Many-Particle Thermal Invisibility and Diode From Effective Media

Invisibility has recently been achieved in optics, electromagnetics, acoustics, thermotics, fluid mechanics, and quantum mechanics; it was realized through a properly designed cloak structure with unconventional (anisotropic, inhomogeneous, and singular) material parameters, which limit practical applications. Here, we show, directly from the solution of Laplace’s equation, that two or more conventional (isotropic, homogeneous, and non-singular) materials can be made thermally invisible by tailoring the many-particle local-field effects. Our many-particle thermal invisibility essentially serves as a new class of invisibility with a mechanism fundamentally differing from that of the prevailing cloaking-type invisibility. We confirm it in simulation and experiment. As an application, the concept of many-particle thermal invisibility helps us propose a class of many-particle thermal diodes: the diodes allow heat conduction from one direction with invisibility, but prohibit the heat conduction from the inverse direction with visibility. This work reveals a different mechanism for thermal camouflage and thermal rectification by using composites, and it also suggests that besides thermotics, many-particle local-field effects can be a convenient and effective mechanism for achieving similar controls in other fields, e.g., optics, electromagnetics, acoustics, and fluid mechanics. [DOI: 10.1115/1.4039910]

1 Introduction

Invisibility means an object cannot be seen or detected, and it is always a fascinating topic in fantasy or science fiction. Since the last decade, it has genuinely entered the realm of scientific research [1,2]. So far, invisibility has attracted enormous research interests in optics [1,4–8], electromagnetics [2,9–14], acoustics [11,15–17,19], thermotics [18,20–29], fluid mechanics [30,31], and quantum mechanics (matter waves) [11,32,33]. In the duration, cloak structures (say, designed with conformal mapping [1] or coordinate transformation [2]) dominate the existing invisibility research [2,4–9,11–17,19,21–33], which are convenient for theoretical settings. However, such cloaks for achieving invisibility require unconventional material parameters that need to satisfy one or more of the following three properties: (i) anisotropy (e.g., the parameter like a dielectric constant or thermal conductivity is a tensor, possessing different values of radial and tangential components), (ii) inhomogeneity (say, the parameter has a spatial gradation profile in the cloak region), and (iii) singularity (e.g., the parameter should be zero or infinite at certain positions). Such requirements (i)–(iii) usually could not be satisfied by naturally occurring materials or chemical compounds, thus significantly limiting the practical applications. To be free from or to compromise with (i)–(iii), researchers have proposed various schemes, see Refs. [1,3–9,11–17,19,21–33]. For example, anisotropy in design invisibility could be overcome by conformal transformation optics using isotropic inhomogeneous medium [1,3]. Gomöry et al. [12] designed a special cylindrical superconductor-ferromagnetic bilayer that can exactly cloak uniform static magnetic fields [12], hence yielding an exact magnetic cloak. Exact bilayer thermal cloaks have been similarly proposed [25–27].
Nevertheless, even though such exact bilayer cloaks \cite{12,25–27} are independent of condition (i) and (ii), they still require condition (iii): the magnetic permeability \cite{12} or thermal conductivity \cite{25–27} of the inner layer must be zero. In general, most schemes \cite{4–9,11–17,19,21–33} based on cloak structures are faced with the same problem: they could not be free from all the (i)–(iii) due to the underlying mechanisms, thus limiting applications. These limitations inspire us that it is necessary to resort to another mechanism. Accordingly, here we propose a many-particle system instead, which is different from the cloak structure. By choosing many-particle local-field effects appropriately, we shall show that our system exhibiting thermal invisibility is free from (i)–(iii) indeed, which satisfies isotropy (the thermal conductivity is a scalar), homogeneity (objects have a random distribution in the region), yielding homogeneous equivalent thermal conductivities of the subregions everywhere, and nonsingularity (thermal conductivities have a finite nonzero value). Compared to the previous random medium method that needs complex algorithm optimization \cite{34,35}, this mechanism is easy to conduct. The concept of many-particle thermal invisibility further yields a class of many-particle thermal diodes with invisibility.

