this fluctuation transition phenomenon remains. Our finding is in contrast to the textbook knowledge about continuous phase transitions, which states that fluctuations will rise abnormally around a transition point since the correlation length becomes infinite. Thus, we call this continuous phase transition *anomalous continuous phase transition*.

2. Methods

2.1. Model

To proceed, let us first introduce the abstract resource-allocation system of our interest. The system contains a repeated game. In the game, there are N agents facing two rooms labeled as Room 1 and Room 2. Each room has a certain amount of resources denoted respectively as M_1 and M_2, with $M = M_1 + M_2$. Both M_1 and M_2 are fixed during the repeated game. Note that our system is close to the El Farol bar problem [31], because both systems can also be seen as an asymmetric minority game (in the original minority game [6], the values of M_1 and M_2 are the same, which is only the case of an unbiased

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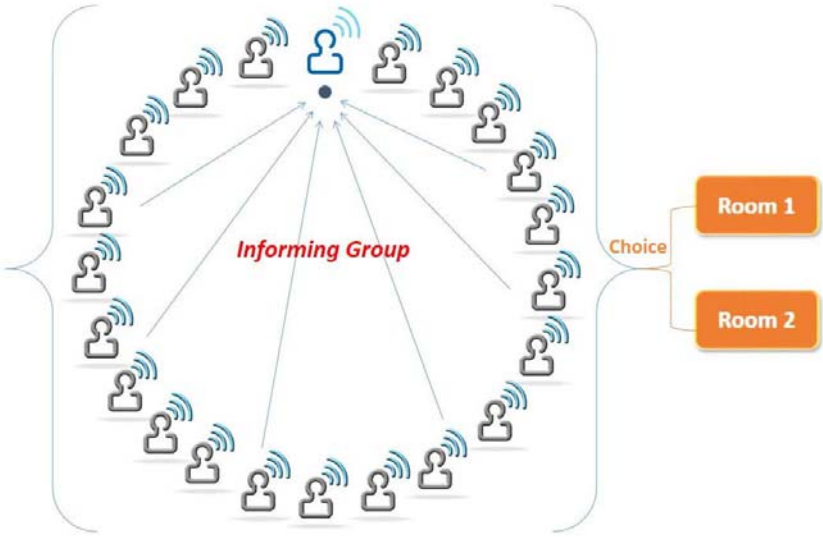

Figure 1. A schematic diagram of our model.

resource distribution). Agents decide at each time step on which room to enter and then divide the resources in it evenly. Here, the number of agents entering Room 1 or Room 2 at a time step is denoted as N_1 or N_2 respectively ($N = N_1 + N_2$). Obviously, the goal for every agent is the same, namely, to choose the room where they can obtain more resources. The exact values of M_1 and M_2 are unknown to all the agents. Moreover, they do not know the global values of M_1/N_1 and M_2/N_2 at each time step in the game either (these are the amounts of resources actually distributed to one agent in each room). So an agent can evaluate his/her performance only by viewing information from his/her informers. In the model, every other agent has a probability k to become an informer of Agent i. Hence, all the agents form a directed random network, which is fixed during the game. We call the group formed by Agent i's informers his/her informing group. Agent i cannot receive any information from outside his/her informing group. At each time step, suppose that $M_1/N_1^i \geq M_2/N_2^i$, where N_1^i or N_2^i is the number of agents containing Agent i and his/her informing group members that enter Room 1 or Room 2, then Room 1 is the winning room for Agent i. However, in other agents' eyes, Room 2 may be the winning room according to information from their own informing groups. A schematic diagram of our model is shown in figure 1. The process described above can also be easily understood by the following interpretation. Since Agent i cannot know the exact winning room (i.e., global information) at each time step, he/she has to estimate which room is winning through sampling analysis (i.e., a limited information): he/she will select an agent into his/her sample with a probability k at the beginning of the game and then the sample is fixed; at each time step, he/she will update his/her strategies' scores based on the winning room estimated using the sample data. Now it is obvious that if $k = 1$, all the agents can obtain global information. As k decreases, the information an agent can get becomes less and less compared to the global information. In particular, $k = 0$ means Agent i's informing group contains nobody, which means now Agent i obtains no information from the other agents. Therefore, we may say that the value of k represents the system's information

Table 1. A particular strategy.

