

# Engineering the accurate distortion of an object's temperature-distribution signature

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**Abstract.** It is up to now a challenge to control the conduction of heat. Here we develop a method to distort the temperature distribution signature of an object at will. As a result, the object accurately exhibits the same temperature distribution signature as another object that is predetermined, but actually does not exist in the system. Our finite element simulations confirm the desired effect for different objects with various geometries and compositions. The underlying mechanism lies in the effects of thermal metamaterials designed by using this method. Our work is of value for applications in thermal engineering.

## 1 Introduction

Thermal energy is not only everywhere in nature, but also an outcome of many other types of energy like electrical energy, solar energy, nuclear energy and mechanical energy. Therefore, it is particularly important to control heat transfer at will. However, it is up to now a challenge to control the conduction of heat because this conduction obeys the diffusion equation, a partial differential equation describing density dynamics in a material with diffusion [1]. In 2008, Fan et al. [2] first adopted the coordinate transformation approach to propose a class of thermal metamaterials with novel thermal properties that cannot be found in nature or chemical compound; their work provides a different way to steer heat conduction. As a result, a lot of thermal metamaterials with novel thermal properties have come to appear, such as cloaks (which are used to let heat flow around an object as if the object does not exist) [2–18], concentrators (which are used to concentrate heat into a specific region) [4,5], inverters (which are used to apparently let heat flow from the region of low temperature to the region of high temperature) [2,19], and rotators (which are used to rotate the flow of heat as if it comes from a different angle) [5,19].

In the area of heat conduction, a temperature distribution signature can be used to identify an object under temperature gradient. In this sense, if one can distort the temperature distribution signature of Object A into that of Object B by using some methods (note here Object B is predetermined, and actually does not exist in the system), Object A will no longer be found according to the detected temperature distribution signature. In other

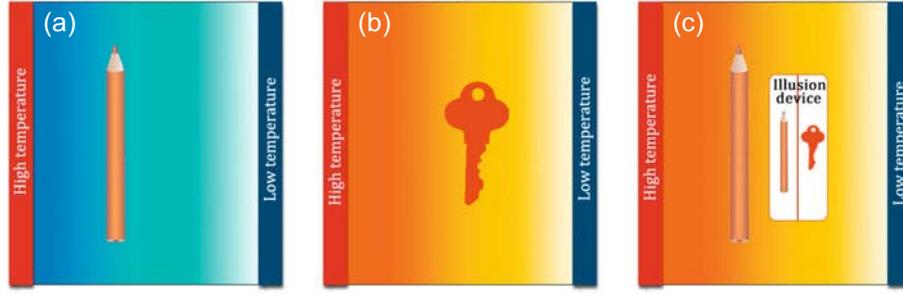
words, Object A is mistakenly believed to be Object B, thus yielding a type of thermal illusion (the thermal counterpart of optical illusion [20]).

In this work, we shall develop the coordinate transformation approach for heat conduction [2,3], and propose a kind of device made of thermal metamaterials, which causes Object A to accurately possess the same temperature distribution signature as Object B; see Figure 1. Our finite element simulations in two dimensions confirm the desired effect for different objects with various geometries and compositions. This kind of device paves a different way for controlling heat conduction as expected.

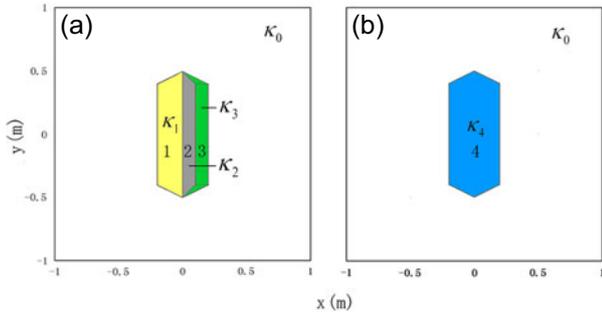
## 2 Theory

To proceed, we plot Figure 1, which schematically shows the thermal illusion under our consideration. In detail, Figure 1a displays the temperature distribution signature of a pencil (Object A); Figure 1b depicts the temperature distribution signature of a key (Object B); Figure 1c is same as Figure 1a, but we add an illusion device. As a result, with the help of the illusion device, the *pencil* outside the illusion device in Figure 1c apparently has the same temperature distribution signature as the *key* in Figure 1b, thus yielding the thermal illusion. Clearly, the illusion device is the key to obtain this kind of thermal illusion. But, how to design the illusion device? We offer relevant details in Figure 2. Figure 2 just shows the illusion device occupying Regions 2–3 embedded in a background of thermal conductivity  $\kappa_0$ , which can be used to produce the desired thermal illusion: the medium of  $\kappa_1$  occupying Region 1 apparently produces the same temperature distribution signature as the medium of  $\kappa_4$

