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2016 EPL 116 18008

(http://iopscience.iop.org/0295-5075/116/1/18008)

A Letters Journal Exploring the Frontiers of Physics

October 2016

EPL, **116** (2016) 18008 www.epljournal.org
doi: 10.1209/0295-5075/116/18008

Complex network approach to classifying classical piano compositions

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received 5 August 2016; accepted in final form 29 October 2016
published online 23 November 2016

PACS 89.75.Fb – Structures and organization in complex systems
PACS 89.75.Hc – Networks and genealogical trees
PACS 89.75.Kd – Patterns

Abstract – Complex network has been regarded as a useful tool handling systems with vague interactions. Hence, numerous applications have arised. In this paper we construct complex networks for 770 classical piano compositions of Mozart, Beethoven and Chopin based on musical note pitches and lengths. We find prominent distinctions among network edges of different composers. Some stylized facts can be explained by such parameters of network structures and topologies. Further, we propose two classification methods for music styles and genres according to the discovered distinctions. These methods are easy to implement and the results are sound. This work suggests that complex network could be a decent way to analyze the characteristics of musical notes, since it could provide a deep view into understanding of the relationships among notes in musical compositions and evidence for classification of different composers, styles and genres of music.

Introduction. – Complex network has been verified as an effective tool analyzing systems with a large number of interacting units. Its advantage is that the topological structure of a complex network can be used to describe system characteristics. The topology reveals connections and interactions among system units, which cannot be fully expressed by traditional statistics. Due to this feature, various kinds of complex systems have been studied by constructing corresponding networks, obtaining results regarding universalities (for example, power-law distribution of node degrees in scale-free networks [1], short edge length and high clustering coefficient in small-world networks [2,3], etc.) as well as diversities among such networks. Recently, some work has reported the applications of complex network on researches concerning literal words [4–7], epidemic propogation [8–11], data mining [12], and music [13].

As an oldest form of art, music expresses particular emotions of the composer or a certain group of people. Some of the previous researches revealed statistical properties within and among pieces of music [14–19]. Voss and Clarke [20] analyzed some music compositions and found a $1/f$ spectral density at frequencies down to the inverse of the length of the piece of music as a property of pleasing music. Hsü and Hsü [21] numerically analyzed a few compositions and found that the musical effects can be expressed as deviations from fractal geometry [22,23]. Dabby [24] proposed a chaotic mapping for generating musical variations, which produces changes between note pitches in pieces of music based on the sensitivity of chaotic trajectories to initial conditions. However, all these works failed to reveal the sequencial properties of specific notes, which directly determine the melody and rythm of a piece. Some attempts are also made to classify musical genres by audio signals and rhythm spectra [25,26]. However, most of them focused on the universal properties of different pieces of music. The differences among pieces, composers, and genres of music remain poorly explored. Furthermore, statistical properties of musical notes (pitches, durations, etc.) are not enough to represent the melodies' features. Moreover, some works

Table 1: Basic information of the compositions and composers analyzed.

Composer	No. of compositions	Total No. of notes
Mozart	503	336430
Beethoven	179	534854
Chopin	88	70490

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Table 2: The frequencies (Hz) of all 88 piano keys. The top row shows the notes' letter-names and the leftmost column shows their octaves.

	C	$^{\#}C$	D	$^{\#}D$	E	F	$^{\#}F$	G	$^{\#}G$	A	$^{\#}A$	B
0										27.50	29.14	30.87
1	32.70	34.65	36.71	38.89	41.20	43.65	46.25	49.00	51.91	55.00	58.27	61.74
2	65.41	69.30	73.42	77.78	82.41	87.31	92.50	98.00	103.8	110.0	116.5	123.5
3	130.8	138.6	146.8	155.6	164.8	174.6	185.0	196.0	207.7	220.0	233.1	246.9
4	261.6	277.2	293.7	311.1	329.6	349.2	370.0	392.0	415.3	440.0	466.2	493.9
5	523.3	554.4	587.3	622.3	659.3	698.5	740.0	784.0	830.6	880.0	932.3	987.8
6	1047	1109	1175	1245	1319	1397	1480	1568	1661	1760	1865	1976
7	2093	2217	2349	2489	2637	2794	2960	3135	3322	3520	3729	3951
8	4186											

Fig. 1: (Color online) The five-line staff for the beginning melody of Mozart's famous "Turkish March" (the piano sonata No. 11 in A major, K. 331).