2 Many-Particle Thermal Invisibility

2.1 Two-Dimensional Case

2.1.1 Theory. For a circular particle with thermal conductivity \( \kappa \) and radius \( R_0 \) embedded in an infinite uniform matrix with \( \kappa_m \), the solution of Laplace’s equation, \( \nabla^2 T = 0 \), in polar coordinates \( (r, \theta) \) is

\[
T_{\text{inner}}(r, \theta) = A_0 + B_0 \ln(r) + \sum_{n=1}^{\infty} \left[ A_n \cos(n \theta) + B_n \sin(n \theta) \right] r^n,
\]

\[
T_{\text{outer}}(r, \theta) = A'_0 + B'_0 \ln(r) + \sum_{n=1}^{\infty} \left[ A'_n \cos(n \theta) + B'_n \sin(n \theta) \right] r^n,
\]

where \( T_{\text{inner}}(r, \theta) \) and \( T_{\text{outer}}(r, \theta) \) denote the distribution of temperature in the particle and matrix, respectively. Here, \( A_0, B_0, A_n, B_n, C_{\text{inner}} D_{\text{inner}}, A'_0, B'_0, A'_n, B'_n, C_{\text{outer}} \), and \( D_{\text{outer}} \) are undetermined coefficients. The accompanying boundary conditions are

\[
T_{\text{inner}}(r = 0) < \infty,
\]

\[
T_{\text{outer}}(r \to \infty, \theta) \to T_0 + c_0 r \cos(\theta),
\]

\[
T_{\text{inner}}(r = R_0) = T_{\text{outer}}(r = R_0),
\]

\[
\kappa_m \frac{\partial T_{\text{outer}}}{\partial r} \bigg|_{r=R_0} = \kappa \frac{\partial T_{\text{inner}}}{\partial r} \bigg|_{r=R_0}.
\]

Here, \( T_0 \) denotes the temperature at \( r = 0 \) and \( c_0 \) is a constant depending on the applied temperature gradient. Then, we obtain

\[
T_{\text{outer}}(r, \theta) = T_0 + c_0 r \cos(\theta) - \frac{\kappa - \kappa_m}{\kappa + \kappa_m} c_0 \frac{R_0^2}{r} \cos(\theta) \quad (1)
\]

So, we are allowed to write the Clausius–Mossotti factor \( z \) as

\[
z = \frac{\kappa - \kappa_m}{\kappa + \kappa_m} \quad (2)
\]

which describes the degree of thermal contrast between the circular particle and matrix. Clearly, \( z = 0 \) corresponds to zero thermal contrast, i.e., the particle and matrix share the same material. Accordingly, a larger \( z \) indicates a larger contrast between the particle and matrix.

To proceed, we consider a two-dimensional (2D) square system, which contains a central square area and an environment occupied by a material with thermal conductivity \( \kappa_m \) surrounding the central square area. \( n \) kinds of circular particles, each with thermal conductivity \( \kappa_n \) and area fraction \( p_n \) \((n = 1, 2, \ldots)\), occupy the whole central square area with a random distribution, thus yielding a many-particle system. In the presence of an external temperature gradient, if the existence of the central square area does not disturb the temperature distribution or heat flow in the environment, the central square area is thermally invisible. For this purpose, we need to set the central square area to possess a special effective thermal conductivity that must be equal to \( \kappa_m \). In this regard, what we need is to let the thermal contrasts (Clausius–Mossotti factor) of all the particles within the central square area be canceled out. Thus, we obtain

\[
\langle \text{Clausius – Mossotti factor} \rangle_{2D} = 0 \quad (3)
\]

where \( \langle \cdots \rangle_m \) denotes the area average of \( \cdots \) over the central square area. Equation (2) owns at least two features. On the one hand, when the thermal conductivities of the particles, \( \kappa_n \), satisfy Eq. (2), the effective thermal conductivity of the central square area equals \( \kappa_m \). This is because the many-particle local-field effects lead to the overall disappearance of the thermal contrasts between all the particles and the environment. In other words, for a specific particle, when we apply Eq. (1) (for one particle in an infinite uniform matrix) to Eq. (2) (for many particles in a random distribution), we have assumed the specific particle to be embedded in an effectively uniform matrix (which is composed of many other particles). Accordingly, the Clausius–Mossotti relation Eq. (1) can be applied to Eq. (2). On the other hand, if the central square area is divided into plenty of subareas, each including many particles distributed randomly, the equivalent thermal conductivities of the subareas are still equal to \( \kappa_m \) according to Eq. (2). That is, these subareas become homogeneous indeed.

