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Y. Gao\textsuperscript{1,2} and J. P. Huang\textsuperscript{2(a)}

\textsuperscript{1}Department of Applied Physics, School of Science, Shanghai Second Polytechnic University - Shanghai 201209, China
\textsuperscript{2}Department of Physics, State Key Laboratory of Surface Physics, and Key Laboratory of Micro and Nano Photonic Structures (Ministry of Education), Fudan University - Shanghai 200433, China

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Abstract – All the thermal cloaks reported in the literature can be used to thermally hide an object inside the cloak. However, a common limitation of this kind of thermal cloaks is that the cloaked object cannot feel the external heat flow since it is located inside the cloak; thus we call these cloaks “conventional thermal cloaks”. Here we manage to overcome this limitation by exploiting a class of unconventional thermal cloaks that enable the cloaked object to feel the external heat flow. Our finite-element simulations in two dimensions show the desired cloaking effect. The underlying mechanism originates from the complementary effect of thermal metamaterials with negative thermal conductivities. This work suggests a different method to design thermal devices where heat conduction can be controlled at will.

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Introduction. – The past decades have witnessed the fact that it is more flexible for people to manipulate light waves, acoustic waves and even seismic waves than heat conduction. This may be because such kinds of waves satisfy the wave equations while heat conduction satisfies the diffusion equation. Therefore, how to manipulate heat conduction at will is up to now a challenge. If successful, many potential applications could become true, for example, in solar collectors and chip cooling.

With an attempt to freely manipulate heat conduction, in 2008 we \cite{1} started from the form invariant of the thermal conduction equation in distorted coordinates, and proposed a kind of thermal cloaks for steady-state heat flow (namely, the temperature, $T$, is not a function of time) by using anisotropic and inhomogeneous (graded) materials. The thermal cloak can hide an object inside the cloak from the detection by measuring the distribution of external heat flux. This behavior is because the cloak and object do not affect the distribution of temperature outside the cloak as if they do not exist. Narayana and Sato \cite{2} experimentally realized this cloaking effect by using forty alternating layers of two materials, one of which has a high thermal conductivity and the other has a low thermal conductivity. In addition, while some theoretical researchers proposed to design such thermal cloaks by using a simplified method based on homogeneous materials \cite{3}, other theoretical \cite{4} or experimental \cite{5,6} researchers also extended the thermal cloaks from steady-state heat flow to unsteady-state heat flow (i.e., $T$ is a function of time). However, all the reported thermal cloaks \cite{1–12} have a common limitation: the cloaked object cannot feel the external heat flow because it is located inside the cloak. In other words, the object hidden inside the cloak has to be “blind”. For clarity, we call these thermal cloaks \cite{1–12} “conventional thermal cloaks”. A similar limitation also appears, but is solved \cite{13} in optical/electromagnetic cloaks. Clearly, if the limitation is overcome in thermal cloaks, the cloaking effect would be more practicable, especially, in the eye of the cloaked object. This statement is because the object can then feel external heat flow but cannot be detected by measuring the distribution of temperature outside the region of the cloak and object. That is, the object itself is no longer “blind”. For comparison, we call such thermal cloaks “unconventional thermal cloaks”. In what follows, we shall propose a recipe for an unconventional thermal cloak, which can hide an object outside the cloak.

Transformation thermophysics. – Let us start by investigating the thermal conduction equation. Without loss of generality, we omit the source term and obtain the steady-state conduction equation for the steady-state heat
where $\kappa$ is a thermal conductivity. Because conduction equations are invariant in their form under the transformation from an original (regular) coordinate to a transformed (distorted) one [14], we achieve the thermal conductivity tensor, $\kappa'$, of the material in the transformed coordinate [1,7],

$$\kappa' = \frac{M\kappa_0 M^t}{\det(M)},$$

in terms of the thermal conductivity in the original coordinate, $\kappa_0$. Here $M$ is the Jacobian transformation matrix between the original and distorted coordinates, $M^t$ denotes the transposed matrix of $M$, and $\det(M)$ is the determinant of $M$. In general, eq. (2) enables us to obtain the thermal conductivity tensor of the material in the coordinates with various types of transformations.

Now we are allowed to design the unconventional thermal cloak of interest. In this work, for simplicity, we consider cylindrical cases with coordinate parameters $(r, \theta, z)$ in the original coordinate and $(r', \theta', z')$ in the transformed coordinate; the cloak is schematically shown in fig. 1. For our purpose, we attempt to fold a big annulus into a small one while keeping their common surface unchanged. In detail, as shown in fig. 1, the large circle with radius $r = r_c$ is linearly changed into a small one with $r = r_a$. Meanwhile, the complementary material is obtained by folding the large annulus (with radius $r$ satisfying $r_b < r < r_c$) into a small one $(r_a < r < r_b)$. As a result, the object, which is to be hidden, should be put in Region C (as indicated in fig. 1), while Region B of fig. 1 corresponds to the desired thermal cloak. Then it becomes necessary to determine the material parameters for the cloak occupying Region B (fig. 1). For this purpose, besides $\theta' = \theta$ and $z' = z$, we consider the following coordinate transformation:

