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REVIEW ARTICLE

Progress in physical properties of Chinese stock markets

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In the past two decades, statistical physics was brought into the field of finance, applying new methods and concepts to financial time series and developing a new interdiscipline "econophysics". In this review, we introduce several commonly used methods for stock time series in econophysics including distribution functions, correlation functions, detrended fluctuation analysis method, detrended moving average method, and multifractal analysis. Then based on these methods, we review some statistical properties of Chinese stock markets including scaling behavior, long-term correlations, cross-correlations, leverage effects, antileverage effects, and multifractality. Last, based on an agent-based model, we develop a new option pricing model — financial market model that shows a good agreement with the prices using real Shanghai Index data. This review is helpful for people to understand and research statistical physics of financial markets.

Keywords econophysics, Chinese stock market, statistical method, statistical physics

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Contents

1 Background

In the past two decades, physicists have entered the field of finance, applying new methods and concepts to financial time series [1] and developing a new interdiscipline "econophysics". New results have sprung out, which challenge the traditional "golden rules" in quantitative finance. For example, Mandelbrot [2] and Fama [3] indicated that the distributions of returns do not obey the Gaussian distribution. In 1963, Mandelbrot proposed the Levy model for return distribution [2], which was extended to stock markets [4]. Mantegna and Stanley discovered that the probability density of stock price returns is truncated Levy distribution [5]. And, many work show that power-law decay in return distributions widely exist in different parts of the world [6–13]. Chinese stock market, as a big emerging market in the world, has also causes concern for Chinese researchers. A lot of statistical analyses of the Chinese stock data have been done in the last decade [10–12, 14–36]. In previous studies, researchers revealed some peculiar physical properties of Chinese stock markets by using some methods from statistical physics.

In order to easily learn the development in this field, here we introduce some commonly used methods in econophysics and review several classical statistical properties of Chinese stock market. In each section, we introduce some different statistical methods and peculiar properties, including distribution functions, correlation functions, detrended fluctuation analysis (DFA) method, and detrended moving average (DMA) method, which present peculiar physical properties such as scaling behavior, autocorrelation, cross correlation, multifractal, etc. In these methods, the analysis on the probability density of the stock price returns/the volume traded at each transaction/the volatility interval is most widely adopted [17, 18, 20, 31–36]. Besides the distributions, the correlation functions are also interesting features of financial time series. The short-term autocorrelation of price returns and the long-term autocorrelation of absolute returns are verified by many academics [6, 7, 10]. And the cross correlations between different stocks are also studied by many researchers [37–40]. Based on multifractal theory, the DFA and DMA method are also widely adopted to study stock markets [10, 17, 24, 25, 41–49]. In addition, option pricing as the prediction for future stock prices is also an important issue. A new method of financial market model (FMM) for treating the option pricing is proposed and shows a good agreement with the prices using real data of Shanghai Composite Index.

Then, we introduce different methods and statistical properties of Chinese stock market one by one.

2 Distributions

2.1 Distribution function

The probability density of price returns is the earliest to be studied. In 1900, Bachelier [50] stated that market price would follow a random walk and the distribution of price returns would be Gaussian. 60 years later, Mandelbrot [2] argued that, according to his research on cotton markets, the probability density is a Levy-stable one, in which the tails decay weaker than $1/r^3$. Later, researchers replaced the Levy-stable distribution with a power-law decay, in which the tails approximately obey $1/r^4$. This result was later verified by many other researchers on different markets [6–10]. The definition of the probability density function (PDF) is

$$P(r = x) \equiv \lim_{\delta x \to 0} \frac{N(x - \frac{1}{2}\delta x < r < x + \frac{1}{2}\delta x)}{N(-\infty < r < +\infty) \cdot \delta x}$$

Here N(window) means the number of data points lying in the window. In a real research, we should be careful about the size of δx. If δx is too small, there will be no data point in the window.

If the number of data points is small, PDF curve will be hard to get. A substitute for PDF is the cumulative distribution function (CDF), which is also widely used by physicists [6, 7, 11, 12]. The CDF is defined as

$$P(r > x) \equiv \frac{N(r > x)}{N(-\infty < r < +\infty)}, \quad x \geqslant 0$$

$$P(r < x) \equiv \frac{N(r < x)}{N(-\infty < r < +\infty)}, \quad x < 0$$

From the definitions of PDF and CDF, it seems that we can get the PDF from CDF by first derivative. This is mathematically true. For example, if CDF obeys $P(r > x)\ x^{-\alpha}$, then PDF follows $P(r = x)\ x^{-\alpha-1}$. But if CDF is very complex, we can not simply get PDF in this way.

2.2 Scaling behavior

The distribution functions are basic but powerful analysis tools, which can be applied to many areas. For instance, researchers have used this kind of tools to study price returns [17], the interval of volatility [51, 52], the number of transactions in time interval and the bid-ask spread in each transaction for Chinese stock market.

For the initial stage of Chinese markets, researchers investigated the CDF of daily returns of 141 Chinese stocks in the Shanghai and Shenzhen Stock Exchange from January 1994 to December 2001 [11]. They observed that the distribution of daily returns follows the power-law rule and found that the positive and negative tails show asymmetry in Fig. 1.

After 2006, Chinese stock market experienced a bull market and a bear market which was caused by subprime crisis. Some researchers tested the power-law rule under different market phases and found out the similar distribution curves of price's returns in bull market and bear market [17]. See Fig. 2.

Further, researchers analyzed the distributions of stock returns at different microscopic timescales using ultra high frequency data [18, 19]. Figure 3 shows the distribution of returns at different timescales. The curves in the right panel are shifted for clarity. We notice that the tail of PDF is fatter with the decrease of timescale and the tail of CDF for large timescale decays faster than the tail with small timescale. Besides, some researchers analyzed the distributions of intraday returns for Chinese stock market at different timescales ($\Delta t = 1$, 5, 15, and 30 min), and found that for small time scales, the intraday return distributions are not universal in Chinese stock markets compared to mature markets but might be universal at large time scales [53].