Exogenous situation	Choice
1	0
2	1
3	1
.	.
.	.
.	.
$P-1$	0
P	1

concentration, $0 \leq k \leq 1$. Note that our system's framework is closer to reality than that of [32]. In [32], market states are not affected by agents' production or exchange behaviors so that an agent can even get sure information of which state the market will be in before his/her decision is made.

Now for our agent-based model, what is left is the design of the agents' decision-making process. In order to test our system under a variety of M_1/M values, we adopt the design from the market-directed resource-allocation game [12], which models biased or unbiased resource distribution problems satisfactorily. In the model, every agent will create S strategies before the game starts. Every strategy has two columns, and a particular one is shown in table 1. The left column represents P possible exogenous situations that agents may face, which are labeled as $1, 2, \ldots, P$, accordingly. For every situation, the right column offers a choice respectively; here the number 1 is for the choice of Room 1 and number 0 is for the choice of Room 2. When an agent creates a strategy, he/she will first randomly choose an integer L that satisfies $0 \leq L \leq P$ according to a uniform distribution, and then fill the number 1 in the right column of the strategy with a probability L/P, and hence 0 with probability $(P-L)/P$ (here we can see the difference in the strategy creation process between the market-directed resource-allocation game [12] and the minority game [6]. In the minority game, the right column of a strategy is filled by 1 or 0 with equal probabilities, i.e., both with a probability of 0.5). The strategies are fixed once they are created initially. At each time step, a particular exogenous situation will be picked randomly from 1 to P with uniform probability. Every agent will use his/her highest-scored strategy to choose rooms under the current situation. If an agent has more than one strategy that ties for the highest score, he/she will randomly select one from them. After each time step, every agent will then assess the performance of his/her strategies based on the partial information obtained from his/her informing group. For example, to Agent i, if Room 1 is winning, all his/her strategies that offer the right choice at the given situation will receive one added point.

It can be seen that, in our model, there exist two kinds of randomness: a fast one, which appears in picking an exogenous situation at each time step and in an agent's selection of a strategy from the ones that tie for the highest score; and a quenched one, which appears in the strategy creation and network generation processes at the beginning of the repeated game (both the strategies and the directed network are then fixed during the game). In order to distinguish these two kinds of random processes when doing statistical analysis,

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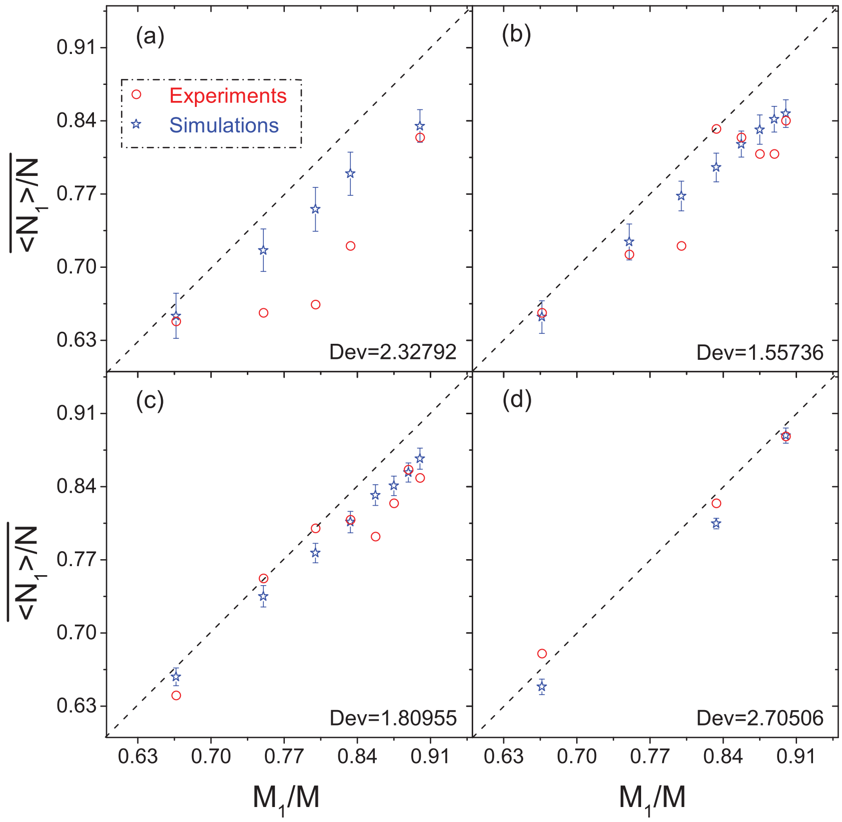