$$r' = \frac{r_a}{r_c} r \quad (0 < r < r_c),$$

$$r' = -\frac{r_b - r_a}{r_c - r_b}(r - r_b) + r_b \quad (r_b < r < r_c).$$

Regarding this transformation, the Jacobian transformation matrix, $M$, is given by

$$M = \begin{pmatrix}
\frac{\partial r'}{\partial r} & \frac{\partial r'}{\partial \theta} & \frac{\partial r'}{\partial z} \\
\frac{\partial r}{\partial r} & \frac{\partial r}{\partial \theta} & \frac{\partial r}{\partial z} \\
0 & 0 & 1
\end{pmatrix}. \quad (5)
$$

So, we obtain the thermal conductivity tensor, $\kappa_1$, for Region A (as displayed in fig. 1),

$$\kappa_1 = \frac{M\kappa_0 M^t}{\det(M)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (\frac{\kappa_0}{r_a})^2 \end{pmatrix}. \quad (6)$$

Similarly, we obtain the thermal conductivity tensor, $\kappa_2$, for Region B (in fig. 1),

$$\kappa_2 = \frac{M\kappa_0 M^t}{\det(M)} = \begin{pmatrix} \kappa_{2,rr} & 0 & 0 \\ 0 & \kappa_{2,\theta\theta} & 0 \\ 0 & 0 & \kappa_{2,zz} \end{pmatrix}. \quad (7)$$

where $\kappa_m$ is the thermal conductivity of the material located in Region C (as shown in fig. 1), and the three diagonal components are respectively given by

$$\kappa_{2,rr} = \frac{r_b(r_c - r_a)\kappa_m}{r_b(r_c - r_a)} + \kappa_m,$$
$$\kappa_{2,\theta\theta} = \frac{r(r_b - r_a)\kappa_m}{r_b(r_c + r) - r_c r},$$
$$\kappa_{2,zz} = \frac{(r_b - r_a)[-r_a r_b + r_b(r_c + r) - r_c r] \kappa_m}{r(r_c - r_a)^2}.$$

**Simulation results.** We are now in a position to perform finite-element simulations in two dimensions by using the commercial software, COMSOL Multiphysics. Figure 1 is a schematic graph showing four regions (A, B, C, and D), which are divided by three radii: $r_a$, $r_b$, and $r_c$. The unconventional thermal cloak occupies Region B (fig. 1). For this purpose, besides $\theta' = \theta$ and $z' = z$, we consider the following coordinate transformation:

$$r' = \frac{r_a}{r_c} r \quad (0 < r < r_c),$$

$$r' = -\frac{r_b - r_a}{r_c - r_b}(r - r_b) + r_b \quad (r_b < r < r_c).$$

Regarding this transformation, the Jacobian transformation matrix, $M$, is given by

$$M = \begin{pmatrix}
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\frac{\partial r}{\partial r} & \frac{\partial r}{\partial \theta} & \frac{\partial r}{\partial z} \\
0 & 0 & 1
\end{pmatrix}. \quad (5)
$$

So, we obtain the thermal conductivity tensor, $\kappa_1$, for Region A (as displayed in fig. 1),

$$\kappa_1 = \frac{M\kappa_0 M^t}{\det(M)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (\frac{\kappa_0}{r_a})^2 \end{pmatrix}. \quad (6)$$

Similarly, we obtain the thermal conductivity tensor, $\kappa_2$, for Region B (in fig. 1),

$$\kappa_2 = \frac{M\kappa_0 M^t}{\det(M)} = \begin{pmatrix} \kappa_{2,rr} & 0 & 0 \\ 0 & \kappa_{2,\theta\theta} & 0 \\ 0 & 0 & \kappa_{2,zz} \end{pmatrix}. \quad (7)$$

where $\kappa_m$ is the thermal conductivity of the material located in Region C (as shown in fig. 1), and the three diagonal components are respectively given by

$$\kappa_{2,rr} = \frac{r_b(r_c - r_a)\kappa_m}{r_b(r_c - r_a)} + \kappa_m,$$
$$\kappa_{2,\theta\theta} = \frac{r(r_b - r_a)\kappa_m}{r_b(r_c + r) - r_c r},$$
$$\kappa_{2,zz} = \frac{(r_b - r_a)[-r_a r_b + r_b(r_c + r) - r_c r] \kappa_m}{r(r_c - r_a)^2}.$$

