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RESEARCH ARTICLE

Robustness of critical points in a complex adaptive system: Effects of hedge behavior

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In our recent papers, we have identified a class of phase transitions in the market-directed resource-allocation game, and found that there exists a critical point at which the phase transitions occur. The critical point is given by a certain resource ratio. Here, by performing computer simulations and theoretical analysis, we report that the critical point is robust against various kinds of human hedge behavior where the numbers of herds and contrarians can be varied widely. This means that the critical point can be independent of the total number of participants composed of normal agents, herds and contrarians, under some conditions. This finding means that the critical points we identified in this complex adaptive system (with adaptive agents) may also be an intensive quantity, similar to those revealed in traditional physical systems (with non-adaptive units).

Keywords complex adaptive system, phase transition, resource allocation, hedge behavior, agent-based simulation

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1 Introduction

The most fundamental difference between traditional physical systems and complex adaptive systems (CASs) lies in their basic units. CASs consist of adaptive agents, while traditional physical systems consist of non-adaptive units. Thus the dynamic mechanics of CASs do not simply follow the rules as traditional physical systems [1–7]. Insights into the dynamics of a CAS are often gained by focusing on fluctuations which are a characteristic of all kinds of economic systems. Here we adopt a model "market-directed resource allocation game (MDRAG)" to observe emergent properties of CASs [8] including resource allocation systems [1–3, 9–11]. In our recent papers [1–3], we have identified a class of phase transitions in the MDRAG, and found that there exists a critical point at which the phase transitions occur. The critical point is given by a certain resource ratio at which the system reaches equilibrium which is the most stable state where the resources are the most effectively allocated and the total utilities of the system are maximal [1]. By previous researches, we have known that herd behavior or contrarian behavior always change the location of the critical point. That is, the minimal fluctuation of systems would not appear at the original resource ratio in the presence of either herd or contrarian behavior.

Hedge behavior is also a kind of self-organization in CASs. Here we introduce hedge behavior into the MDRAG to analyze the effects of hedge behavior on the whole system's fluctuations. By performing computer simulations of agent-based modeling [1, 12, 13] and theoretical analysis, we report that the critical point is robust against various kinds of human hedge behavior where the numbers of herds and contrarians can be varied widely. The critical point also helps to reveal the robustness of the most stable state (corresponding to the minimal fluctuation) of the CAS when hedge behavior exists or disappears. This finding means that the critical points we identified in this CAS with adaptive agents may also be an intensive quantity, similar to those revealed in traditional physical systems with non-adaptive units.

2 Agent-based simulations

In order to observe the influence of system's fluctuations caused by hedge behavior, we structured an agent-based model and carried out a series of computer simulations based on the MDRAG which is an extended version of the minority game [10]. This MDRAG can be described as follows. There will be two rooms: Room 1 and Room 2, each of which owns its resource marked as M_1 and M_2. Here, $M_1 \geqslant M_2$. Some agents see the two rooms, but do not know the amount and ratio of resources in two rooms. Everyone needs independently to choose to enter one of the two rooms. After all agents finished their choices, the resources in Room 1 are equally divided by those who entered Room 1, while the resources in Room 2 are equally divided by the agents who entered Room 2. If the agents in Room 1 get more per capita resource than the agents in Room 2, we regard them (choosing Room 1) as winners, and vice versa. In this model, we label the number of agents in Room 1 and Room 2 N_1 and N_2, respectively. If $M_1/N_1 > M_2/N_2$, then agents in Room 1 win, and vice versa.

In the MDRAG, the system will reach a most stable state [1] at the critical point. Under the assumption that changing the behaviors of agents might change the system equilibrium, if we could observe how the introduction of different behaviors would change the resource allocation, the model will be optimal. Based on the previous researches [2, 3, 14, 15], we know that herd behavior and contrarian behavior in the MDRAG could respectively shift the critical point to opposite directions, accordingly affecting the fluctuation of the system. Hence, we introduced herd behavior and contrarian behavior as a kind of hedge behavior.