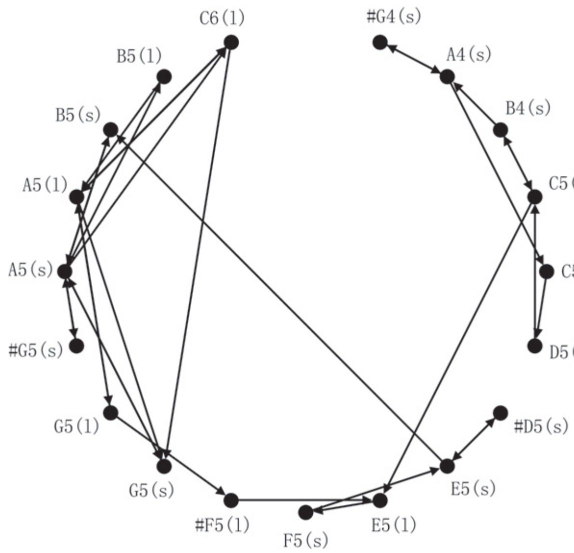

Fig. 2: The directed network constructed from the melody shown in fig. 1. Notes that do not appear are not displayed. The l's in the parenthesis stand for long-duration notes and the s's stand for short-duration notes.

of classification using networks concepts have been carried out recently, such as the identification of extreme events and transitions in spatio-temporal systems [27], characterization of experimental two-phase flow patterns [28], and estimation of coupling directions in cardiorespiratory systems [29]. These induce us to apply complex network to the analyses of music and to develop some new classification methods.

In this paper, we construct and analyze networks of classical piano compositions of Mozart, Beethoven and Chopin. The simple statistical and scaling properties of the data are presented in our former research [30]. We find that, despite some common properties and distributions of general networks, distinctions among pieces and composers are prominent. Based on these distinctions, we propose two new classification methods, which could provide deep insight into the correlation between the relationships among notes [31] and styles of music pieces.

Methods. – We know that each composition is composed of continuous musical notes with various lengths of each note. For convenience of mathematical disposition, any particular composition can be transferred to a time series of notes. To be specific, we can draw the time series on an xy coordinate with the x-axis represengting time and the y-axis representing pitches of notes. The fluctuations of notes constitute different melodies, and further, different compositions. We extract 770 pieces of piano music of Mozart, Beethoven, and Chopin from kern humdrum music data base [32] as files of MIDI format, which contain whole digital information of the compositions. Only the main themes (the right-hand part of piano compositions) are analyzed for simplicity. The overall information of our data is shown in table 1.

We choose to analyze piano music because the possible notes within any composition are fixed, since there are always 88 keys on a piano [33], which are labeled by their letter-names and octaves (an octave is a series of eight notes in a musical scale). We neglect pauses in our analyses because their mechanism of emergence is somewhat different from that of notes. So it is convenient to correspond note pitches (in frequencies) in the MIDI files to

Table 3: Statistical properties of the pitches for Mozart, Beethoven and Chopin.

	Mean of pitches (Hz)	Pitch fluctuations			
		Mean (Hz)	Std. Dev. (Hz)	Kurtosis	Skewness
Mozart	435.45	−0.240	118.99	11.11	0.296
Beethoven	416.33	0.784	139.67	16.45	−0.322
Chopin	414.04	0.584	159.83	17.69	0.177

Table 4: Parameters of the note networks for Mozart, Beethoven and Chopin.

	Number of connected nodes	Number of edges	Clustering coefficient
Mozart	134	5791	0.8743
Beethoven	145	9052	0.8989
Chopin	153	5402	0.7423

specific keys with their letter-names and octaves. Table 2 shows the correspondence between the labels and pitch frequencies of all 88 keys on a piano.

Two main descriptions of a musical note are its pitch and duration. Here, we simply split all notes of a specific composer into long-duration notes and short-duration notes. For this purpose, we find out the median of duration among these notes. The notes with shorter durations than this median are classified as "short notes" of his/her, and the notes with longer durations than the median are classified as "long notes", *i.e.*, half of a composer's notes are short, and half are long. Due to the fact that numerous lengths of actual durations exist, this simplification is reasonable and helps to construct networks that are not too sparse.