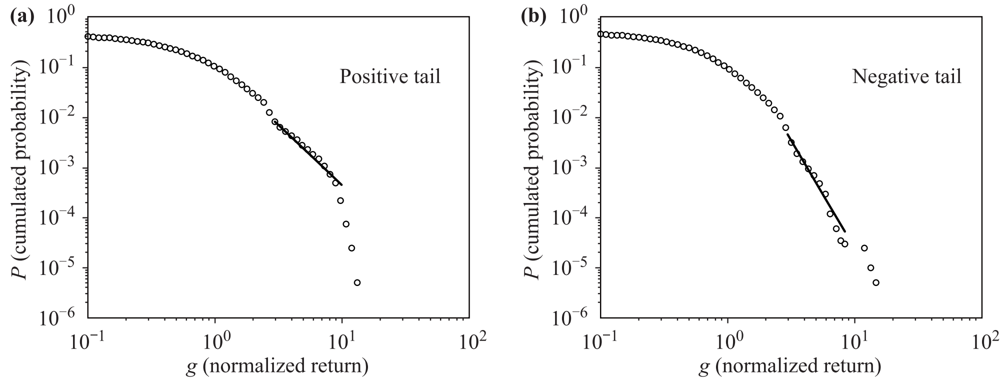

Fig. 1 CDF of normalized returns of of 104 Chinese stocks in the Shanghai and Shenzhen Stock Exchange from January 1994 to December 2001: **(a)** positive tail and **(b)** negative tail. The solid line is a power-law fit with $\alpha = 2.44 \pm 0.02$ for positive tail and $\alpha = 4.29 \pm 0.04$ for negative tail. Reproduced from Ref. [11].

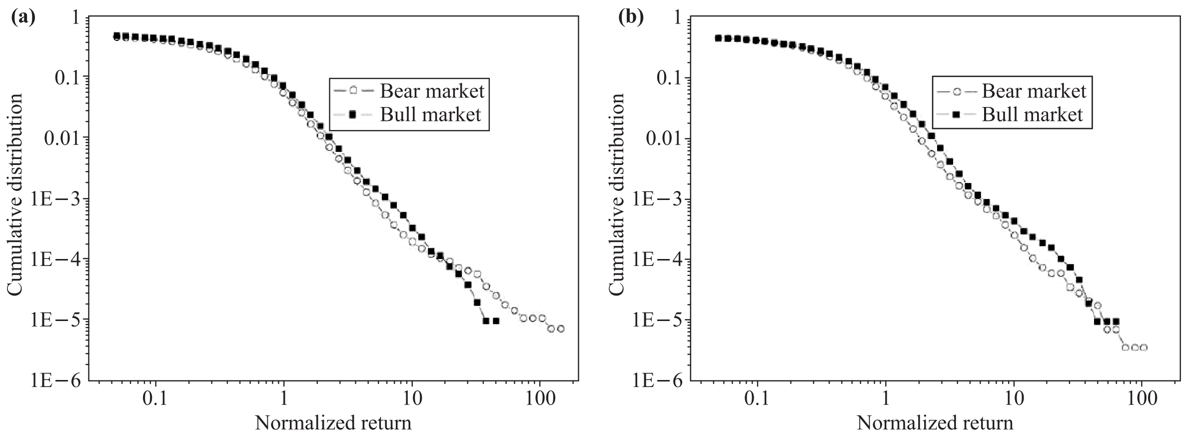

Fig. 2 CDF of normalized returns for Shanghai Index under bull market and bear market: **(a)** positive tail with $\alpha = 2.29 \pm 0.05$ and **(b)** negative tail with $\alpha = 2.18 \pm 0.04$. Reproduced from Ref. [17].

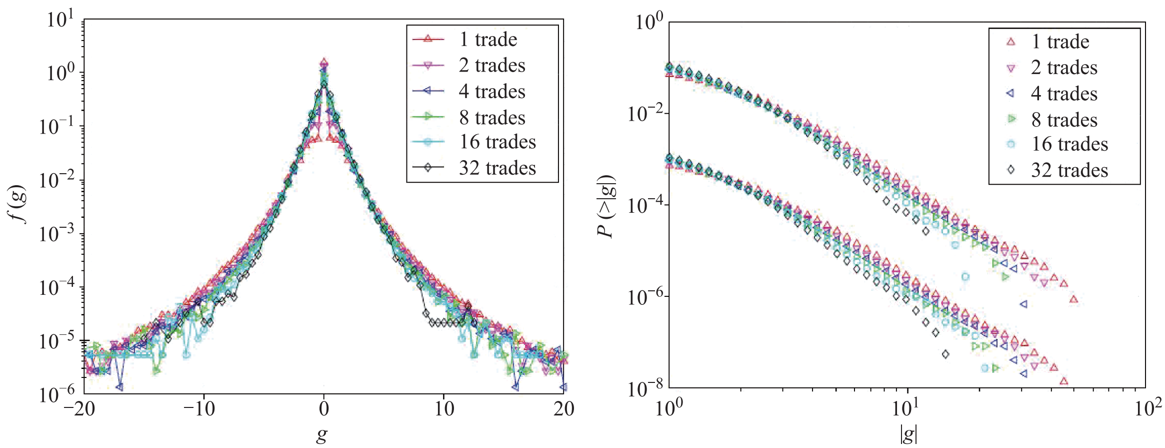

Fig. 3 Distribution of event-time returns under different timescales $\Delta t = 1, 2, 4, 8, 16, 32$ transactions. In right CDF figure, upper cluster is positive returns while lower is negative returns. Reproduced from Ref. [18].

Except for the stock returns, trading volumes and sizes are also important variables. In the work [20], researchers investigated the distributions of trading volumes and sizes. Figure 4 shows the distributions of trade sizes of 22 stocks on Shenzhen exchange in year 2003, while Fig. 5 displays the distributions of trading volumes. Their tails both show a power-law rule.