In our agent-based model, there are N_n normal agents, N_h imitators and N_c contrarians. We define $\beta_1 = N_h/N_n$ and $\beta_2 = N_c/N_n$. The total number of all agents is $N = N_n + N_h + N_c = N_1 + N_2$. Each normal agent decides to enter one of the two rooms based on its strategy table. At the start, each normal agent has S strategy tables. Each of strategies determined by a preference index L and has P potential situations [1]. L characterizes the heterogeneity of preferences of normal agents, where $L \in [0, P]$. At the end of a round, each normal agent will score the S strategy tables in its hand. Then, the best strategy table will be used for the next round. When the computer gives the current situation, P_i ($P_i \in [1, P]$), at every round, each normal agent enters the room according to its best strategy table. The table of a strategy's left column represents P potential situations and the right column is filled with 1 and 0, which represent entering Room 1 and Room 2, respectively. The probability of 1 or 0 is determined by a certain preference index of L. The rule of constructing the right column of the table is that for a probability of L/P the right column is filled with 1 and for probability of $(P-L)/P$ the right column is filled with 0.

Meanwhile, the system may secretly add some contrarians and imitators generated by the computer in certain rounds. Imitators and contrarians have no strategy tables, whose decisions at a certain round are based on the choices of normal agents. In every round, every imitator or contrarian will randomly choose several normal agents as its group from all normal agents. The logic is as follows: An imitator will choose to enter the room where a majority of the normal agents in its group choose to enter. If most of the normal agents in its group choose to enter Room 1, then it will choose to enter Room 1, and vice versa. Contrarians choose to enter the room where a minority of the normal agents in its group chooses to enter. If most of the normal agents in its group choose to enter Room 1, then it will choose to enter Room 2, and vice versa. After one round of the option, we remark the total number of agents in Room 1 and Room 2 including normal agents, contrarian and imitators, labeled as N_1 and N_2, respectively. If $M_1/N_1 > M_2/N_2$, then agents in Room 1 win, and vice versa.

Based on the mentioned model, computer simulations are carried out with 100 normal agents for simulation parameters, S=11 and P=121. In the process we changed the parameters including the ratio of resource distribution between the two rooms and the ratio of contrarians and imitators. 400 time steps of simulation will be lasted under each parameter set. In order to scrutinize the effect of hedge behavior on the whole resource allocation system, we calculated the fluctuation of the systems. The fluctuation of the systems can be defined as $\frac{\sigma^2}{N} = \frac{1}{2N}\sum_{i=1}^{2}\langle(N_i - \tilde{N}_i)^2\rangle$, the smaller of the value is, the closer the system approaches to the equilibrium, because the optimal resource population is given by $\tilde{N}_i = M_i N/(M_1+M_2)$. The previous studies indicated that the resource allocation system exhibits a phase transition as M_1/M_2 varies. In other words, at the critical point, $(M_1/M_2)_t$, $\frac{\sigma^2}{N}$ reaches the lowest value which means the system becomes the most stable. See Fig. 1. The systems reach the most stable state at M_1/M_2=11 without imitators and contrarians. At this moment the critical point is $(M_1/M_2)_t$=11. But the fluctuation of the system changes when herd behavior is introduced. Likewise, contrarian behavior can also affect the system' fluctuations. However, if we introduce herd and contrarian behaviors simultaneously, we found the proper hedge proportion that makes the critical point unchanged by adjusting simultaneously the strength of the two different behaviors.