Following the aforementioned classification of notes, any one of the notes we analyze can be distributed into one of the $88 * 2 = 176$ categories. These 176 classification items are exactly the nodes of our complex networks. Thus, for a melody, every time the jump from note i to note j (whether the duration is long or short) occurs, the weight of the correspongding directed edge is increased by 1. One thing to note here is that a musical note may repeat itself in melodies, so edges could originate from and point toward the same node. By doing so, any compositon can be chronologically expressed by end-to-end edges on unified network nodes. The superposition of all such networks of a composer's compositions constitutes the musical network for his/her own. We illustrate our network construction of the beginning melody of Mozart's "Turkish march" as an example in fig. 1 and fig. 2.

Results. – We denote the pitch of time t as $f(t)$ ($t = 1, 2, 3, \dots, N$), with N being the length in notes of the composition. Thus, the pitch fluctuation $Z_f(t)$, which describes the pitch change between adjacent notes, can be defined as

$$Z_f(t) = f(t+1) - f(t), \qquad t = 1, 2, 3, \dots, N-1. \quad (1)$$

First, let us take a glimpse at the statistical properties for the pitches of the composers obtained in our former research [30], see table 3. Here, pitch fluctuations are calculated by subtracting the frequency of the former note from that of the latter note. The results indicate that the differences among the three composers are subtle. So it is a tough task to distinguish them from each other using only statistics of pitches. That is the reason why we seek complex network for help.

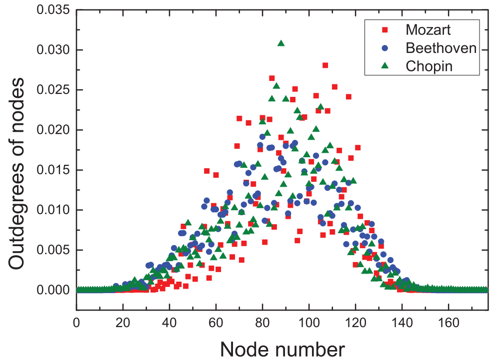

Fig. 3: (Color online) Outdegrees of 176 network nodes for Mozart (red squares), Beethoven (blue dots), and Chopin (green triangles).

Before displaying the characteristics of the musical networks, we number the 176 nodes for clarity: the node representing the note with the n-th lowest pitch is labeled "$2n-1$" if its duration is short, or "$2n$" if its duration is long. Under such setup, we calculate the number of connected nodes, number of edges, clustering coefficient, and node degree [34] and edge weight distributions of the networks of the three composers, respectively. The clustering coefficients are calculated as in [35], with the generalized clustering coefficients obtained by using the geometric mean method for defining triplet values. The parameters of the networks constructed are shown in table 4. Since indegree and outdegree of a node have minor distinctions, here we only present the outdegree of each node so that every edge is counted once. When calculating the edge weights, we standardize the data by dividing the original edge weights by the total number of note fluctuations of the relevant composer. Thus, the outdegree of each node is the sum of all standardized weights of edges that originate from the node. The outdegree distributions of nodes for the three networks are shown in fig. 3, and the intensities of edge weights (the weights of particular edges divided by total number of adjacent note jumps of the composer) for the three composers (Mozart, Beethoven and Chopin) are shown as heat maps in fig. 4.

In table 4, we can see that the highest number of notes (153 out of total 176) appear in Chopin's compositions, while it is 134 for Mozart and 145 for Beethoven. However, Chopin used fewer kinds of note jumps (5402) than the others, and the clustering coefficients show the same pattern. Meanwhile, Beethoven's compositions are the most jumping and diversified, considering much more kinds of note jumps than Mozart's and Chopin's.

Since the outdegrees of nodes actually equal to the occurring probabilities of the musical notes in each of the three composers' compositions, fig. 3 shows this usage habits of the composers. We note that notes with the highest frequencies are mainly located in the central area of the piano, and are mostly used by Mozart and Chopin. Meanwhile, quite a number of other notes are least used by Mozart. This indicates that Mozart had a "clearer cut stand on what to love and what to hate" compared to Beethoven and Chopin. By contrast, note usages of Beethoven are mostly moderate. However, this phenomenon can also be caused by the restrictions of the instrument at Mozart's time.

Because of the fact that the nodes of different networks are exactly the same, we draw heat maps of the three networks to clearly demonstrate the difference among edge weights. Here, some of the aforementioned statistical properties of notes can be verified. Mozart's pitch fluctuations have the smallest standard deviation and kurtosis, which corresponds to a constrained, narrow belt along the diagonal of the heat map without an unambiguous central line. On the contrary, Chopin's network exhibits a relatively dispersive pattern and a distinct central line.