3 Correlations

3.1 Long-term autocorrelation

The autocorrelation function is another important feature of financial time series, which is also a powerful tool

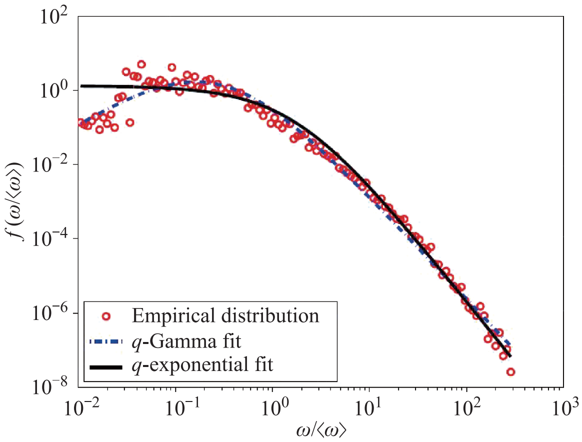

Fig. 4 The distributions of trade sizes. The corresponding tail exponents using two fit methods are 2.03 and 2.22, respectively. Reproduced from Ref. [20].

to test Bachelier's random walk assumption [6, 7]. It is defined as

$$\rho(\tau)=\frac{E(B_t-E(B_t))(B_{t+\tau}-E(B_{t+\tau}))}{\sqrt{D(B_t)D(B_{t+\tau})}}$$

Here $E(x)$ is the expectation of x, and $D(x)$ is the standard deviation. When τ is small enough, $E(B_t)$ is close to $E(B_{t+\tau})$, and $D(B_t)$ is close to $D(B_{t+\tau})$, so the above equation can be reduced to

$$\rho(\tau)=\frac{E(B_tB_{t+\tau})-E^2(B_t)}{D(B_t)}$$

which is widely used by most researchers in calculating the autocorrelation function [6, 7, 10].

These years many researchers applied this method to Chinese stock market and scrutinized the correlations of Chinese stock. Some researchers calculated the autocorrelation function of Shanghai and Shenzhen Index log-return $r(t)$, and absolute log-return $|r(t)|$ under bull market and bear market [17]. Figure 6 and Fig. 7 show that the bull market has a stronger long-term autocorrelation than the bear market. Moreover, the autocorrelation of absolute returns for Chinese stock market obeys a power-law decay.

3.2 Cross-correlation

Another interesting correlation function is the cross correlations [37–39]. Suppose we have time series of stock

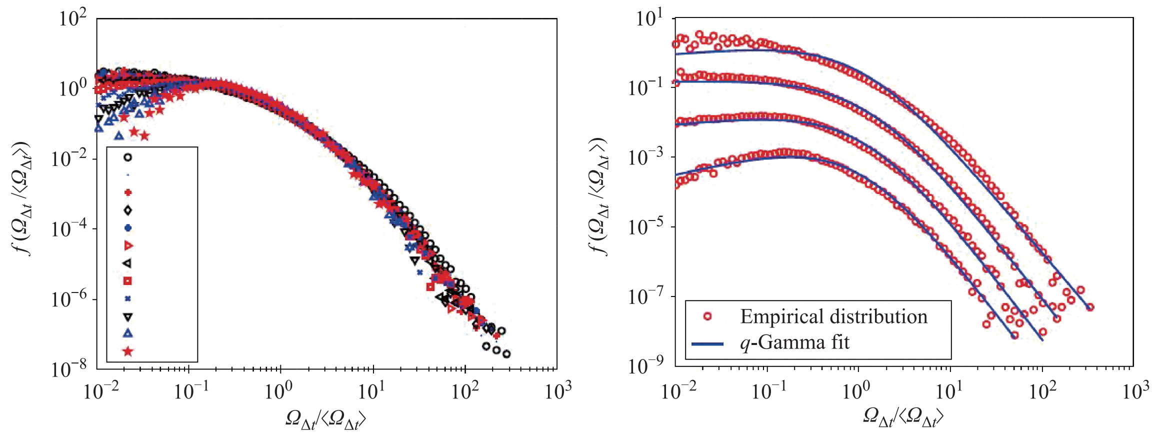

Fig. 5 The distributions of the normalized trading volumes at different timescales. The timescales in **(b)** from top to bottom are Δt = 1 min, 5 min, 15 min, and 60 min, respectively. Reproduced from Ref. [20].

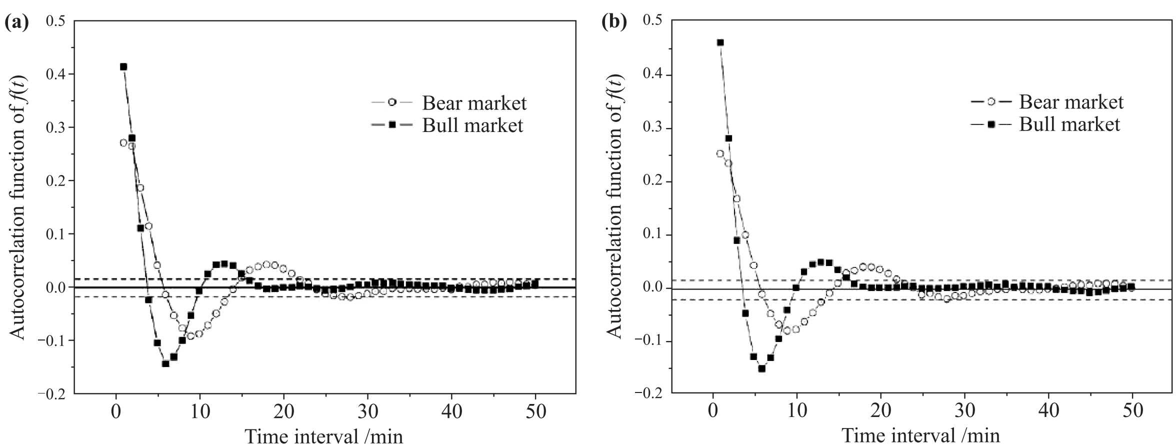

Fig. 6 The autocorrelation of log-return $r(t)$ in bull market and bear market: **(a)** Shanghai Index and **(b)** Shenzhen Index. Reproduced from Ref. [17].