Figure 2. $\overline{\langle N_1\rangle/N}$ versus M_1/M for $k=$ (a) 0.2, (b) 0.52, (c) 0.76 and (d) 1. The simulated ensemble contains 50 systems, each of which has the same parameters: $N=25$, $S=8$, and $P=16$. Each system evolves for 600 time steps (the first half for stabilization, which is enough for system relaxation, and the second half for statistics). The blue stars show the ensemble average of $\langle N_1\rangle/N$, i.e., $\overline{\langle N_1\rangle/N}$ and error bars are added. The red circles show the experimental results. For each data point, one system with the associated parameters is conducted for 15 time steps and $\langle N_1\rangle/N$ is calculated on the last 5 time steps shown in tables 2–5 in order to avoid the relaxation time steps. The number of human participants recruited in the laboratory system is the same as used in the simulations, i.e., $N=25$. The diagonal dashed line with slope = 1 indicates the optimal (balanced) state: $\langle N_1\rangle/N = M_1/M$.

we use $\langle\cdots\rangle$ to denote the time average on the quantity $\cdots$ for the fast random processes, and use $\overline{\cdots}$ to represent the ensemble average on the quantity $\cdots$ for the quenched random processes.

In the simulations, we set $S=8$ and $P=16$. Based on the previous theoretical analysis [23], it can be calculated that now $(\langle N_1\rangle/N)_{\max} = 1-(1/P)\sum_{\tilde{L}=1}^{P}[(\tilde{L}/(P+1))^S] = 0.911$. Moreover, for $M_1/M \le (\langle N_1\rangle/N)_{\max}$, namely, $M_1/M \le 0.911$, the system in which all agents get access to global information (i.e., $k=1$) can reach the optimal (balanced) state where $\langle N_1\rangle/N = M_1/M$. Hence, we vary the value of M_1/M from 2/3 to 9/10 in the simulations.

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Table 2. Experimental data of N_1/N for different values of M_1/M for $k = 0.2$ within the 15 time steps.

	N_1/N				
Time step	$M_1/M = 2/3$	$=3/4$	$=4/5$	$=5/6$	$=9/10$
1	0.6	0.56	0.56	0.72	0.8
2	0.6	0.68	0.68	0.76	0.8
3	0.64	0.72	0.64	0.68	0.8
4	0.56	0.6	0.76	0.64	0.72
5	0.68	0.76	0.76	0.76	0.84
6	0.64	0.6	0.76	0.76	0.76
7	0.52	0.76	0.72	0.76	0.84
8	0.6	0.68	0.72	0.72	0.76
9	0.6	0.48	0.64	0.76	0.84
10	0.56	0.68	0.72	0.68	0.8
11	0.64	0.64	0.84	0.76	0.76
12	0.68	0.56	0.64	0.68	0.84
13	0.56	0.72	0.64	0.72	0.88
14	0.68	0.72	0.56	0.76	0.84
15	0.68	0.64	0.64	0.68	0.8
Average[a]	0.648	0.656	0.664	0.72	0.824

[a] The last five time steps are taken to calculate the time average value of N_1/N, i.e., $\langle N_1\rangle/N$ shown in figure 2.

2.2. Experiment

It is well known that for a targeted system, there can be many different designs in the agent-based modeling method. So how to validate a model is crucial. One way is to compare simulated results with field data [33]–[38], and the other is to do controlled experiments in laboratory systems [12], [23]–[25], [39]. In this research, we prefer the latter, and recruit 25 students from the Department of Physics at Fudan University as the human participants. The game settings are the same as our agent-based model except that artificial agents are now replaced by human players. Two kinds of monetary rewards are provided to give the students incentives: (A) each participant will be given one virtual point if he/she chooses the right room at any time step and, after the experiments, the accumulated virtual points a participant wins will be exchanged into cash under the ratio: 1 point = 1 Chinese Yuan; (B) bonuses of 150, 100, and 50 Chinese Yuan are also offered to the three best-performing players, respectively. The rules and rewards are made clear to the students before the experiments [40]. A round of repeated pre-games with 15 time steps is also offered to the students to familiarize them with the rules. Each set of parameters (i.e., k and M_1/M) is then carried out with the students for one round with 15 time steps. Note that each set of values of k and M_1/M is selected randomly before each round of repeated games so that the students can hardly figure out whether his/her experience gained in the last game can be used in the next one.

