

## ELECTROROTATION OF COLLOIDAL SUSPENSIONS

J. P. HUANG and K. W. YU

*Department of Physics, The Chinese University of Hong Kong, Shatin, NT, Hong Kong*

G. Q. GU

*Department of Physics, The Chinese University of Hong Kong, Shatin, NT, Hong Kong  
 College of Information Science and Technology, East China Normal University  
 Shanghai 200 062, China*

Received 9 August 2002

We investigate the pressure effect on the electrorotation (ER) spectrum of living cell suspensions by considering the particle shape effect. Based upon the spectral representation theory, we consider coated oblate spheroidal particles and present a theoretical study of ER spectrum. Analytic expressions for the characteristic frequency as well as the dispersion strength can be obtained, thus simplifying the fitting of experimental data. From the theoretical analysis, we find that the cell shape, coating as well as material parameters can change the ER spectrum. We demonstrate good agreement between our theoretical predictions and experimental data on human erythrocytes suspensions.

### 1. Introduction

It is important to investigate the frequency-dependent responses of colloids and biological cells, to ac electric fields, which yields valuable information on the structural (Maxwell-Wagner) polarization effects.<sup>1,2</sup>

In the dilute limit, the ER of individual cell can be predicted by ignoring the mutual interaction between the cells.<sup>3</sup> However, the cells may aggregate in the plane of the rotating external field. In this case the Brownian motion can be neglected, and the cell system becomes non-dilute even though it is initially dilute. Based on the multiple image method,<sup>4</sup> we recently studied the ER spectrum of two approaching spherical particles.<sup>5</sup> Then we discussed the high concentration system to investigate the local field effect.<sup>6</sup>

The hydrostatic pressure effect may change the cell (e.g. human erythrocytes<sup>7</sup>) shape from perfect sphere to oblate spheroid. Thus, to discuss the pressure effect on the dielectric behavior of cells, we may consider the cell shape effect instead.

### 2. Formalism

Consider a suspension of spheroidal particles of complex dielectric constant  $\tilde{\epsilon}_1$  coated with a shell of  $\tilde{\epsilon}_s$  dispersed in a host medium of  $\tilde{\epsilon}_2$ , where  $\tilde{\epsilon} = \epsilon + \sigma/i2\pi f$

with  $f$  being the frequency of the applied field  $\mathbf{E} = E\hat{\mathbf{z}}$ . The depolarization factors of the oblate spheroidal particles satisfy a sum rule

$$L_z + 2L_{xy} = 1$$

where

$$L_z(1/3 < L_z < 1) \text{ and } L_{xy}(= L_x = L_y)$$

are the depolarization factors along the  $z$ - and  $x$ - (or  $y$ -) axes of the oblate spheroidal particle, respectively.  $L_z = L_{xy} = 1/3$  just indicates sphere.

The ER behavior is based on the interaction between a rotating electric field  $\mathbf{E}$  and the induced dipole moment  $\mathbf{M}$ . In circularly polarized fields, the axis of lowest polarizability, namely  $z$ -axis of the present particle, should be oriented perpendicular to the plane of field rotation.<sup>10</sup> In the steady state, the frequency-dependent rotation speed  $\Omega(f)$ , which results from the balance between the torque and the viscous drag, is given by<sup>3</sup>

$$\Omega(f) = -F(\epsilon_2, E, \eta) \text{Im}[\tilde{b}_{xy}],$$

where  $\text{Im}[\cdots]$  indicates taking the imaginary parts of  $[\cdots]$ , and  $F$  is a coefficient which is proportional to the square of magnitude of field  $E^2$ , but being inversely proportional to the dynamic viscosity  $\eta$  of the host medium. For a single coated spheroidal particle, the dipole factor  $\tilde{b}_{xy}$  is given by<sup>11</sup>

$$\tilde{b}_{xy} = \frac{1}{3} \frac{(\tilde{\epsilon}_s - \tilde{\epsilon}_2)[\tilde{\epsilon}_s + L_{xy}(\tilde{\epsilon}_1 - \tilde{\epsilon}_s)] + (\tilde{\epsilon}_1 - \tilde{\epsilon}_s)y[\tilde{\epsilon}_s + L_{xy}(\tilde{\epsilon}_2 - \tilde{\epsilon}_s)]}{(\tilde{\epsilon}_s - \tilde{\epsilon}_1)(\tilde{\epsilon}_2 - \tilde{\epsilon}_s)yL_{xy}(1 - L_{xy}) + [\tilde{\epsilon}_s + (\tilde{\epsilon}_1 - \tilde{\epsilon}_s)L_{xy}][\tilde{\epsilon}_2 + (\tilde{\epsilon}_s - \tilde{\epsilon}_2)L_{xy}]} \quad (1)$$

where  $y$  is the volume ratio of the core to the whole coated spheroid.