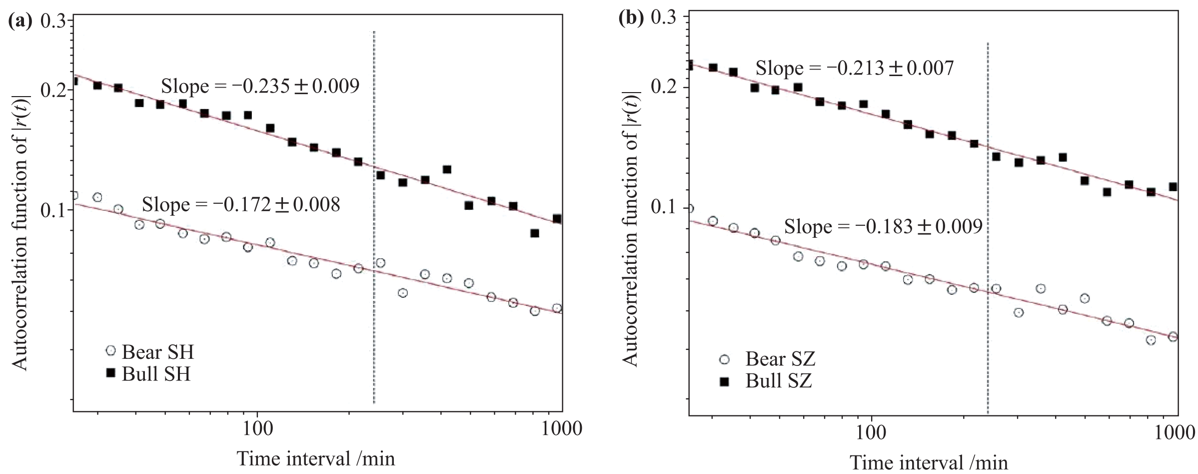

Fig. 7 The autocorrelation of the absolute log-return $|r(t)|$ in bull market and bear market: **(a)** Shanghai Index and **(b)** Shenzhen Index. The dashed line represents the time interval of one trading day (240 min). Reproduced from Ref. [17].

prices G_i for $i = 1, 2, 3, \cdots, N$. The correlation matrix C is defined as

$$C_{ij} \equiv \frac{\langle G_i G_j \rangle - \langle G_i \rangle \langle G_j \rangle}{\sigma_i \sigma_j}$$

Here $\langle \cdots \rangle$ represents a time average over the whole period, σ_i denotes the standard deviation of stock i. Much work can be done on the correlation matrix C, for example, testing its eigenvalues.

In 2009, some researchers analyzed the cross-correlation matrix C of Chinese stock returns and found that the Chinese stock market as a developing market shows much stronger correlations than American and Indian stock markets [21]. See Fig. 8.

3.3 Leverage effect

In order to further investigate the evolution of stock returns from historical time series, some researchers focused on the return-volatility correlation and observed a leverage effect and an antileverage effect in Chinese indices [10, 22]. They defined the return-volatility correlation function as $L(t) = [\langle r(t')|r(t'+t)|^2\rangle - \langle r(t')\rangle\langle|r(t')|^2\rangle]/Z$, $Z = \langle|r(t')|^2\rangle^2$, and calculated $-L(t)$ with daily data and minutely data. In Fig. 9, a positive $L(t)$ for Chinese indices is observed in the positive time direction, which indicated an antileverage effect, while German market exist a leverage effect (a negative $L(t)$).

3.4 Detrended fluctuation analysis method and long-term correlation

The detrended fluctuation analysis (DFA) method is based on the multifractal theory [54, 55]. It is designed to analyze the long-term correlation by many researchers [6, 7, 10, 56]. Suppose $B(t)$ is a time series, we define $C(t)$ as

$$C(t') = \sum_{t''=1}^{t'} [B(t'') - \langle B \rangle]$$

where $\langle \cdots \rangle$ means the average value. t' in $C(t')$ runs in

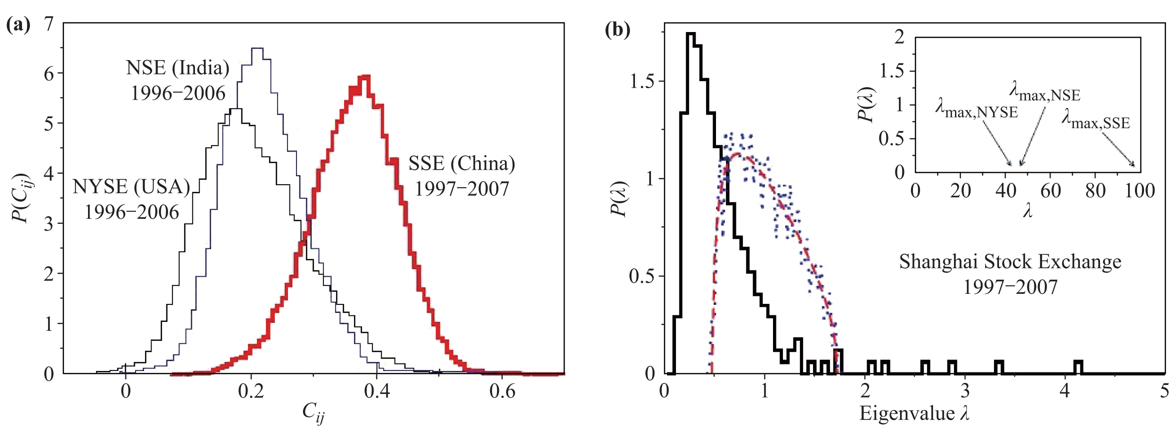

Fig. 8 The PDF of the elements of the cross-correlation matrix C for three countries: China, India, and USA. **(b)** The PDF of the eigenvalues of the correlation matrix C for Shanghai Stock Exchange (*solid line*). Reproduced from Ref. [21].

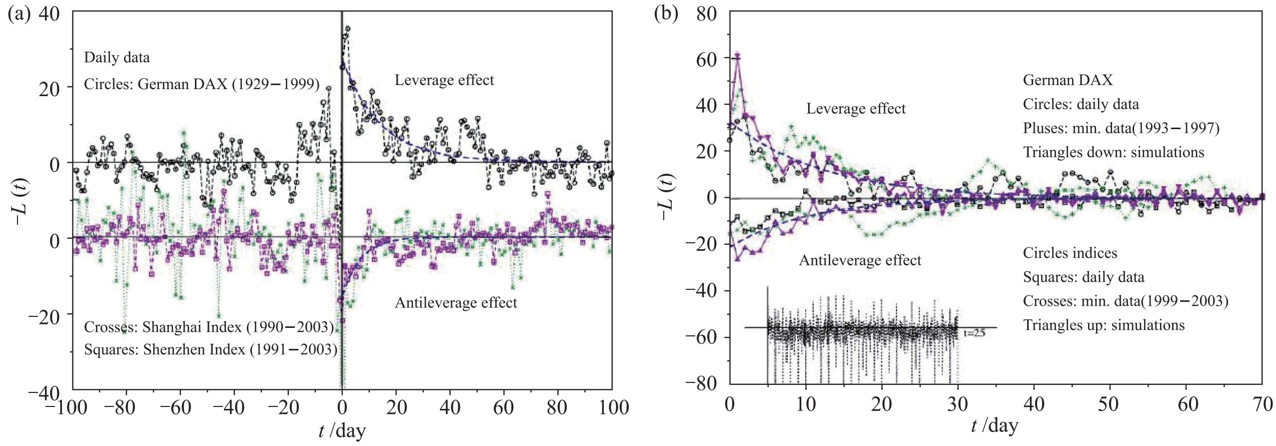

Fig. 9 **(a)** $-L(t)$ with daily data. Dashed lines: an exponential fit $L(t) = c\exp(-t/\tau)$. For leverage effects, $c = -27$ and $\tau = 15$; for antileverage effects, $c = 15$ and $\tau = 7$. **(b)** $-L(t)$ with daily and minutely data. For leverage effects, $c = -33$ and $\tau = 13$; for antileverage effects, $c = 21$ and $\tau = 10$. Reproduced from Ref. [22].