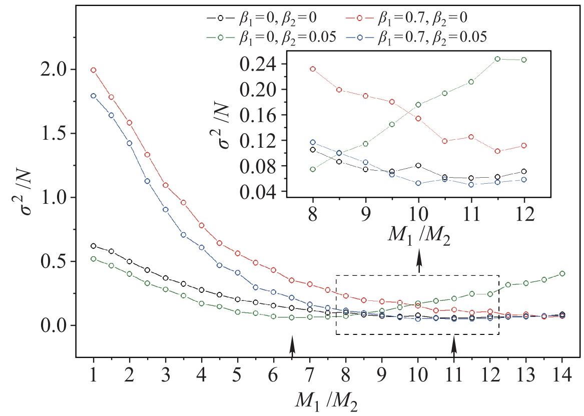

Fig. 1 Simulation results for fluctuation σ^2/N in different M_1/M_2 from 1 to 14 with normal agents (*black*), herd behavior (*red*), contrarian behavior (*green*), and hedge behavior (*blue*). The circles in dashed frame are magnified above the red arrow because the overlapped circles are hard to distinguish. Three black arrows on the horizontal axis indicate the four critical points of the most stable state of the four curves. The arrow at M_1/M_2=6.5 indicates the most stable state of green curve while the arrow at M_1/M_2=13.5 indicates the most stable state of red curve and the arrow at M_1/M_2=11 indicates the most stable state of black and blue curves. Note the two critical points of the most stable state of black and blue curves are well overlapped, and thus denoted by one arrow only, which clearly shows that suitable hedge behavior could let the system (with hedge behavior) have the same most stable state as the original system (without hedge behavior). For each parameter set, simulations are run for 100 times, each over 400 time steps (the last 200 time steps for statistics).

As we all know, the adaptive agents are the most remarkable feature of CASs. In order to analyze the robustness of critical points under hedge behavior, we need to scrutinize the microscopic behaviors of the system, i.e., the performance of each agent. Here, we observe their performance by normal agents' preference. We define the preference of each normal agent as the average rate of entering Room 1 or Room 2 in last 200 statistical time steps of simulations. We select three resource ratios to observe: M_1/M_2=1 that represents a uniform (or unbiased) distribution, M_1/M_2=3 that stands for a small bias, and M_1/M_2=14 that indicates an extreme bias. Figures 2 and 3 show the statistical results of simulations under different resource ratios and numbers of imitators and contrarians. We note that the average preference of all normal agents varied with M_1/M_2, which can be clearly seen in Fig. 2. This showed the global environmental adaptability of normal agents. It is noted that in the case with hedge behavior, the proportions of normal agents' preferences to two rooms are mainly the same. It indicates that the adaptability of normal agents results in consistent microscopic performances of systems in both cases, which further brings the robustness of critical points under the existence of hedge behavior. In Fig. 3 we can see that the preference of every normal agent is different no matter in which environment of resources or behaviors. In this system, normal agents show heterogeneity of preferences, which has a remarkable influence on achieving the balance of CASs. It implies that even if individuals are different, they can still present consistency as a whole.

From Fig. 1, we infer that for any strength of one behavior, a proper hedge proportion exists to make critical