Without loss of generality, we first consider the \( n = 2 \) case that the central square area includes two materials with \( \kappa_1 \) and \( \kappa_2 \). According to Eq. (2), we set \( \kappa_1 > \kappa_m > \kappa_2 \). The material of \( \kappa_1 \) tends to expel isotherms and attract heat flux lines (Fig. 1(c)) while the other material of \( \kappa_2 \) has an opposite effect (Fig. 1(d)). Thus, the distortions of heat flux lines contributed by the two materials can be canceled out at certain area fractions according to Eq. (2). Figure 1(a), together with Fig. 1(j), shows the result of finite element simulations based on the commercial software COMSOL MULTIPHYSICS. When compared with Fig. 1(b) for the case of pure environmental material, the temperature distribution in the environment shown in Fig. 1(a) has almost not been disturbed by the presence of the central square area containing two different materials. Thus, the thermal invisibility is achieved (Fig. 1(a)).

If we extend the central square area from two materials (Fig. 1(a) to three materials (Figs. 2(a) and 2(b)) whose thermal conductivities are set to satisfy Eq. (2) as well, the same thermal invisibility behavior appears as well (see Fig. 2(a)). Similarly, more kinds of materials \((n > 2)\) can be added into the central square area, and same thermal invisibility behaviors can always come to appear as long as the thermal conductivities and area fractions of these materials satisfy Eq. (2).

2.1.2 Experiment. We further perform an experiment to confirm the above-mentioned simulation results. The experimental setup is depicted in Fig. 1(e); detailed experimental methods can be found in experimental details. We use an infrared camera to detect the distribution of temperature in the experimental sample. Figure 1(f), together with Fig. 1(j), depicts the case of the mixture of coalesced polydimethylsiloxane and copper \((T_d)\) in an environment of brass \((H62)\), and it shows a satisfactory thermal invisibility indeed, which echoes with the simulation result shown in Fig. 1(a) (a quantitative comparison is shown in Fig. 1(j)).
Fig. 1 Two-dimensional results of (a)–(d) finite element simulations under the boundary condition of heat insulation and (f)–(j) experiments based on (e) the experimental setup (where HOT and COLD represent the hot and cold source, respectively): the color surface in (a)–(d), (f)–(j) denotes the distribution of temperature; the white lines in (a)–(d) represent the isothermal lines. (a) and (f) show a 20 cm × 20 cm system (the experimental sample of (f) is depicted in the upper right corner of (e)), which owns a central square area (6.7 cm × 6.7 cm) containing the first material (as shadowed in the central square area of (a)) of thermal conductivity 0.15 W/(m K) and area fraction 36.4% randomly embedded in the second material of 400 W/(m K) and 63.6%; the first (second) material can be seen as an assembly of a kind of circular particles with different sizes; outside the central square area is the environment occupied by the material of \( \kappa_m = 109 \) W/(m K). (b) and (g) show the case of the pure environmental material \( \kappa_m = 109 \) W/(m K), and their central squares (6.7 cm × 6.7 cm) denote only the position for the sake of comparison. (c) and (h) are same as (a) and (f), respectively, but the central square area only includes the material of \( \kappa_m = 400 \) W/(m K). (d) and (i) are also same as (a) and (f), respectively, but the central square area only contains the material of 0.15 W/(m K). (j) shows the quantitative comparison of temperature distribution between simulation and experiment, for the structure of (a) and (f). The “I–V” represent the five positions as depicted in (e).
understanding the experimental thermal invisibility (Fig. 1(f)), we also plot more relevant experimental comparisons (see Figs. 1(g)–1(i), which agree well with Figs. 1(b)–1(d) (simulation results), respectively).

In view of the good agreement between our simulation (Figs. 1(a)–1(d)) and experiment (Figs. 1(f)–1(i)) (see also Fig. 1(j)), now we may conclude that Eq. (2) is indeed useful for achieving many-particle thermal invisibility.

2.2 Three-Dimensional Case. Figures 1, 2(a), and 2(b) show only two-dimensional cases according to Eq. (2). For completeness, we can obtain three-dimensional Clausius–Mossotti factor \( x \) as

\[
x = \frac{\kappa - \kappa_m}{\kappa + 2\kappa_m}
\]

and three-dimensional invisibility can be similarly achieved using

\[
\langle\text{Clausius – Mossotti factor}\rangle_{3D} = 0
\]

where \( \langle\cdots\rangle_{3D} \) denotes the volume average of \( \cdots \) over the whole central cubic volume. In Eq. (4), the Clausius–Mossotti factor is for three dimensions. As a model demonstration, we perform finite element simulations for a three-dimensional case, (see Figs. 2(c) and 2(d)). The central cubic volume contains two materials with thermal conductivities 0.15 W/(m K) and 155.6 W/(m K) and volume fractions 20% and 80% (as is shown in (d)), which are randomly embedded in each other; either of the two materials is also an assembly of a kind of spherical particles with different sizes; outside the central cubic volume is also the environment of \( \kappa_m = 109 \) W/(m K).