time interval from 1 to T. Then the time interval $(1, T)$ is divided into windows with a size of t. $C(t')$ is fitted to a linear function $C_t(t')$ in each window. Eventually, the DFA function is obtained as

$$F(t) = \sqrt{\frac{1}{T}\sum_{t'=1}^{T}[C(t') - C_t(t')]^2}$$

Usually, the DFA method can give a more precise analysis on the long-term correlation, as we can see in previous work [10, 17, 24, 25]. Figure 10 is an example of DFA on the long-term correlation from Ref. [10], and shows that long-term correlation of the bull market is stronger than the bear market.

3.5 Detrended moving average method

The detrended moving average (DMA) method is an algorithm based on moving average technique, which is also widely employed to analyze long-term correlation [41–43]. The difference with DFA is that the linear function $C_t(t')$ is replaced by a moving average function over a moving window in the DMA method $\tilde{C}_t(t')$. Previous studies have shown that the DMA method is comparable to DFA and slightly outperforms DFA under some situations [44].

4 Multifractality

4.1 Multifractal detrended fluctuation analysis method

Many studies have shown that foreign financial markets exhibit multifractal behavior [23, 57–59]. For years, some researchers reported that multifractality is also observed in Chinese stock market [24–30]. In order to observe the multifractal nature, many different methods have been applied. Based on DFA method, multifractal detrended fluctuation analysis (MFDFA) is commonly adopted, which is a generalization of DFA:

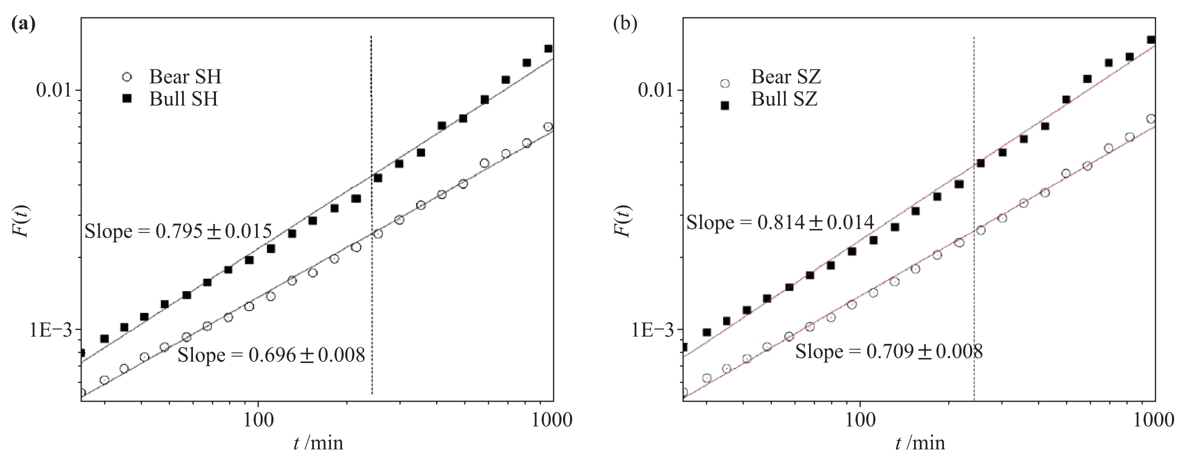

Fig. 10 The DFA function of Chinese stock indices under bull market and bear market: **(a)** Shanghai Index and **(b)** Shenzhen Index. The dashed line corresponds to one trading day (240 min). Reproduced from Ref. [17].

$$F(t) = \left(\frac{1}{T} \sum_{t'=1}^{T} [C(t') - C_t(t')]^2 \right)^{\frac{1}{q}}$$

If q=2, it is a DFA function. In previous studies, $F(t)$ shows a power-law decay if time series has Scale characteristics, $F(t) \propto t^{h(q)}$. Here h is the Hurst exponent. Theoretically, if $0.5 < h < 1$, $B(t)$ is long-term correlated in time; if $0 < h < 0.5$, $B(t)$ is temporarily anti-correlated; if $h = 0.5$, $B(t)$ obeys Gaussian white noise; if $h > 1.0$, the time series is unstable.

The scaling exponent $\tau(q)$ is commonly adopted to characterize the multifractality, $\tau(q) = qh(q) - D_f$, where D_f is the fractal dimension, with $D_f = 1$ in time series analysis. When $\tau(q)$ presents nonlinearity, multifractal behavior exists in time series. Based on the Legendre transformation, the singularity exponent α and multifractal spectrum $f(\alpha)$ is obtained as, $\alpha = d\tau(q)/dq$, $f(\alpha) = q\alpha - \tau(q)$. Figure 11 shows that Hurst exponent $h(q)$ decrease with q and $\tau(q)$ presents nonlinearity [27]. It is worth noting that the singularity widths in multifractal spectrum for Chinese stock markets is wider than New York stock exchange, which indicates that the Chinese stock shows stronger multifractality than New York.

4.2 Multifractal detrending moving average method

Similar to MFDFA, multifractal detrending moving average (MFDMA) method, which is an extension of DMA, is also generalized to analyze multifractal behavior of time series [24, 45]. In previous studies, researchers found that MFDMA method outperforms the MFDFA for multifractal analysis [45]. For the MFDMA method, its fluctuation function follows the same form:

$$F_q(n) = \left(\frac{1}{T} \sum_{t'=1}^{T} [C(t') - C_t(t')]^2 \right)^{\frac{1}{q}}$$

where n is the window size (corresponding t in MFDFA), and $C_n(t')$ follows the form in the DMA method as, $\tilde{C}_n(t') = \frac{1}{n} \sum_{k=-\lfloor (n-1)\theta \rfloor}^{\lceil (n-1)(1-\theta) \rceil} y(t'-k)$. θ is a position parameter and varies from 0 to 1. Figure 12 shows the multifractal analysis of intraday Shanghai Stock Exchange Composite Index [45]. Figure 13 shows the comparison of multifractal analysis between MFDFA and MFDAM [24].