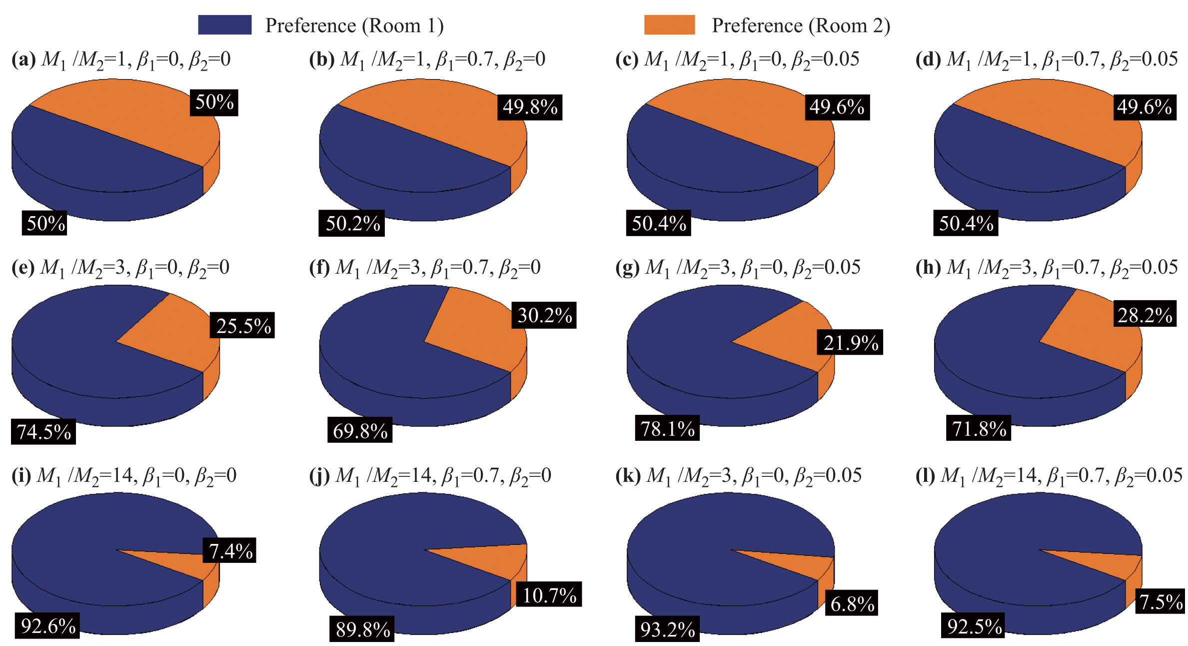

Fig. 2 The average preference of all normal agents for the 12 parameter sets: M_1/M_2=1 **(a–d)**, 3 **(e–h)**, and 14 **(i–l)** with β_1=0 and β_2=0 **(a, e, i)**, β_1=0.7 and β_2=0 **(b, f, j)**, β_1=0 and β_2=0.05 **(c, g, k)**, and β_1=0.7 and β_2=0.05 **(d, h, l)**. For each parameter set, simulations are run for 400 time steps (the first 200 time steps for equilibration and the last 200 rounds for statistics).