Based on the spectral representation,<sup>12,13</sup> we can obtain the analytic expression for the characteristic frequency at which the maximum ER velocity occurs. Let  $\tilde{\epsilon}_1 = \tilde{\epsilon}_2(1 - 1/\tilde{s})$ , and assume a real number  $x = \tilde{\epsilon}_s/\tilde{\epsilon}_2$ ,<sup>3</sup> we obtain

$$\tilde{b}_{xy} = \Pi + \frac{F_1}{\tilde{s} - s_1} \quad (2)$$

where  $\Pi$  denotes the nonresonant part<sup>13</sup> which vanishes in the limit of unshelled spheroidal cells. Regarding the expressions for various quantities  $s_1$ ,  $F_1$  and  $\Pi$ , we refer the reader to Ref. 3. After substituting  $\tilde{\epsilon} = \epsilon + \sigma/i2\pi f$  into Eq. (2), we rewrite  $\tilde{b}_{xy}$  after simple manipulations

$$\tilde{b}_{xy} = \left(\Pi + \frac{F_1}{s - s_1}\right) + \frac{\delta\epsilon}{1 + if/f_c} \quad (3)$$

with<sup>2,8</sup>

$$s = (1 - \epsilon_1/\epsilon_2)^{-1} \text{ and } t = (1 - \sigma_1/\sigma_2)^{-1},$$

where the dispersion magnitude and characteristic frequency admit respectively<sup>3</sup>

$$\delta\epsilon = F_1 \frac{s - t}{(t - s_1)(s - s_1)}, \quad f_c = \frac{1}{2\pi} \frac{\sigma_2}{\epsilon_2} \frac{s(t - s_1)}{t(s - s_1)}. \quad (4)$$

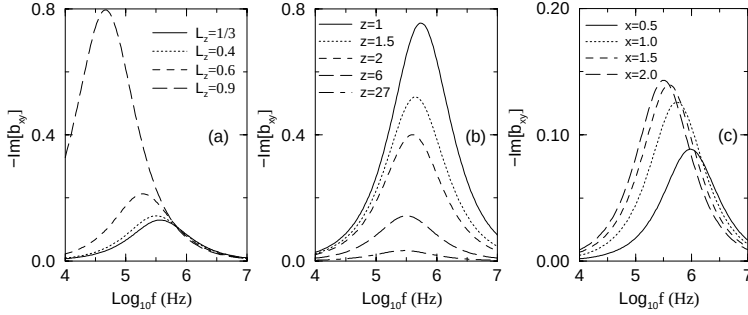


Fig. 1. The dependence of  $-Im[b_{xy}]$  on frequency for  $s = 1.1$ ,  $t = -0.005$ ,  $\epsilon_2 = 80\epsilon_0$ ,  $\sigma_2 = 2.9 \times 10^{-5} S/m$ . (a)  $z = 6$ ,  $x = 2$ ; (b)  $L_z = 0.4$ ,  $x = 2$ ; (c)  $z = 6$ ,  $L_z = 0.4$ .

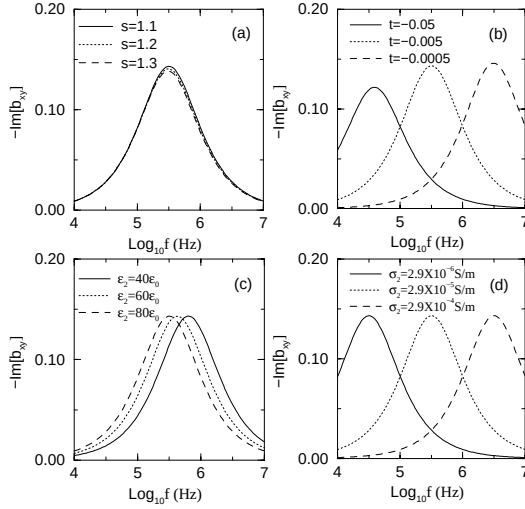


Fig. 2. The dependence of  $-Im[b_{xy}]$  on frequency for  $x = 2$ ,  $z = 6$ ,  $L_z = 0.4$ ; (a)  $t = -0.005$ ,  $\epsilon_2 = 80\epsilon_0$ ,  $\sigma_2 = 2.9 \times 10^{-5} S/m$ ; (b)  $s = 1.1$ ,  $\epsilon_2 = 80\epsilon_0$ ,  $\sigma_2 = 2.9 \times 10^{-5} S/m$ ; (c)  $t = -0.005$ ,  $s = 1.1$ ,  $\sigma_2 = 2.9 \times 10^{-5} S/m$ ; (d)  $t = -0.005$ ,  $s = 1.1$ ,  $\epsilon_2 = 80\epsilon_0$ .