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Table 3. Experimental data of N_1/N for different values of M_1/M for $k = 0.52$ within the 15 time steps.

	N_1/N							
Time step	$M_1/M = 2/3$	$=3/4$	$=4/5$	$=5/6$	$=6/7$	$=7/8$	$=8/9$	$=9/10$
1	0.64	0.64	0.56	0.8	0.76	0.8	0.76	0.88
2	0.6	0.68	0.72	0.48	0.76	0.64	0.76	0.6
3	0.6	0.6	0.68	0.72	0.88	0.76	0.76	0.72
4	0.68	0.68	0.8	0.76	0.68	0.84	0.84	0.88
5	0.52	0.64	0.72	0.84	0.84	0.8	0.76	0.72
6	0.72	0.84	0.72	0.52	0.72	0.64	0.84	0.84
7	0.64	0.68	0.72	0.68	0.8	0.76	0.76	0.68
8	0.44	0.88	0.72	0.84	0.84	0.84	0.76	0.8
9	0.56	0.6	0.72	0.68	0.6	0.8	0.8	0.84
10	0.68	0.8	0.72	0.76	0.72	0.76	0.76	0.76
11	0.68	0.68	0.68	0.88	0.76	0.68	0.72	0.72
12	0.56	0.68	0.72	0.84	0.8	0.76	0.84	0.84
13	0.64	0.8	0.64	0.8	0.84	0.88	0.8	0.84
14	0.68	0.76	0.76	0.84	0.88	0.84	0.84	0.92
15	0.72	0.64	0.8	0.8	0.84	0.88	0.84	0.88
Average[a]	0.656	0.712	0.72	0.832	0.824	0.808	0.808	0.84

[a] The last five time steps are taken to calculate the time average value of N_1/N, i.e., $\langle N_1\rangle/N$ shown in figure 2.

3. Results

In figure 2, the simulated ensemble contains 50 systems. Each system evolves for 600 time steps. The first 300 time steps are for stabilization and the second 300 are used to calculate the time average of N_1/N, namely $\langle N_1\rangle/N$. Then $\overline{\langle N_1\rangle/N}$ can be obtained by taking the ensemble average for the 50 systems. The blue stars in figure 2 show $\overline{\langle N_1\rangle/N}$ versus M_1/M under four different values of k. The error bars represent the standard deviation (denoted as SD_m) among the 50 systems' $\langle N_1\rangle/N$ values. The experimental results are given in tables 2–5. For each set of k and M_1/M, $\langle N_1\rangle/N$ (represented as red circles in figure 2) is calculated on the basis of the last five experimental time steps in order to avoid relaxation time. In both simulations and experiments, $N = 25$. Deviations between the simulated and experimental results in each sub-figure (denoted as Dev) are defined as the average of $|(\langle N_1\rangle/N)_e - (\overline{\langle N_1\rangle/N})_m|/SD_m$ over all the points (here $|\cdots|$ denotes the absolute value of $\cdots$; the subscript m stands for the simulations and e for the experiments). For the four sub-figures, the values of Dev are (a) 2.33, (b) 1.56, (c) 1.81 and (d) 2.71, respectively. Suppose that in the simulated ensemble, $\langle N_1\rangle/N$ follows a Gaussian distribution, then the experimental values of $\langle N_1\rangle/N$ fall in (a) 98%, (b) 88.2%, (c) 93% and (d) 99.4% confidence interval accordingly. In the experiments, there always exist some uncontrolled factors, such as mood swings of the students during the games. So for figure 2, it can be said that our agent-based model is a good mimic of the human

Table 4. Experimental data of N_1/N for different values of M_1/M for $k = 0.76$ within the 15 time steps.