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**Fig. 1.** Schematic graph showing the concept of thermal illusion proposed in this work; the bar on the left (or right) with “High temperature” (or “Low temperature”) denotes the heat source of high (or low) temperature. (a) The blue gradient denotes the temperature distribution signature of a pencil (an object); (b) the orange gradient represents the temperature distribution signature of a key (another object); (c) same as (a), but we add an illusion device that is indicated by the white area. As a result, the temperature distribution signature outside the illusion device is different from that in (a), but same as that in (b). In other words, with the help of the key of the illusion device, the *pencil* outside the illusion device apparently has the same temperature distribution signature as the *key* in (b), thus yielding a thermal illusion.



**Fig. 2.** Schematic graph showing how to design the illusion device occupying Regions 2 and 3 embedded in a background of thermal conductivity  $\kappa_0$ , for producing the thermal illusion, namely, the medium of  $\kappa_1$  occupying Region 1 apparently produces the same temperature distribution signature as the medium of  $\kappa_4$  occupying Region 4. Here Regions 1–2 both have a shape of trapezoid, Region 3 contains one rectangle and two triangles, and Region 4 is in the shape of hexagon, which has the same shape, size and location as the total area of Regions 1–3. For achieving this kind of thermal illusion, the medium of  $\kappa_2$  (complementary medium) occupying Region 2 thermally cancels the medium occupying Region 1, and the medium of  $\kappa_3$  (restoring medium) occupying Region 3 thermally takes place of the medium occupying Region 4. Note that the origin of coordinates is located at the center of the right boundary of Region 1.

occupying Region 4. Here Regions 1–2 both have a shape of trapezoid, Region 3 contains one rectangle and two triangles, and Region 4 is in the shape of hexagon, which has the same shape, size and location as the total area of Regions 1–3. For achieving this kind of thermal illusion, the medium of  $\kappa_2$  (complementary medium) occupying Region 2 thermally cancels the medium occupying Region 1, and the medium of  $\kappa_3$  (restoring medium) occupying Region 3 thermally takes place of the medium occupying Region 4. Then, we are in a position to obtain  $\kappa_2$  and  $\kappa_3$ , in order to obtain the illusion device. Without loss of generality, we consider the heat conduction equation

for the steady state:

$$\nabla \times (\kappa \nabla T) = 0, \quad (1)$$

where  $\kappa$  is the thermal conductivity and  $T$  is the temperature. Since equation (1) remains form invariant under coordinate transformations, we may directly apply the coordinate transformation approach to equation (1) [2]. As a result, the new thermal conductivity  $\kappa'$  in the transformed coordinates satisfies the following relation [2,3],

$$\kappa' = \frac{J \kappa J^t}{\det(J)}, \quad (2)$$

where  $J$  is the Jacobian transformation matrix between the original and distorted coordinates,  $J^t$  is the transposed matrix of  $J$ , and  $\det(J)$  is the determinant of  $J$ . As shown in Figure 2a, Region 2 should be used to thermally cancel Region 1, which can be achieved by folding the geometry of Region 1 into Region 2. So, according to equation (2), the thermal conductivity of Region 2,  $\kappa_2$ , is given by:

$$\kappa_2 = \frac{J_{12} \kappa_1 J_{12}^t}{\det(J_{12})}. \quad (3)$$

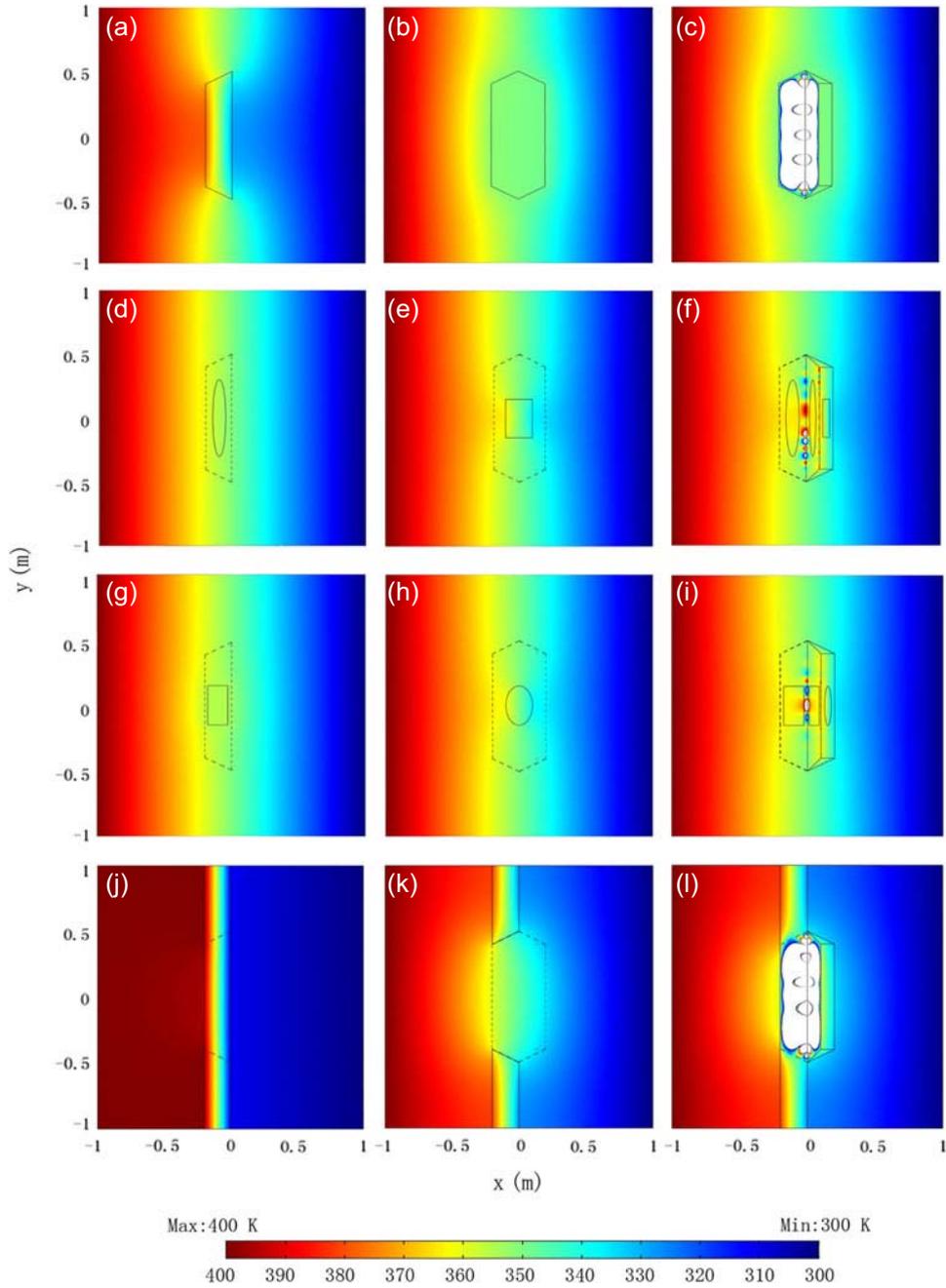
Here,  $J_{12}$  is the Jacobian transformation matrix which is determined by the coordinates transformation between Region 1 [with coordinates  $(x_1, y_1)$ ] and Region 2 [with coordinates  $(x_2, y_2)$ ],

$$J_{12} = \begin{pmatrix} \frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial y_1} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial y_1} \end{pmatrix}. \quad (4)$$

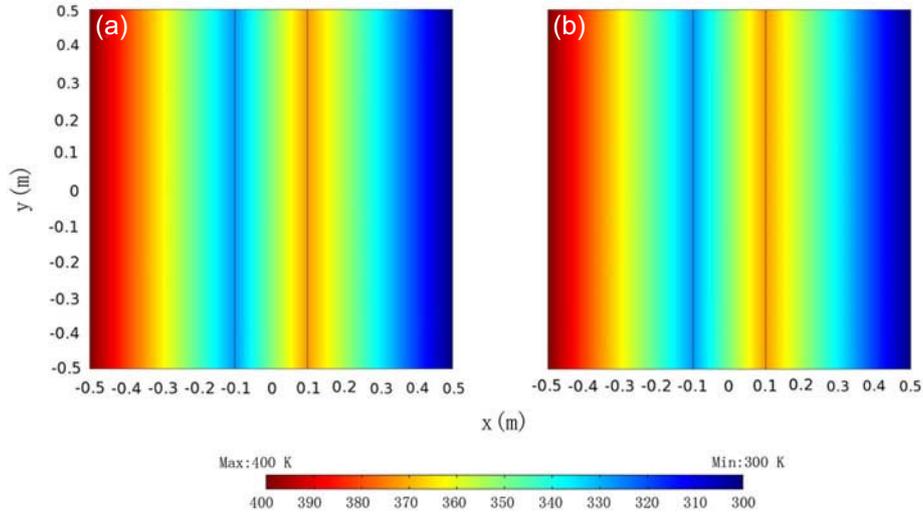
On the other hand, since Region 3 of Figure 2a is used to replace the whole area corresponding to Region 4 of Figure 2b, according to equation (2), the thermal conductivity of Region 3,  $\kappa_3$ , is given by:

$$\kappa_3 = \frac{J_{43} \kappa_4 J_{43}^t}{\det(J_{43})}, \quad (5)$$

where  $J_{43}$  is the Jacobian transformation matrix determined by the coordinates transformation between



**Fig. 3.** Simulation results of temperature distribution signatures in two dimensions. Regions 1–4 of Figure 2 apply to (a)–(l) herein as well, and dashed lines indicate the position of both Region 1 in (d, f, g, i, j, l) and Region 4 in (e, h, k). (a) Region 1 within the background of  $\kappa_0 = 40 \text{ W}/(\text{mK})$  is fully occupied by Object A of  $\kappa_A = 1 \text{ W}/(\text{mK})$  in the shape of trapezoid, (b) Region 4 within the same background is fully occupied by Object B of  $\kappa_B = 400 \text{ W}/(\text{mK})$  in the shape of hexagon, and (c) is same as (a), but involves an illusion device that occupies Regions 2–3. (b) and (c) show the same temperature distribution signature outside either Region 4 of (b) or Regions 1–3 of (c), thus yielding the desired thermal illusion. (d–f) is same as (a–c), but Object A (or Object B) becomes an elliptical (or rectangular) object of  $\kappa_A = 400 \text{ W}/(\text{mK})$  (or  $\kappa_B = 1 \text{ W}/(\text{mK})$ ), which lies in the background and partially occupies Region 1 (or Region 4). (g–i) is same as (d–f), but Object A (or Object B) has a shape of rectangle (or ellipse) instead. (j) shows the background with a 0.2 m-width thermal wall of  $\kappa_A = 1 \text{ W}/(\text{mK})$ , which separates the space into two disconnected parts. In (j), the thermal wall just serves as Object A, which contains Region 1. (k) shows Region 4, which only involves the same material as the background. That is, (k) contains Object B, which is actually the truncated thermal wall and lies outside Region 4. (l) is same as (j), but we add an illusion device to Regions 2–3. Similarly, (k) and (l) show the same temperature distribution signature outside either Region 4 of (k) or Regions 1–3 of (l), thus yielding the thermal illusion as well. The white area in (c), (f), (i) and (l) denotes the temperature whose value exceeds the bounds of color bars.



**Fig. 4.** Simulation results of temperature distribution signatures in two dimensions. The left and right boundaries of the whole simulation box are set to be 400 K and 300 K, respectively. (a) shows an rectangular region (ranged from  $x = -0.1$  m to 0.1 m) of  $\kappa_1 = -80$  W/(mK) within the background of  $\kappa_0 = 100$  W/(mK). The geometry of (b) is as same as (a), but we change  $\kappa_1$  to be 80 W/(mK) and set the temperature at  $x = -0.1$  m or 0.1 m to be 327.3 K or 372.7 K by acting external work appropriately. Clearly, (b) has the same temperature distribution signature as (a), thus showing a feasible method on how to realize apparently negative thermal conductivities in reality.

Region 3 [with coordinates  $(x_3, y_3)$ ] and Region 4 [with coordinates  $(x_4, y_4)$ ],

$$J_{43} = \begin{pmatrix} \frac{\partial x_3}{\partial x_4} & \frac{\partial x_3}{\partial y_4} \\ \frac{\partial y_3}{\partial x_4} & \frac{\partial y_3}{\partial y_4} \end{pmatrix}. \quad (6)$$