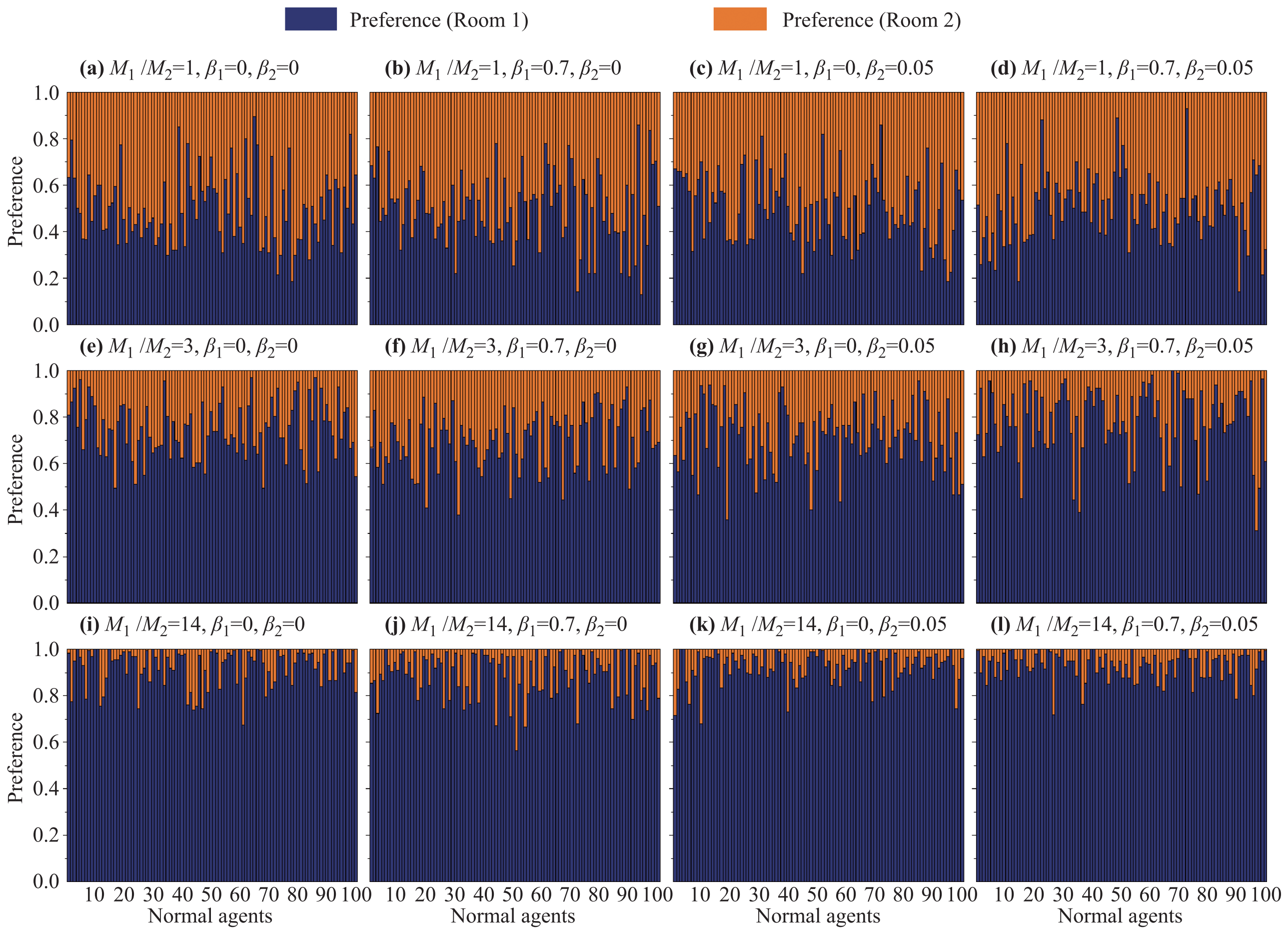

Fig. 3 The preference of each normal agent for the 12 parameter sets: M_1/M_2=1 (**a–d**), 3 (**e–h**), and 14 (**i–l**) with β_1=0 and β_2=0 (**a**, **e**, **i**), β_1=0.7 and β_2=0 (**b**, **f**, **j**), β_1=0 and β_2=0.05 (**c**, **g**, **k**), and β_1=0.7 and β_2=0.05 (**d**, **h**, **l**). For each parameter set, simulations are run for 400 time steps (the first 200 time steps for equilibration and the last 200 rounds for statistics).

points of the system robust. Also, by adjusting the hedge proportion, the resource allocation system could reach the most stable state in any satisfied resource ratio. Thus, it is very helpful for optimizing resources allocation under any resource distributions. Therefore we carried out more comprehensive simulations. The simulation results are shown in Fig. 4.

In Fig. 4 each color represents the most stable equilibrium state (i.e., with the critical point, $(M_1/M_2)_t$, that corresponds to the minimum value of $\frac{\sigma^2}{N}$) in a certain combination of β_1 and β_2. We can see that, for the case without imitators and contrarians, system can reach equilibrium at the critical point, $(M_1/M_2)_t$=11. Herd behavior makes the critical point shift to a larger M_1/M_2, while contrarian behavior makes the critical point shift to a smaller M_1/M_2, which means that either herd behavior or contrarian behavior always changes the system's fluctuation. However, if we adjust the two behaviors simultaneously, then this kind of hedge behavior can make critical points robust no matter how many agents exist. Besides, the system can achieve the optimal state at any resource ratio by adjusting the values of β_1 and β_2