Time step	N_1/N							
	$M_1/M = 2/3$	$=3/4$	$=4/5$	$=5/6$	$=6/7$	$=7/8$	$=8/9$	$=9/10$
1	0.52	0.8	0.8	0.76	0.84	0.64	0.84	0.68
2	0.88	0.44	0.64	0.84	0.6	0.84	0.64	0.8
3	0.6	0.64	0.8	0.76	0.88	0.76	0.8	0.8
4	0.56	0.76	0.68	0.72	0.68	0.8	0.8	0.88
5	0.76	0.64	0.64	0.88	0.72	0.8	0.8	0.76
6	0.68	0.76	0.76	0.72	0.84	0.88	0.8	0.84
7	0.6	0.68	0.68	0.8	0.8	0.72	0.92	0.88
8	0.56	0.68	0.84	0.76	0.68	0.84	0.8	0.8
9	0.72	0.68	0.68	0.8	0.88	0.76	0.84	0.88
10	0.52	0.6	0.76	0.72	0.8	0.72	0.76	0.84
11	0.6	0.76	0.8	0.76	0.92	0.68	0.8	0.76
12	0.68	0.76	0.76	0.88	0.92	0.84	0.84	0.88
13	0.64	0.76	0.76	0.8	0.64	0.88	0.8	0.96
14	0.68	0.68	0.84	0.84	0.64	0.88	0.92	0.8
15	0.6	0.8	0.84	0.76	0.84	0.84	0.92	0.84
Average[a]	0.64	0.752	0.8	0.808	0.792	0.824	0.856	0.848

[a] The last five time steps are taken to calculate the time average value of N_1/N, i.e., $\langle N_1\rangle/N$ shown in figure 2.

system. Thanks to the flexibility of the agent-based modeling method, we further extended the number of agents to $N = 1001$ and the ensemble size to 500. The simulated $\overline{\langle N_1\rangle/N}$ versus k is shown in figure 3. It can be seen that even for a small value of information concentration, e.g. $k = 0.2$, the system can still almost reach the optimal (balanced) state where $\langle N_1\rangle/N = M_1/M$. This means that the invisible hand can still influence the system when only a small fraction of the information is distributed to every agent.

For the system discussed in figure 3, the other important property is the fluctuation level, which is given by

$$f = \frac{1}{2N}\sum_{i=1}^{2}\langle(N_i - \langle N_i\rangle)^2\rangle \equiv \frac{1}{N}\langle(N_1 - \langle N_1\rangle)^2\rangle. \tag{1}$$

The ensemble average of the simulated system's fluctuation level, denoted as $\overline{f}$, is shown in figure 4. It can be seen that $\overline{f}$ declines slightly when the information concentration k increases from zero. This is normal, so we interpret it as a normal phase. But after that, $\overline{f}$ climbs up abnormally, which means that now agents getting more information will be more harmful to the system's stability (a higher fluctuation level means a lower stability of the system); this is in contrast to the belief that more information is better, so we interpret this as an abnormal phase. The phase transition can be explained by the competition between the following two processes happening in the system. (1) Uncertainty-reducing process: as k increases, Agent i's informing group becomes larger; in other words, this

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Table 5. Experimental data of N_1/N for different values of M_1/M for $k = 1$ within the 15 time steps.

	N_1/N		
Time step	$M_1/M = 2/3$	$=5/6$	$=9/10$
1	0.6	0.8	0.8
2	0.88	0.76	0.88
3	0.8	0.76	0.8
4	0.6	0.8	0.84
5	0.68	0.72	0.72
6	0.6	0.76	0.92
7	0.64	0.88	0.84
8	0.68	0.8	0.84
9	0.64	0.64	0.6
10	0.64	0.72	0.76
11	0.68	0.8	0.8
12	0.68	0.84	0.92
13	0.64	0.84	0.92
14	0.68	0.8	0.88
15	0.72	0.84	0.92
Average[a]	0.68	0.824	0.888

[a] The last five time steps are taken to calculate the time average value of N_1/N, i.e., $\langle N_1\rangle/N$ shown in figure 2.

means the sample size that Agent i uses to estimate the winning room becomes larger. So the estimation error will be reduced, which makes Agent i more confident about his/her choices. (2) Uncertainty-increasing process: owing to the presence of the directed network, if Agent j is an informer of Agent i, then uncertainties in Agent j's choices can affect Agent i and increase Agent i's own choice fluctuations. Now if Agent i is also in Agent j's informing group (i.e., the two agents now become a pair of mutual informers), then the increased choice fluctuations of Agent i can be further transferred back to Agent j to affect his/her choice uncertainties. So in other words, choice uncertainties in the system can be magnified when facing mutual informing pairs. As we know, when k increases, the number of informing pairs appearing in the system will rise so that the overall fluctuation level will tend to increase. As a result, a continuous phase transition between a normal phase and an abnormal phase begins to appear as k increases from 0 to 1. The lowest $\overline{f}$ point is labeled as the transition point and the associated critical value of k is denoted as k_c. Detailed information around the transition point is shown in the insets of figure 4.