2.3 Discussion. A feature of our design is that heat can flow cross the invisible region, namely, the central square area (Figs. 1(a), 1(f), and 2(a)) or the central cubic volume (Fig. 2(c)). Although the isotherms and heat flux lines in the invisible region

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**Fig. 2** (a) Two-dimensional and (c) three-dimensional finite element simulation results of the (b) two-dimensional and (d) three-dimensional structure, respectively: the color surface in (a) and (c) denote the pathway and strength of heat flux, respectively. In (a) and (c), the boundary condition is also set at heat insulation. (a) depicts a 20 cm \( \times \) 20 cm system; its central square area (6.7 cm \( \times \) 6.7 cm) includes three different materials (randomly distributed with each other, as is shown in (b)) with thermal conductivities 0.15 W/(m K), 270.5 W/(m K) and 400 W/(m K) and area fractions 33.3%, 33.3%, and 33.4%; each of the three materials is also occupied by a kind of circular particles with different sizes; outside the central square area is the environment occupied by the material of \( \kappa_m = 109 \) W/(m K). (c) displays three orthogonal planes of a 20 cm \( \times \) 20 cm \( \times \) 20 cm system; its central cubic volume (6.7 cm \( \times \) 6.7 cm \( \times \) 6.7 cm) contains two materials with thermal conductivities 0.15 W/(m K) and 155.6 W/(m K) and volume fractions 20% and 80% (as is shown in (d)), which are randomly embedded in each other; either of the two materials is also an assembly of a kind of spherical particles with different sizes; outside the central cubic volume is also the environment of \( \kappa_m = 109 \) W/(m K).
may be distorted due to the random distribution of materials, the temperature field within the environment is not disturbed, thus being called invisibility. Note that the small distortion at the close proximity of the invisible region is caused by the nonuniform distribution of the circular or spherical particles, which can be overcome if one adopts particles as many as possible. Now we can say that the interior localized distortions in such invisible regions can cancel out each other as a result of many-particle local-field effects, and then they do not affect the exterior temperature field in the environment. This is the reason why the thermal invisibility can be achieved (Figs. 1(a), 1(f), 2(a), and 2(c)). So, similar invisible functions can be realized via other designs, such as with periodic structures and particles having different shapes/sizes. Previous works about thermal cloaks (e.g., Ref. [36]) determine thermal conductivities by using coordinate transformation. In this work, the effective medium method is applied to approximately realize the desired thermal conductivity because one cannot find it in the thermal diode: simulation results. (a) Schematic graph showing the diode structure, where the left part has a central square area containing material A ($\kappa_A = 0.15$ W/(m K), say, polydimethylsiloxane) randomly embedded in material B ($\kappa_B = 400$ W/(m K), say, red copper) with area fraction 36.4%. Materials C and D occupy the other areas with temperature-dependent thermal conductivities following the second expression of Eq. (3) in Ref. [29], $\kappa_C = \kappa_j + \left(\kappa_s - \kappa_j\right)/\exp\left(-\left(T - T_c\right)/\left(1.0\right) + 1\right)$ and $\kappa_D = \kappa_j + \left(\kappa_s - \kappa_j\right)/\exp\left(-\left(T - T_c\right)/\left(1.0\right) + 1\right)$ with $\kappa_j = 0.026$ W/(m K) (e.g., air), $\kappa_s = 100$ W/(m K) (e.g., brass) and $T_c = 298$ K, which may be experimentally realized with the aid of shape-memory alloy according to the design depicted in Fig. 2 of Ref. [29]. (b) The distribution of temperature (color surface) and isotherms (white lines) in the thermal diode for high flux $J_H = 1.31 \times 10^5$ W/m$^2$. (c) Same as (b), but the positions of the heat source and cold source are exchanged, thus showing the case of low flux $J_L = 3.88 \times 10^4$ W/m$^2$. The corresponding rectification ratio $(J_H - J_L)/(J_H + J_L) = 94\%$. may be distorted due to the random distribution of materials, the temperature field within the environment is not disturbed, thus being called invisibility. Note that the small distortion at the close proximity of the invisible region is caused by the nonuniform distribution of the circular or spherical particles, which can be overcome if one adopts particles as many as possible. Now we can say that the interior localized distortions in such invisible regions can cancel out each other as a result of many-particle local-field effects, and then they do not affect the exterior temperature field in the environment. This is the reason why the thermal invisibility can be achieved (Figs. 1(a), 1(f), 2(a), and 2(c)). So, similar invisible functions can be realized via other designs, such as with periodic structures and particles having different shapes/sizes. Previous works about thermal cloaks (e.g., Ref. [36]) determine thermal conductivities by using coordinate transformation. In this work, the effective medium method is applied to approximately realize the desired thermal conductivity because one cannot find it in
natural materials. Our work is different from the existing researches since we theoretically design the device via the effective medium method from the very beginning. In addition, it is also worth mentioning that our thermal invisibility corresponds to the fact that the hidden objects are able to feel the temperature gradient or heat flow, thus yielding the idea of thermal camouflage.