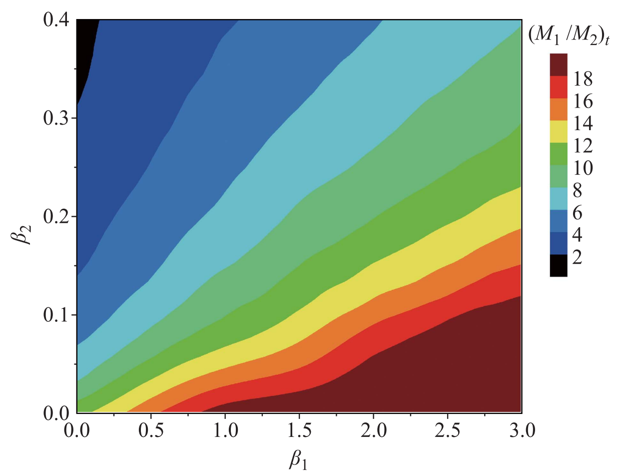

Fig. 4 Critical point, $(M_1/M_2)_t$, as functions of β_1 and β_2 as a result of the computer simulations with 100 normal agents. For each parameter set, simulations are run for 100 times, each over 400 time steps (the last 200 time steps for statistics).

appropriately.

Such adjustment of hedge behavior can surely give references to CASs. We found that the critical point of the system is robust and independent of the total number of agents composed of normal agents, imitators and contrarians, under some conditions. Also the resource allocation system could be controlled to reach the most stable state (corresponding to a certain critical point) in any resource bias through adjusting the proportion of two kinds of agents, β_1 and β_2.

3 Theoretical analysis

The further understanding of critical point $(M_1/M_2)_t$ versus β_1 and β_2 is necessary for understanding the microscopic mechanism in the system. For this purpose, we conduct a theoretical analysis.

For the fixed values of S and P, the system could reach the most stable state only at the critical point (i.e. a specifically ratio of resource ratio, $(M_1/M_2)_t$) under the situation without contrarians and imitators. In order for the resource allocation systems to reach the optimal states at any ratio of resource distribution, we have to adjust the values of β_1 and β_2, thus changing critical points, $(M_1/M_2)_t$, accordingly.

3.1 The properties of critical points

(a) MDRAG: All normal agents use the strategy $(L_i)_{\max}$ with the largest preference in their hand. Meanwhile, the ratio of numbers of agents in two rooms is equal to the ratio of resource distribution. Then we have

$$N_1 = \sum x_i$$

where the choice of agent i is denoted as x_i=1 (Room 1) or 0 (Room 2). Next, we obtain

$$\frac{\langle N_1\rangle}{N_n} = \frac{\sum\langle x_i\rangle}{N_n} = \frac{\sum_i^{N_n}(L_i)_{\max}}{PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_t \tag{1}$$

where $\langle\cdots\rangle$ denotes the averaged value of $\cdots$.

(b) MDRAG + Imitators + Contrarians: All normal agents use $(L_i)_{\max}$. An imitator follows the majority of its group to choose x_h, while every contrarian follows the minority in his group to choose x_c. Thus we obtain

$$\frac{\langle N_1\rangle}{N} = \frac{\sum_i^{N_n}(L_i)_{\max} + P\sum_h^{N_h}\langle x_h\rangle + P\sum_c^{N_c}\langle x_c\rangle}{(1+\beta_1+\beta_2)PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_{t'} \tag{2}$$

where $\beta_1 = \frac{N_h}{N_n}$ and $\beta_2 = \frac{N_c}{N_n}$.

3.2 Solve $\sum_i^{N_n}(L_i)_{\max}$, $\sum_h^{N_h}\langle x_h\rangle$ and $\sum_c^{N_c}\langle x_c\rangle$

(a) For normal agent i with S strategies, the probability of L_i taking 0 to P is $\frac{1}{P+1}$. Then, the probability of $(L_i)_{\max}$ being a certain value of L is

$$p(L) = \left(\frac{L+1}{P+1}\right)^S - \left(\frac{L}{P+1}\right)^S$$

If N_n is large enough, there is

$$\sum_i^{N_n}(L_i)_{\max} = \sum_{L=0}^{P} N_n p(L) L = PN_n\left[1-\frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S\right] \tag{3}$$

Plugging Eq. (3) into Eq. (1), under the situation without contrarians and imitators, there is

$$\frac{\langle N_1\rangle}{N_n} = 1-\frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S = \left(\frac{M_1}{M_1+M_2}\right)_t \equiv m_n \tag{4}$$

According to Eq. (4), it is known that if P and S are fixed, we can solve the critical point denoted by $(M_1/M_2)_t$ without contrarians and imitators.