4.3 Multifractal detrended cross-correlation analysis method

In addition, the multifractal detrended cross-correlation analysis (MFDCCA) method is worth mentioning, which is also a helpful method with a lot of applications to financial markets in recent years [46–49]. The detrended cross-correlation analysis (DCCA) was proposed by Podobnik and Stanley in 2008 to analyze long-range cross-correlations between two nonstationary time series [46]. Then, based on DCCA, Zhou found that it can

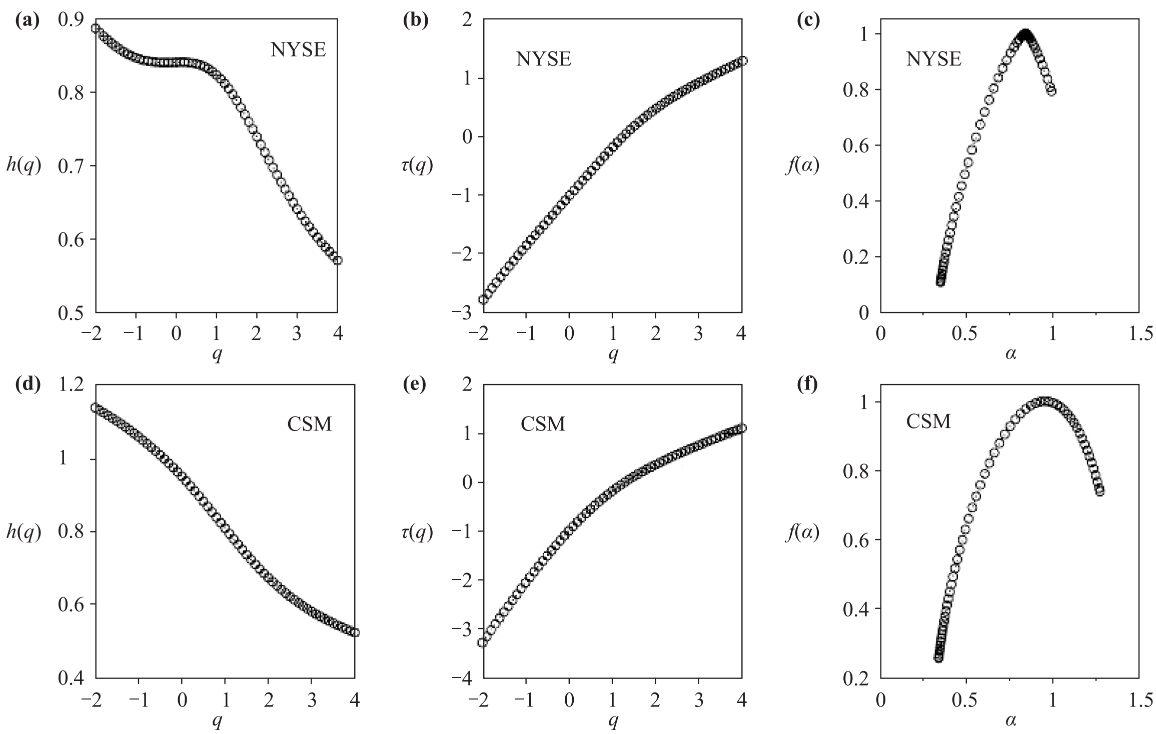

Fig. 11 Multifractal analysis of stock time series: **(a–c)** New York Stock Exchange and **(d–f)** Chinese Stock Market. Reproduced from Ref. [27].

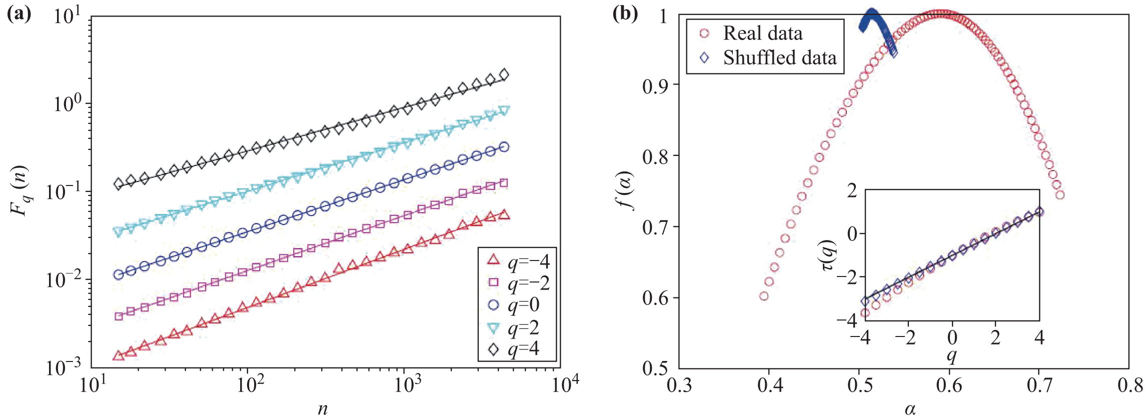

Fig. 12 Multifractal analysis of intraday time series of Shanghai Stock Exchange Composite Index. Reproduced from Ref. [45].