In the traditional continuous phase transition theory [17], it is stated that around a transition point, since the correlation length becomes infinite, fluctuations will increase greatly. In contrast, in figure 4, it can be seen that, at the transition point, the ensemble fluctuations of the fluctuation level, expressed as $\sigma^2(f) = \overline{f^2} - (\overline{f})^2$, remain at a low value with a magnitude of only 10^{-5}. However, in the abnormal phase, where k is large, $\sigma^2(f)$ increases rapidly as k increases, and for $M_1/M \geq 7/8$, $\sigma^2(f)$ even begins to decline clearly when k increases further.

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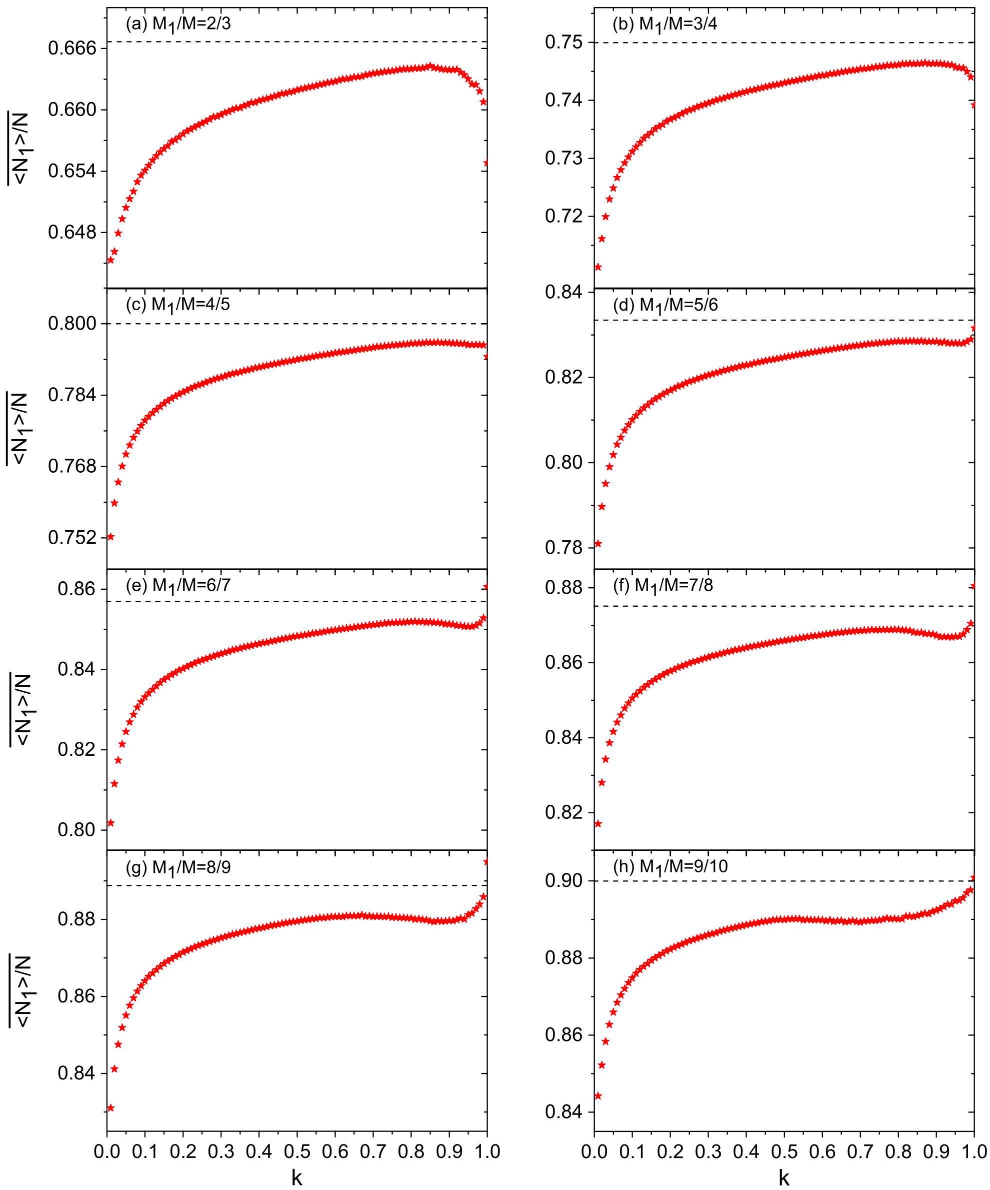