Besides the difference among various composers' genres and rhythms, the reason for distinct outdegree distributions of nodes can be traced back to the evolution of piano over the years. Some hardware factors of the instrument can come into play, such as the touch of keys, loudness, sustain time of the notes, repetition of notes [36], which also determine the structures of the compositions. Therefore, the network method cannot only distinguish a particular composer from the others, but also be used to analyze distinctions of compositions caused by different ages and composing instruments, etc.

Through the analyses of the networks above, despite some universalities discovered by early researches, there are obvious distinctions among the three networks. This induces us to propose new classification methods for any pieces of music based on the connections between notes, since it is more precise to describe melodies by note jumps instead of mere notes. We elucidate the methods for classification and corresponding results in the next section.

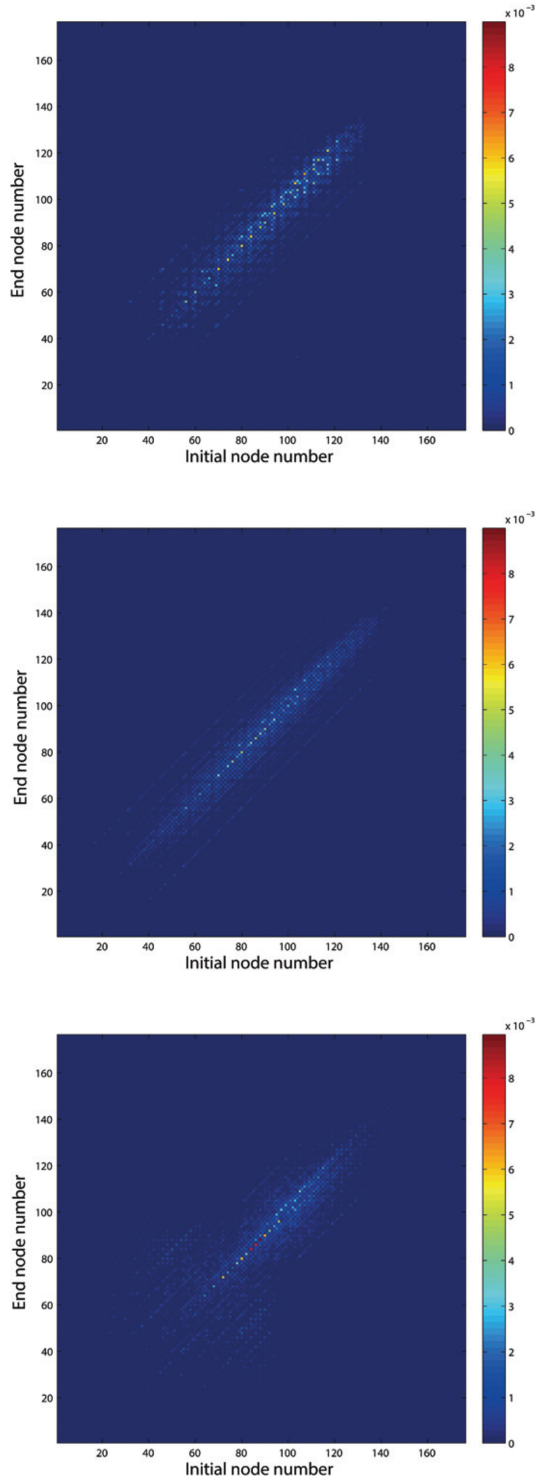

Fig. 4: (Color online) Edge weights of the network constructed for Mozart, Beethoven, and Chopin as heat maps. The horizontal axis represents the initial node of the edge, and the vertical axis represents the end node of the edge. The colors in the maps indicate the occuring probability of the corresponding edges (*i.e.*, adjacent note jumps).

Table 5: Classification results of all the compositions analyzed by using the complete distance method.

		Percentage classified as the style of		
		Mozart	Beethoven	Chopin
The compositions of	Mozart	82.90%	14.51%	2.58%
	Beethoven	15.64%	73.18%	11.17%
	Chopin	5.68%	20.45%	73.86%

Table 6: Results of the presence-or-absence classificatoin method.