### 3. Numerical Result

In Fig. 1, the effect of  $L_z$ ,  $z = 1/y$  and  $x$  on  $-Im[b_{xy}]$  are investigated, respectively. There is always a peak, the location of which is just the characteristic frequency  $f_c$ . As  $L_z$  (or  $x$ ) increases,  $f_c$  becomes red-shift, while the corresponding peak value increases as well. However, a larger  $z$  yields smaller peak value, while  $f_c$  is changed weakly.

In Fig. 2, we discuss the effect of  $s$ ,  $t$ ,  $\epsilon_2$  and  $\sigma_2$  on  $-Im[b_{xy}]$ . The  $s$  effect is small so that it can be neglected. Smaller  $|t|$  leads to larger  $f_c$ ; at the same time, increasing peak value appears. Increasing  $\epsilon_2$  yields decreasing  $f_c$ , and the related

peak value is unchanged. For increasing  $\sigma_2$ ,  $f_c$  increases concomitantly, but the peak value remains unchanged as well.

In Fig. 3, the experimental data is extracted from an experiment on human erythrocytes.<sup>9</sup> Obviously, there is a good agreement between our theory and this experiment. During this fitting, we model the cell as oblate spheroid ( $L_z = 0.65$ ).

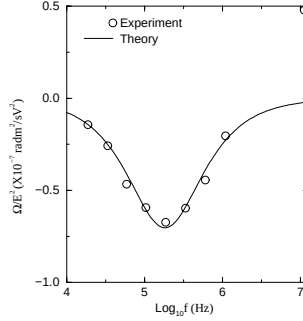


Fig. 3. A fitting on human erythrocyte data ( $\sigma_2 = 12.5 \times 10^{-3}$ ). Theory:  $s = 100$ ,  $t = 1.00783$ ,  $\epsilon_2 = 80\epsilon_0$ ,  $z = 3$ ,  $x = 0.008$ ,  $L_z = 0.65$ ,  $F/E^2 = 135$ .

#### 4. Conclusion

We have presented a theoretical study of ER based on the spectral representation theory. From the theoretical analysis, we find that cell shape and the coating can change the characteristic frequency. By adjusting the cell shape, dielectric properties and the thickness of the coating, it is possible to obtain good agreement between our theoretical predictions and the experimental data on human erythrocytes suspensions.

#### Acknowledgments

This work was supported by the Research Grants Council of the Hong Kong SAR Government under grant CUHK 4245/01P.

#### References

1. For a review, see J. Gimsa and D. Wachner, *Biophys. J.* **77**, 1316 (1999).
2. Jun Lei, Jones T. K. Wan, K. W. Yu and Hong Sun, *J. Phys.: Condens. Matter* **13**, 3583 (2001).
3. J. P. Huang and K. W. Yu, *J. Phys.: Condens. Matter* **14**, 1213 (2002).
4. K. W. Yu and Jones T. K. Wan, *Comput. Phys. Commun.* **129**, 177 (2000).
5. J. P. Huang, K. W. Yu and G. Q. Gu, *Phys. Rev.* **E65**, 021401 (2002).
6. J. P. Huang, K. W. Yu and G. Q. Gu, *Phys. Lett.* **A300**, 385 (2002).
7. K. Asami and T. Yamaguchi, *Annals of Biomedical Engineering* **27**, 427 (1999).

8. Jun Lei, Jones T. K. Wan, K. W. Yu and Hong Sun, *Phys. Rev.* **E64**, 012903 (2001).
9. J. Gimsa, Th. Schnelle, G. Zechel, and R. Glaser, *Biophys. J.* **66**, 1244 (1994).
10. J. Gimsa, *Bioelectrochemistry* **54**, 23 (2001).
11. L. Gao, Jones T. K. Wan, K. W. Yu and Z. Y. Li, *J. Phys.: Condens. Matter* **12**, 6825 (2000).
12. D. J. Bergman, *Phys. Rep.* **43**, 379 (1978).
13. K. P. Yuen and K. W. Yu, *J. Phys.: Condens. Matter* **9**, 4669 (1997).