(b) We know that normal agents still use their strategy with $(L_i)_{\max}$ at critical points after introducing contrarians and imitators into the system. Therefore, for normal agents, there is

$$\frac{\langle N_{n1}\rangle}{N_n} = 1-\frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S = \left(\frac{M_1}{M_1+M_2}\right)_t \equiv m_n$$

Approximately, when imitator h and contrarian c respectively choose k normal agents, the probability that they got the normal agents who came into Room 1 is denoted as $\frac{\langle N_{n1}\rangle}{N_n} = \left(\frac{M_1}{M_1+M_2}\right)_t \equiv m_n$. Then, the probability that x_c equals to 1 or 0 is shown in the following table:

x_c	Probability
1	$\sum_{q=0}^{y} C_k^q\left[\left(\frac{M_1}{M_1+M_2}\right)_t\right]^q\left[\left(\frac{M_2}{M_1+M_2}\right)_t\right]^{k-q} \equiv m_c$
0	$1-m_c$

Here $y = \frac{k-1}{2}$, and k is odd. Then we have

$$\langle x_c\rangle = m_c = \sum_{q=0}^{y} C_k^q (m_n)^q (1-m_n)^{k-q} \tag{5}$$

So, $\langle x_h\rangle = 1-m_c$.

(c) Plugging Eq. (5) into Eq. (2) yields

$$\frac{\langle N_1\rangle}{N} = \frac{PN_n m_n + PN_h(1-m_c) + PN_c m_c}{(1+\beta_1+\beta_2)PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_{t'}$$

Then, $\frac{\langle N_1\rangle}{N} = \frac{m_n+\beta_1(1-m_c)+\beta_2 m_c}{1+\beta_1+\beta_2} = \left(\frac{M_1}{M_1+M_2}\right)_{t'}$.

By adjusting β_1 and β_2 appropriately, we could get a robust critical point of the system. Besides, the system could reach the most stable state under any resource ratio M_1/M_2. Figure 5 shows $(M_1/M_2)_t$ versus β_1 and β_2 as a result of the theoretical analysis with the same parameters as those adopted in the previous simulations, which echoes with the simulation results shown in Fig. 4.

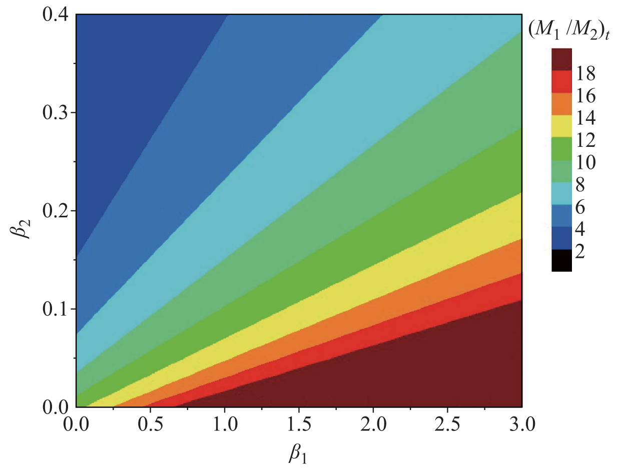

Fig. 5 Critical point, $(M_1/M_2)_t$, as functions of β_1 and β_2 as a result of theoretical analysis.

4 Conclusions

In summary, using agent-based simulations and theoretical analysis, we have investigated the role of hedge behavior in a resource-allocation system. The critical point identified herein helped to reveal the robustness of the most stable state of the system when hedge behavior exists or disappears. These results mean that the critical points we identified in this CAS (complex adaptive system) with adaptive agents may also be an intensive quantity, similar to those revealed in traditional physical systems with non-adaptive units.

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