Figure 3. Simulated results of $\overline{\langle N_1\rangle/N}$ versus k for different values of M_1/M. The values of $\overline{\langle N_2\rangle/N}$ are not shown here because $\overline{\langle N_2\rangle/N} = 1 - \overline{\langle N_1\rangle/N}$. The ensemble contains 500 systems, each of which has the same parameters: $N = 1001$, $S = 8$, and $P = 16$. Each system evolves for 600 time steps (the first half for stabilization, which is enough for system relaxation, and the second half for statistics). The horizontal dashed line shows the value of $\langle N_1\rangle/N$ for the optimal (balanced) state: $\langle N_1\rangle/N = M_1/M$.

The above phase transition phenomenon starts to appear in our system that contains a limited number of agents. So we attempt to analyze the relationship between k_c and N. Because the critical information concentration k_c shows no relationship to M_1/M in figure 4, we average it over different values of M_1/M and obtain $\overline{k}_c$. Figure 5 displays a

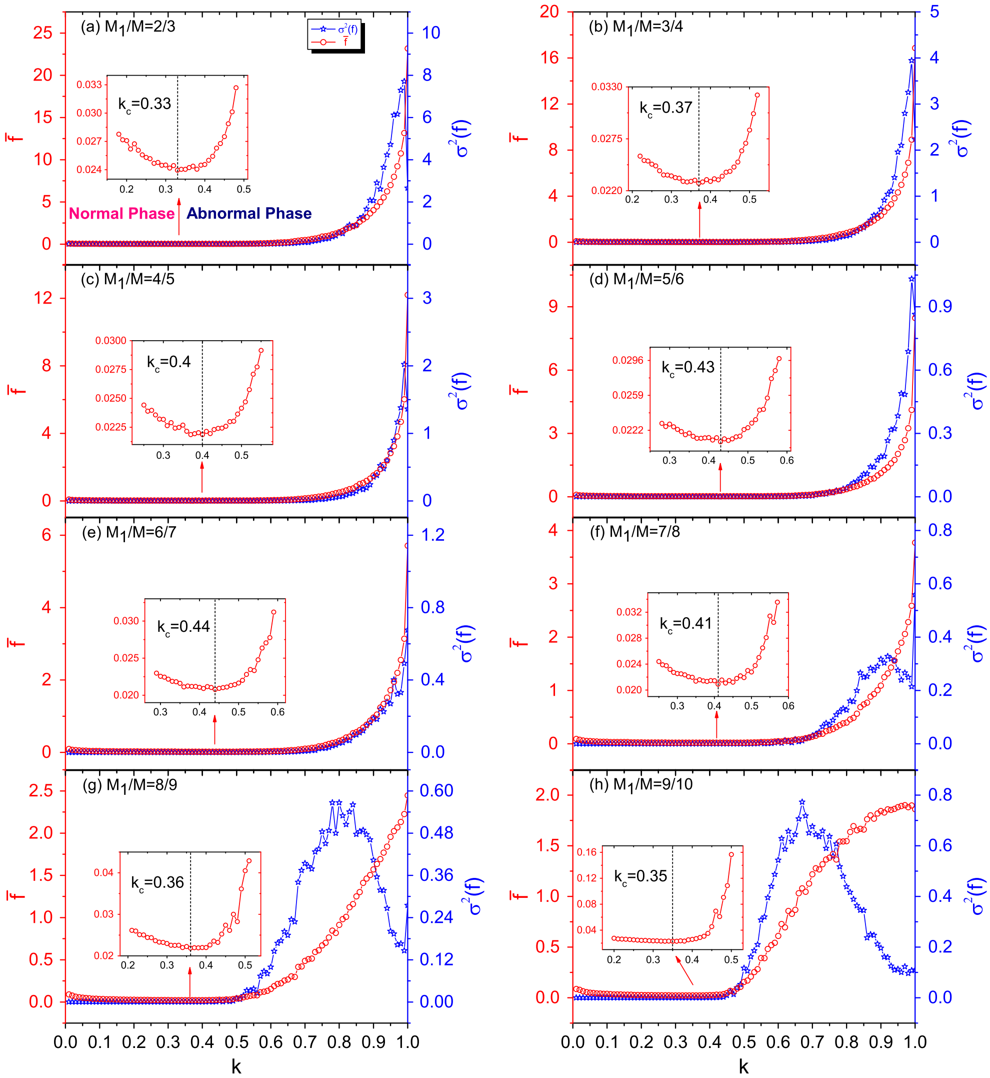

Figure 4. Fluctuations of the simulated system. For the eight M_1/M values, the ensemble averages of the fluctuation level (denoted as $\overline{f}$) under different k values are represented by red circles, while blue stars are for the ensemble fluctuations of the fluctuation level (denoted as $\sigma^2(f)$). $\overline{f}$ goes through a continuous phase transition from a normal phase to an abnormal phase as k increases. The insets give detailed information around the transition point (k_c stands for the critical value of k) accordingly. The ensemble contains 500 systems, each of which has the same parameters: $N = 1001$, $S = 8$, and $P = 16$. Each system evolves for 600 time steps (the first half for stabilization, which is enough for system relaxation, and the second half for statistics).

power-law relationship between $\overline{k}_c$ and N in the tail (i.e., for $N \geq 1001$) as $\overline{k}_c = 447 \times N^{-1}$. Hence, when the system becomes infinitely large, i.e., $N \to +\infty$, this anomalous phase transition phenomenon still exists in the critical situation where every agent has 447 informers on average.

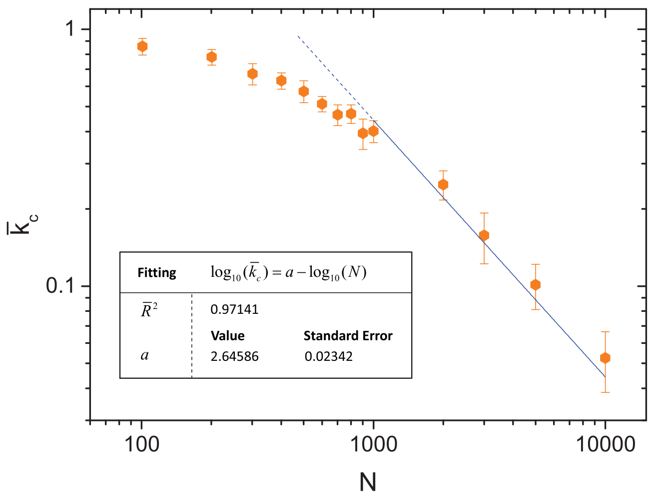