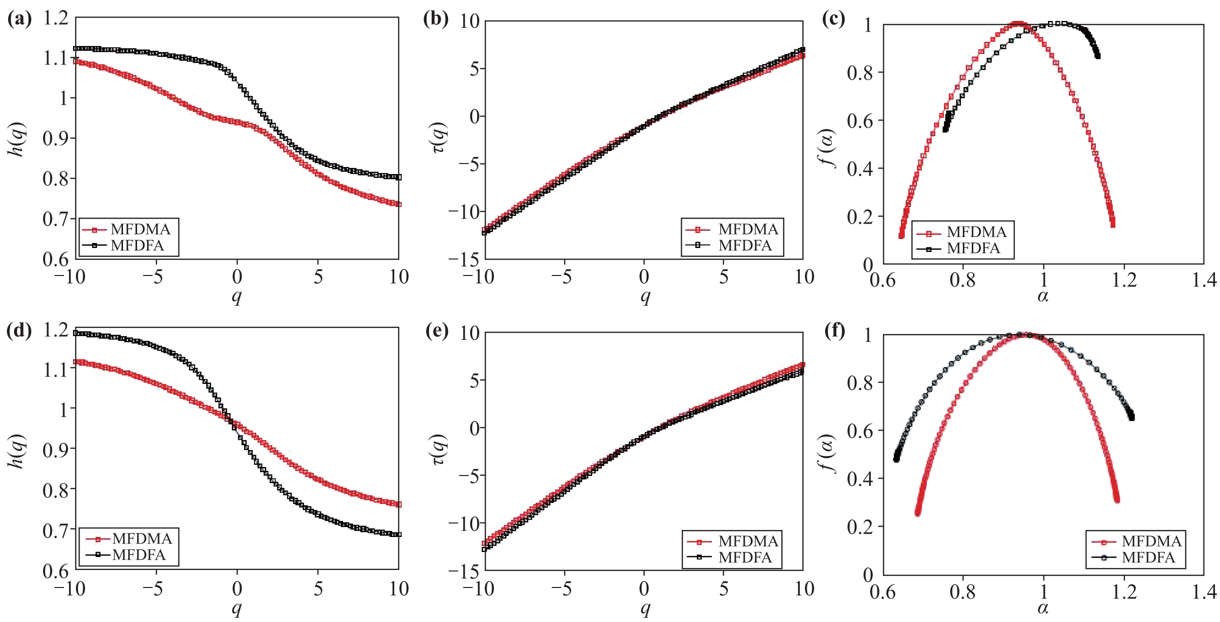

Fig. 13 Multifractal analysis of the intertrade duration time series using MFDMA and MFDFA. **(a–c)** Bao Steel stock; **(d–f)** its warrant. Reproduced from Ref. [24].

be generalized to reveal multifractal nature and proposed the multifractal detrended cross-correlation analysis (MFDCCA) method and its extension [48, 49], which is very useful when applied to one/two dimensional binomial measures, financial price series and multifractal random walks.

5 Option pricing

5.1 Bouchard–Sornette method

Option is a very useful financial derivative [60–62]. A call option gives the holder the right to buy a certain number of underlying assets at a specific strike price on or before expiration date. In contrast, a put option gives the holder the right to sell. The underlying assets can be stocks, indexes, etc. Option pricing as the prediction for future price is also an important issue. Options that can only be executed on the expiration date are called the European options, while the ones that can be executed at any time before the expiration date are called the American options.

For a call option, if the market price of the underlying asset is higher than the strike price, the holder will execute the option to buy the asset at the strike price and then sell it at the market price to gain profits. However, if the market price is lower than the strike price, the holder will choose to let call option expire without execution. For a put option, the operation is opposite. It can be seen that, options can be used to manage risks or to speculate. A most important question is how to price the option, namely, how much an agent has to pay to obtain the option so that he/she can get the right to choose whether to execute or not. A very famous option pricing model is the Black–Scholes formula [63, 64] which gives the price of European options. It assumes that the price of the underlying asset follows geometric Brownian motion. A diffusion equation related to the option's price is then derived. Under certain boundary conditions, the solution of the diffusion equation is yielded:

$$C = S_0 N(d_1) - X\mathrm{e}^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

Here, C is the current European call option value. S_0 is the current price of the underlying asset. $N(d)$ means the probability that a random draw from a standard normal distribution is below the value of d. X is the strike price. r is the risk-free interest rate. T is the time to expiration date. σ is the standard deviation of the annualized continuously compounded rate of returns of the underlying asset. Using the put-call parity theorem, we can get the formula for European put option as well:

$$P = Xe^{-rT}[1 - N(d_2)] - S_0[1 - N(d_1)]$$

In the financial market, it is well known that, the price of stocks or indexes do not follow geometric Brownian motion. They have stylized facts instead [51], such as fat tails of the return distribution, volatility clustering, etc. In order to drop the inaccurate assumption of geometric Brownian motion, Bouchaud and Sornette propose a new method using real market price data to price options [52]. The formula is:

$$V_0 = (1 + r_f)^{-T} \int_0^{\infty} V_T[S_T, X] p[S_T|S_0] \mathrm{d}S_T$$

It can be applied to both European call and put options. In the formula, V_0 is the current option value. r_f is the risk-free interest rate. T is the time to expiration date. X is the strike price. S_0 and S_T means the price of the asset at the current time and at the expiration date respectively. $V_T[S_T, X]$ represents the value of the option given the asset value being S_T and the strike price being X. It is obvious that for call options, $V_T[S_T, X] = \max[S_T - X, 0]$, while for put options, $V_T[S_T, X] = \max[X - S_T, 0]$. $\max[a, b]$operator gives the large one's value between a and b. $p[S_T|S_0]$ is the currently forecast price distribution at the expiration date. The distribution can be estimated using the history price distribution.

Compared with Black-Scholes formula, the Bouchaud–Sornette method is closer to the real market. However, it still has some shortcomings. First, the method uses the history market price to predict the price distribution at the expiration date. But it is known that the market changes quickly. The future's distribution cannot stay the same with the history's. Second, for emerging markets, the history data can be too few to calculate price distributions. Hence, we should try to find a new approach to option pricing. Agent-based models (ABMs) have long been a useful method when simulate complex human systems [65–71]. The bottom-up approach can give a clear view of the underlying dynamics of the systems. Next, we design an ABM for financial market and then combine the ABM with Bouchaud–Sornette method to calculate the option price on the market. After adjusting the parameters carefully, It shows a good agreement with the price using Bouchaud–Sornette method on Shanghai Composite Index (or Shanghai Index). This implies that this new approach has a good potential for option pricing.

5.2 Financial market model method

In the past, lots of researchers proposed many interesting models to model real financial market [72–80], such as minority games [72, 78], simple nonlinear adaptive systems [74], stochastic multi-agent models [76], and order-driven model which is constructed by real stock market order data and its statistical laws [79, 80]. In this part, we design an ABM which is a modification of the market-directed-resource-allocation game [66, 67]. For convenience, we name our model the financial market model (FMM) and we use the terminology from the stock market to describe our model specifically. In FMM, there are N agents making their decisions independently. Each of them has S strategies. An example of the strategies is shown in Table 1. The left column is the market situations, ranging from 1 to P, representing P exogenous situations. The right column gives the decision of the agents facing a certain market situation. $+1$ means that agents predict the price to rise, while -1 for the prediction of price to fall.