**Simulation results.** We are now in a position to perform finite-element simulations in two dimensions by using the commercial software, COMSOL Multiphysics. Figure 1 is a schematic graph showing four regions (A, B, C, and D), which are divided by three radii: $r_a = 0.5$ m, $r_b = 0.7$ m, and $r_c = 0.9$ m (parameters for simulations). Region B is located between $r_a$ and $r_c$, and it denotes the unconventional thermal cloak of our interest. The object, which is to be cloaked, is located in Region C between $r_b$ and $r_c$, a region outside Region B. According to eq. (6), both Region A (with radius $r < r_a$) and Region D (with $r > r_c$) are set to have the same isotropic and homogeneous material (background material) with thermal
Fig. 2: (Color online) Two-dimensional finite-element simulations of temperature distribution. The color surfaces represent temperature; the arrows denote the pathway of heat flux. The temperature is respectively set to be 400 K and 300 K at the left and right boundaries of the squared simulation area with the side length of 2.5 m. (a) Temperature distribution of the background that contains an isotropic and homogeneous material with thermal conductivity $\kappa_0 = 40 \text{W/(m} \cdot \text{K})$. For comparing with (b) and (c), we add a circle of radius $r = r_c$ to label the position only. (b) Temperature distribution for the case where an object of thermal conductivity $\kappa_m = 400 \text{W/(m} \cdot \text{K})$ is used to replace Region C (between $r_a$ and $r_c$ as indicated in fig. 1) of the background displayed in (a). (c) Same as (b), but the cloaking area (Region B) between $r_a$ and $r_b$ is filled with materials according to eq. (7).

Fig. 3: (Color online) The three diagonal components, $\kappa_{2,rr}$, $\kappa_{2,\theta\theta}$, and $\kappa_{2,zz}$, as a function of radius $r$ within Region B ($r_a < r < r_b$). The upper (red) line shows the overlapped values of $\kappa_{2,rr}$ and $\kappa_{2,zz}$. Parameters: $\kappa_m = 400 \text{W/(m} \cdot \text{K})$, $r_a = 0.5 \text{m}$, $r_b = 0.7 \text{m}$, and $r_c = 0.9 \text{m}$. Conductivity $\kappa_0 = 40 \text{W/(m} \cdot \text{K})$. The temperature is respectively set to be 400 K and 300 K at the left and right boundaries of the squared simulation area with the side length of 2.5 m; heat insulation is used for both up and down boundaries. Figure 2 shows the simulation results of temperature distribution for three cases. Details are as follows. Figure 2(a) displays the temperature distribution for the pure background material; the distribution is regular. Then, we replace Region C inside the background material of fig. 2(a) with an object of thermal conductivity $\kappa_m = 400 \text{W/(m} \cdot \text{K})$; see fig. 2(b). Accordingly, when compared to fig. 2(a), fig. 2(b) shows that the distribution of temperature within Region D is significantly affected by the object (which occupies Region C). Next, we use the cloaking material to replace Region B according to eq. (7), and plot fig. 2(c). As a result, fig. 2(c) shows that the temperature distribution within Region D ($r > r_c$) turns out to be the same as that in fig. 2(a). In other words, the thermal cloak (occupying Region B) can be used to hide the object (located in Region C) from the detection by measuring the distribution of external heat flux (within Region D). This behavior also holds for objects (which occupy Region C) with arbitrary shapes like squares or triangles (figures are not shown herein). Clearly the underlying mechanism lies in the complementary effect of materials within Region B, which have a graded, anisotropic, negative thermal conductivity (see fig. 3). Such materials with negative thermal conductivities are called “thermal metamaterials” since they cannot be found in naturally occurring materials or chemical compounds. Regarding this kind of thermal metamaterials, we have to address more. Physically, a negative thermal conductivity corresponds to the fact that heat is transferred from regions of low temperature to regions of high temperature. This process means that, to comply with
the second law of thermodynamics, an external work (say, based on Peltier effects [15,16]) should be performed accordingly. Indeed, researchers have reported a negative thermal conductivity when investigating chains of rotors with mechanical forcing [17]. Nevertheless, if the external work is stable, the negative thermal conductivity obtained should also be stable. In this case, the steady-state conduction equation (eq. (1)) could be used as we have done in this work. On the other hand, if the external work is unstable, the corresponding negative thermal conductivity will also be unstable, which is dependent on time. As a result, the distribution of temperature will evolve with time, and thus an unsteady-state conduction equation should be adopted instead.

Conclusions. – In summary, we have proposed a class of unconventional thermal cloaks. Such a cloak can hide an object, which is located outside the cloaking shell, from the detection by measuring the distribution of external heat flux. A feature of this kind of thermal cloaks is that the cloaked object itself can feel external heat flow. The feature makes it distinctly different from the conventional thermal cloaks where cloaked objects cannot feel external heat flow. This feature mainly originates from the complementary effect of thermal metamaterials with negative thermal conductivities. Achieving this feature is at the cost of a specific design of configurations. Our simulations can be extended from two dimensions to three dimensions as long as material parameters are set appropriately. Thus, by choosing the complementary effect of thermal metamaterials appropriately, this work suggests a different way to design thermal devices where heat conduction can be controlled at will.

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