Figure 5. A log–log plot showing $\overline{k}_c$ versus N. Here, $\overline{k}_c$ is the average of k_c for different values of M_1/M. For each value of N, the ensemble contains 50 systems, each of which has the same parameters: $S = 8$ and $P = 16$. Each system evolves for 600 time steps (the first half for stabilization, which is enough for system relaxation, and the second half for statistics). The blue line fits the tail of the data (i.e., from $N = 1001$ to $N = 10001$) according to $\log_{10}(\overline{k}_c) = a - \log_{10}(N)$; details are given in the inset, where the regression coefficient, $\bar{R}^2 = 0.97141$, indicates a good fit to the tail since $\bar{R}^2 = 1.0$ means a perfect fit [41]. Error bars are added as well.

4. Discussion and conclusions

In this paper, we have designed an agent-based model with partial information for biased resource-allocation problems. A series of controlled human experiments have been conducted to show the reliability of the model design. We have found that, even for a small information concentration, the system can still almost reach the optimal (balanced) state. Furthermore, we have found that the ensemble average of the simulated system's fluctuation level has a continuous phase transition. This means that, in the abnormal phase, too much information can actually damage the system's stability. Hence, returning to the question raised at the beginning, we can say that the invisible hand can operate most efficiently only at the transition point where partial information is obtained by agents.

On the other hand, at the transition point, the ensemble fluctuations of the fluctuation level remain at a low value. When increasing the number of agents, the critical value of information concentration obeys a power-law decay in the tail. This behavior confirms that when the system becomes infinitely large, there still exists this kind of fluctuation transition phenomenon. This finding is in contrast to the textbook knowledge about continuous phase transitions, which also addressed at the beginning, namely, that fluctuations will rise abnormally around a transition point since the correlation length becomes infinite. This may pave the way for investigating the role of human adaptability in further developing traditional physics.

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For these reasons, this work is expected to be of value to a variety of fields, ranging from physics, to economics, to complexity science, and to artificial intelligence.

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