	Number of unique note jumps	Percentage classified as his own	Method 1 + Method 2
Mozart	323	50.30%	91.25%
Beethoven	2645	100%	100%
Chopin	580	65.91%	89.77%

Classification. – The previous sections have introduced the construction method of the musical networks and distinctions are found among different composers. Here, we try to develop two composition classification methods based on these distinguishing networks and composer styles.

The first classification method is called complete distance method, and is operated as follows: for any one of the standardized network, there are totally $176 * 176 = 30976$ possible edges with arbitrary weights, while the nodes in different networks are the same. Thus, for a particular composer, his/her style can be expressed by a point in a 30976-dimension space, with its coordinates being the weights of all the edges. Similarly, any composition can be dealt with accordingly and can be corresponded to a point in the same space. We then calculate the "distance" between the composition (point C) and a composer's style (for instance, Mozart (point C^M)) by

$$D_M = \sqrt{\sum_{n=1}^{30976} (C_n - C_n^M)^2}. \qquad (2)$$

Here C_n denotes the projection of point C on the n-th axis. Finally, we compare the three "distances" from the composition to the three composers (*i.e.*, D_M, D_B and D_C), and the composition is classified as someone's style whose position is closest to this composition. The illustration and summary of the results of this method is shown in table 5 and fig. 5.

Figure 5 shows the classification results for all 88 Chopin's compositions. The blue column of a specific composition denotes the difference between its "distance" from Mozart and Chopin, and the red column that from Beethoven and Chopin. It is reasonable that if both of these two values are greater than zero, *i.e.*, $D_M - D_C > 0$ and $D_B - D_C > 0$, then Chopin's style is the nearest to this composition, and it should be classified as "Chopin style". Otherwise, if D_M or D_B is the smallest, the composition should be classified as "Mozart style" or "Beethoven style". The summarized results of all the compositions analyzed are presented in table 5. Over 73% of the compositions are classified "correctly". It is worth noting that classifying a Chopin's composition as Mozart style is not "incorrect", it may truly acts as the style of Mozart. Meanwhile, compositions of Chopin and Mozart are rarely mixed, since the distance between Mozart and Chopin is the farthest (0.0213 between Beethoven and Chopin, 0.0172 between Mozart and Beethoven, and 0.0290 between Mozart and Chopin).

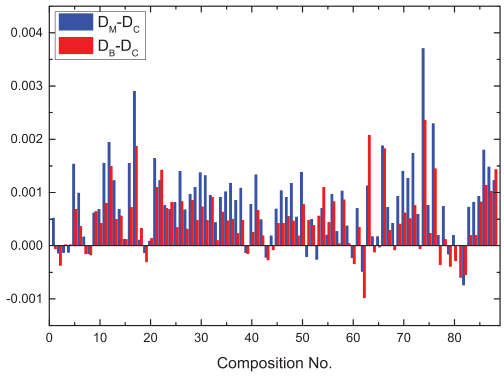

Fig. 5: (Color online) The differences between the distances from Mozart and Chopin (blue columns) and from Beethoven and Chopin (red columns) for the 88 Chopin's compositions.

Another classification method is the presence-or-absence method. For any particular composer, there are specific and unique note jumps in his/her compositions acting as signatures, that do not appear in other's compositions. We are able to classify pieces of music by these specific characteristics. We show the properties and results of this method in table 6. We see that all of Beethoven's compositions have unique note jumps of his. And if we combine the two classification methods, approximately at least 90% of the compositions can be classified "correctly".

Conclusions. – As inspired by the existing extensive research on complex networks in the field of statistical physics, in this work we have proposed, for the first time, a complex network approach to studying classical piano music on the basis of note sequences and lengths in compositions. We have analyzed three networks constructed based on compositions of three composers (Mozart, Beethoven and Chopin). Some distinctions, such as different outdegree distributions and edge weights, among the networks have been found, which have inspired us to develop classification methods for music styles and genres. This work provides a deeper view into the relationships among notes in compositions, and a new, simple and comprehensive way to analyze and classify all kinds of musical styles and genres due to its generality and advantages.

However, our work still stays as a simplified version, and future researches can be focused on: 1) extending the network edges to three or even more consecutive note jumps; 2) analyzing more pieces of music by various composers with various styles and genres; 3) sorting the lengths of the musical notes by more precise categories.

* * *

We acknowledge the financial support by the Science and Technology Commission of Shanghai Municipality under Grant No. 16ZR1445100.

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