### 3 Results

In order to show the desired thermal illusion convincingly, we are in a position to perform finite element simulations (based on commercial software COMSOL Multiphysics). For simulations, we set the following coordinates transformation between Region 1  $(x_1, y_1)$  and Region 2  $(x_2, y_2)$ ,

$$x_2 = -\frac{x_1}{2}, \quad (7)$$

$$y_2 = y_1. \quad (8)$$

Then, the substitution of equations (7) and (8) into equations (3) and (4) yields  $\kappa_2$ . On the other hand, the coordinates transformation between Region 4  $(x_4, y_4)$  and Region 3  $(x_3, y_3)$  is set to be:

$$y_3 = y_4 \text{ for the whole area of Region 3,} \quad (9)$$

$$\frac{1}{4} = \frac{-2y_3 + 1 - x_3}{-2y_4 + 1 - x_4} \text{ for the upper obtuse triangle,} \quad (10)$$

$$\frac{1}{4} = \frac{2y_3 + 1 - x_3}{2y_4 + 1 - x_4} \text{ for the lower obtuse triangle,} \quad (11)$$

$$\frac{1}{4} = \frac{0.2 - x_3}{0.2 - x_4} \text{ for the rectangle of Region 3.} \quad (12)$$

So, plugging equations (9)–(12) into equations (5) and (6) gives  $\kappa_3$ . Figure 3 shows our simulation results; in each

panel of Figure 3, the temperature at the left and right boundary is respectively set to be 400 K and 300 K. In Figure 3a, Region 1 within the background of  $\kappa_0 = 40$  W/(mK) is fully occupied by Object A of  $\kappa_A = 1$  W/(mK) in the shape of trapezoid; In Figure 3b, Region 4 within the same background is fully occupied by Object B of  $\kappa_B = 400$  W/(mK) in the shape of hexagon; Figure 3c is same as Figure 3a, but involves an illusion device that occupies Regions 2–3. Clearly, Figures 3b and 3c show the same temperature distribution signature outside either Region 4 of Figure 3b or Regions 1–3 of Figure 3c, thus yielding the desired thermal illusion.

Then, we discuss the case of different shapes and compositions. Figure 3d–3f is same as Figure 3a–3c, but Object A (or Object B) becomes an elliptical (or rectangular) object of  $\kappa_A = 400$  W/(mK) (or  $\kappa_B = 1$  W/(mK)), which lies in the background and partially occupies Region 1 (or Region 4). Clearly, Figure 3f is the thermal illusion of Figure 3e due to the remote control of the illusion device. Similarly, Figure 3i is the thermal illusion of Figure 3h because of the remote control of the illusion device as well, where the shape of Object A and Object B has been exchanged.

In Figures 3a–3i, both Region 1 and Region 4 are fully or partially occupied by Object A and Object B. Figure 3j–3l shows a different case, where Object A (Fig. 3j) is partially occupied by Region 1 instead and Object B (Fig. 3k) is outside Region 4. Also, Figure 3l is still the thermal illusion of Figure 3k. In other words, the illusion device proposed in this work can be used to get thermal illusions not only for the whole area of Object A (Figs. 3c, 3f, 3i)], but also for a part of Object A (Fig. 3l).

According to Figure 3, we may turn to the conclusion that this illusion device is capable of realizing

various kinds of thermal illusion, which are independent of the shape and composition of Object A and Object B. But, what is the underlying mechanism? As mentioned in Figure 2, this mechanism lies in the complementary and restoring effects of thermal metamaterials. In particular, thermal conductivities of these thermal metamaterials should be not only anisotropic, but also negative, as implied by equations (3) and (5). Since a negative thermal conductivity of a material means that heat flows from the region of low temperature to the region of high temperature, in reality one must apply an external work to make the physics similar to that of negative conductivities as required by the second law of thermodynamics. In fact, an electric refrigerator is just a sort of “material” whose effective thermal conductivity is apparently negative because heat is brought from the interior of the refrigerator (namely, the region of low temperature) to the exterior of the refrigerator (i.e., the region of high temperature) due to the input of electric power. On the same footing, for experimental demonstration of our proposal about thermal illusions, Peltier effects [21, 22] may help to get apparently negative values of thermal conductivities. In this direction, we plot Figure 4 that shows a feasible approach on how to obtain heat flux flowing in a specific region from low temperature to high temperature by acting external work appropriately as if the thermal conductivity were negative. In other words, the presence of external sources makes the physics similar to that of negative conductivities.

## 4 Conclusions

To sum up, by using the coordinate transformation approach for heat conduction [2, 3], we have proposed a kind of thermal illusion device composed of thermal metamaterials. Our finite element simulations in two dimensions have confirmed the desired thermal illusions for different objects with various geometries and compositions: With the help of the illusion device, Object A could accurately exhibit the same temperature distribution signature as Object B (a predetermined object) although Object B actually does not exist. The underlying mechanism originates from the effects of thermal metamaterials designed by us. This work not only proposes a concept of thermal illusion for applications in various fields including military use, but also offers a method to control heat flow at will (e.g., it can even be developed for treating the patterns of local heat flux [23]).

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