Our many-particle thermal invisibility is robust against both out-of-plane infrared cameras (Figs. 1(f) and 1(g)) and in-plane sensors (Fig. 3). Regarding these sensors, they may output electric (or other) signals transformed from temperature signatures according to thermoelectric or pyroelectric effects. Differing from the out-of-plane infrared camera, the existence of in-plane sensors can disturb the distribution of temperature outside the central square area. But, as shown in Fig. 3, this distortion caused by the sensors does not affect the validity of thermal invisibility even though the thermal environment changes from line heat sources (Figs. 3(a)–3(d)) to point heat sources (Figs. 3(e)–3(h)). In other words, the sensors outside become blind because they cannot be used to distinguish a thermal composite and a pure environmental material located in the central square area.

### 3 Thermal Diode With Invisibility

As a model application, our thermal invisibility can help to propose a class of thermal diodes; the research of such a diode was started in 2004 [37] and has received much attention due to potential applications [38]. Here, we propose a different kind of many-particle-system-based thermal diodes (Fig. 4(a)); the diodes allow heat flow from one direction with invisibility (Fig. 4(b)), but prohibit the heat flow from the inverse direction with visibility (Fig. 4(c)). Such a diode with the invisibility or visibility provides the heat rectification with an additional freedom of control, which may be useful in the areas where both thermal camouflage and thermal rectification are needed. Imagine one wants to hide a detector in the thermal diode (such as medium A). This can be realized by adding a compensation material (such as medium B). Then, one can detect thermal signals when the thermal diode works on the ON-state (as we have mentioned in Sec. 2.3 that “the hidden objects are able to feel the temperature gradient or heat flow, thus yielding a one-way invisibility.”). However, when the thermal diode works on the OFF-state and the thermal signals disappear, the invisible function will be invalid and it shows a warning that the thermal diode is turned off.

### 4 Conclusion

In summary, we have experimentally demonstrated that tailoring many-particle local-field effects can cause granular composites to be thermally invisible in an environment. This invisibility has a mechanism essentially differing from that of the well-known cloaking-type invisibility. Our concept of many-particle thermal invisibility has helped to propose a class of many-particle thermal diodes with invisibility. This work provides a new design strategy for thermal camouflage and thermal rectification by using composites. And it also implies that, besides thermotics, many-particle local-field effects could be a useful mechanism for achieving similar controls in other fields like optics/electromagnetics, acoustics, and fluid mechanics, where the electric permittivity and magnetic permeability, mass density and modulus, and diffusion coefficient, respectively, play the same role as the thermal conductivity in thermotics. What one needs to do is to apply effective medium methods to different fields. It may also be useful in implementation of conformal thermal devices in future.

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### Appendix: Experimental Details

The four experimental samples (Figs. 1(f)–1(i)) are manufactured by chemically etching, and they are covered by 0.01 mm thick polydimethylsiloxane films in order to eliminate the reflection. The size of the holes in the copper with conductivity 400 W/(m·K) is in the millimeter scale. Since polydimethylsiloxane is a kind of soft matter, the holes are filled with polydimethylsiloxane (with 0.15 W/(m·K)) and the defects made by air bubbles can be neglected. Additionally, even though there exists a little air, the resulting contact resistance has almost no effect on the overall result because air’s conductivity is close to polydimethylsiloxane’s conductivity. Outside the central square is occupied by brass with 109 W/(m·K). All the samples are placed on 2.0 cm thick plates of expanded polystyrene. A stable heat source is served by a tank filled with hot water (313 K), whose heat capacitance is much higher than the samples. The cold source corresponds to a same tank filled with ice-water mixture (273 K). The room temperature is about 293 K. The samples are detected by using the infrared camera (Flir E60) with a resolution of 0.1 K.

### References