Table 1 An example of a strategy

Market situations	Choices
1	$+1$
2	-1
3	-1
...	...
$P-1$	$+1$
P	$+1$

After entering the market, each agent will score his/her S strategies based on the virtual performance: when a strategy makes a right prediction, 1 point is added to its score whether it is used by the agent or not, otherwise, nothing is given to the strategy. Before each trading, agents will use their highest-scored strategies to make a decision. If several strategies have the highest score, a strategy among them will be chosen randomly. In the following part, we will describe an agent's decision-making process specifically.

Right before the time step t, an agent predicts the price movement from time step t to $t+1$ using his/her best strategy. If the prediction from the strategy is $+1$, which means price will rise, he/she will give a buy order at time step t and a sell order at $t+1$, and both the orders have one unit of stock. Similarly, when he/she predict the price to fall, a sell order will be given at time step t and a buy order at $t+1$. This process will be repeated with the time, so it can be seen that his/her net order at time step $t+1$ is a total effect of two operations, which depend on the predictions on time step t and $t+1$ respectively. For example, if he/she has the same predictions at the two time steps, the net order will be zero, meaning that there is no trading action from the agent at time step $t+1$. This setting can give different agents different time scales to trade on. When all the agents give the net orders at time step $t+1$, the price movement can be obtained by calculating the excess demand:

$$\ln S_{t+1} - \ln S_t = k(\ln N_c - \ln N_p)$$

where N_c and N_p represent the total buy orders and sell orders accordingly. k means the market depth. S_t is the stock price at time step t.

Using the FMM, we can obtain simulated stock price series. By inputting the simulated data into the Bouchaud–Sornette method, we can then price options on the stock market. In the FMM, we set N=1000, P=32, S=6, and k=0.09. Figure 12 shows a simulated stock price series. It exhibits some similarities with the real market data.

Then we use the simulated data to calculate $p[S_T|S_0]$ in the Bouchaud–Sornette method and then the option price can be obtained. For the option, the current stock value is set to be 500 and the expiration date is set as 21 trading days. The results are shown in Figs. 13 and 14 for both call options and put options accordingly. A good agreement is seen, which implies that the FMM model can be used to simulate the real market and then to do the option pricing.

From these results, we designed an ABM called FMM to model the real market and then combined the simulated data with the Bouchaud–Sornette method to do the option pricing. The results show a good agreement with the prices using real Shanghai Index data. This implies that the FMM has a potential to replace the real market data. Hence, in dealing with the emerging markets, the

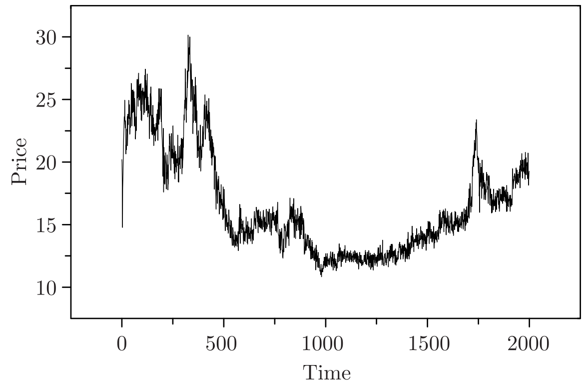

Fig. 14 The simulated stock price series.

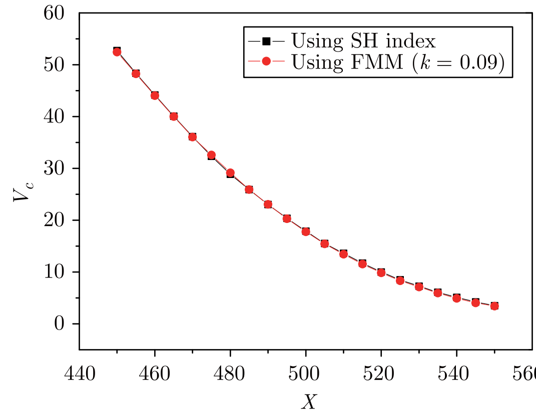

Fig. 15 The price of call options V_c along with different strike prices X using FMM simulated data (*circles*) and Shanghai Index daily data from January 2, 2001 to October 27, 2009 (*squares*).

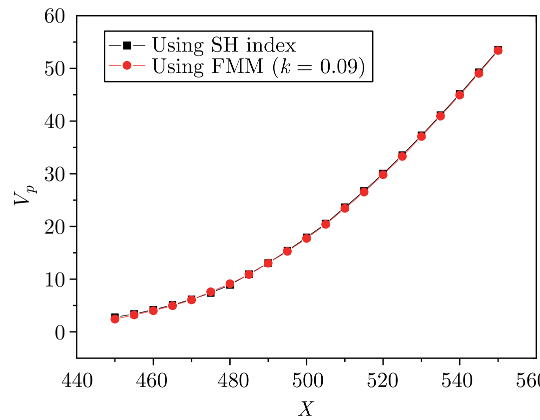

Fig. 16 The price of put options V_p along with different strike prices X using FMM simulated data (*circles*) and Shanghai Index daily data from Jan. 2, 2001 to Oct. 27, 2009 (*squares*).

FMM can be used to generate more data so that price distributions can be calculated and options can be priced.

It should be noted that here is only an initial approach of combining the ABM and Bouchaud–Sornette method to price options. In the future, the FMM should be modified so that it can be initialized using the real market data and generates the future price movements, which are used to calculate the future price distributions. Then we can obtain option values more accurately using the Bouchaud–Sornette method.

6 Summary

In this review, we have introduced several commonly used methods in econophysics including distribution functions, correlation functions, DFA methods, etc. Then, we have reviewed some physical properties of Chinese stock markets such as scaling behavior, correlations, leverage effects, antileverage effects, multifractality, and so on. It is not difficult to learn these methods. However, the fact that people use similar methods to test different objects in order to find empirical regularities, tells us that econophysics needs tough work on large amount of economic data. Certainly, we cannot include all the relevant achievements into this review, say, Fibonacci-like levels revealed in stock price reversals [81]. But, the commonly used methods and statistical properties listed in this review are expected to help people to foster researches on stock markets.

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