Chin. Phys. Lett. **39**, 075201 (2022)

Transformation Plasma Physics

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(Received 24 April 2022; accepted manuscript online 31 May 2022)

Plasma technology has widespread applications in many fields, whereas the methods for manipulating plasma transport are limited to magnetic control. In this study, we used a simplified diffusion-migration approach to describe plasma transport. The feasibility of the transformation theory for plasma transport was demonstrated. As potential applications, we designed three model devices capable of cloaking, concentrating, and rotating plasmas without disturbing the density profile of plasmas in the background. This research may help advance plasma technology in practical fields, such as medicine and chemistry.

DOI: 10.1088/0256-307X/39/7/075201

Plasma, known as the fourth state of matter, is a gaseous mixture of unbound ions, electrons, and reactive radicals that becomes highly electrically conductive.[1] Although plasma is not common on Earth's surface, it can be created artificially by charging gases with direct/alternating current or radio/microwave frequency sources. Due to its unique composition, plasma technology plays a vital role in many fields, including micro/nanoelectronics, chemistry, bio-medicine, aerospace, and material science.[2–5]

Despite extensive theoretical and experimental research, manipulating plasma transport remains a critical challenge. Conventional control of charged particles depends on external magnetic fields. This simple method may limit the accuracy of manipulation. Since the past decade, transformation theory, an approach to replacing space transformation with material transformation, has attracted widespread attention in wave and diffusion systems as a reliable and powerful method of controlling matter[6–8] or energy flow.[9–17] However, it has not yet been introduced to plasma transport, a unique diffusion process. One possible explanation could be the particularly complex motion process in plasmas.

This work utilizes a toy model (diffusion-migration model) to describe plasma transport and design three conceptual devices, i.e., cloak, concentrator, and rotator, to control transient plasma flow based on the transformation theory. Here "cloak" can provide a zero-density gradient inside the device; "concentrator" gives a larger density gradient inside the device; "rotator" can deflect the transport direction of the plasma inside the device. Most importantly, all the devices do not disturb the density profiles of plasmas located in the background. The present results might broaden the horizon of manipulating transient plasma transport and might be helpful to inspire further improvements in plasma physics.

Theory. Compared to the conventional diffusion system, the realistic plasma transport is much more complicated, because the interaction between charged particles and intrinsic local electromagnetic fields affects the transport process a lot. Furthermore, the ionization reaction in plasma can also significantly affect the momentum and energy transfer of particles. In general, the transport of charged particles in plasma is dominated by[18]

$$\partial_t n - \nabla \cdot (D\nabla n) \pm \nabla \cdot (\mu \boldsymbol{E} n) + \nabla \cdot (\boldsymbol{v} n) = S, \quad (1)$$

where n, D, μ, $\boldsymbol{E}$, $\boldsymbol{v}$, and S are the density, diffusivity, mobility, electric field, advective velocity, and external source, respectively. In particular, the sign of the third term (i.e., migration term) is positive for positive particles and negative for negative particles. We only considered electric fields for brevity while ignoring the advective process and the gaseous reaction.[19] Hence, plasma transport can be simplified to a diffusion-migration process. Then according to the Einstein relation, Eq. (1) could be rewritten as

$$\partial_t n - \nabla \cdot (D\nabla n) \pm \nabla \cdot \left[\left(\frac{D\boldsymbol{E}}{T}\right) n\right] = S, \quad (2)$$

where T (in units of V) is assumed to be a constant plasma temperature. According to the transformation theory, the plasma flow can be manipulated by transforming the corresponding parameters when the controlling equation is form-invariant under a coordinate transformation. Next, we demonstrate that Eq. (2) at steady state strictly keeps form invariance after transforming coordinates.

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For steady state, the equation should be written as

$$-\nabla \cdot (D\nabla n) \pm \nabla \cdot \Big[\Big(\frac{D\boldsymbol{E}}{T}\Big)n\Big] = S, \tag{3}$$

where we replaced mobility with diffusivity. Then to obtain intuitive transformed results, we write down the component form of the diffusion-migration equation in a curvilinear space with the corresponding coordinate x_i as follows:[20]

$$-\partial_i(\sqrt{g}D^{ij}\partial_j n) \pm \partial_i\Big[\Big(\frac{\sqrt{g}D^{ij}E_j}{T}\Big)n\Big] = \sqrt{g}S, \tag{4}$$

where g is the determinant of $g_i \cdot g_j$, with g_i and g_j being covariant bases of the curvilinear space. Then we write Eq. (4) in the physical space with coordinate x'_i,

$$-\partial_{i'}\frac{\partial x'_i}{\partial x_i}\Big[\sqrt{g}D^{ij}\frac{\partial x'_j}{\partial x_j}\partial_{j'}n \mp \Big(\frac{\sqrt{g}D^{ij}E_j}{T}\Big)n\Big] = \sqrt{g}S, \tag{5}$$

where $\partial x'_i/\partial x_i$ and $\partial x'_j/\partial x_j$ are the components of the Jacobian matrix J whose determinant $\det J = 1/\sqrt{g}$. Hence, we may reduce Eq. (5) to

$$-\nabla' \cdot \Big[D'\nabla' n \mp \Big(\frac{D'\boldsymbol{E}'}{T}\Big)n\Big] = S', \tag{6}$$

where $D' = JDJ^{\tau}/\det J$, $\boldsymbol{E}' = J^{-\tau}\boldsymbol{E}$, and $S' = S/\det J$. Here, ∇' refers to the differential in the new coordinates x'_i. J is the Jacobian matrix with components $J_{ij} = \partial x'_i/\partial x_j$, J^{τ} is the transpose of J, while $J^{-\tau}$ is the inverse transpose of J. $\det J$ equals the determinant of J. Thus, the steady diffusion-migration equation strictly keeps form invariance for arbitrary coordinate transformations.

However, the case is distinctly different in the transient state. Equation (2) at transient state could be reduced to[21]

$$\frac{1}{\det J}\partial_t n - \nabla' \cdot (D'\nabla' n) \pm \nabla' \cdot \Big[\Big(\frac{D'\boldsymbol{E}'}{T}\Big)n\Big] = S'. \tag{7}$$

Compared with Eq. (2), the metric induced by the coordinate transformation in front of $\partial_t n$ in Eq. (7) cannot be absorbed by materials or field parameters. Hence, the transient plasma transport is not strictly form-invariant under a coordinate transformation except for $\det J = 1$. Nevertheless, considering an approximation, we can remove the unwanted metric and rewrite Eq. (7) as

$$\partial_t n - \nabla' \cdot (D''\nabla' n) \pm \nabla' \cdot \Big[\Big(\frac{D''\boldsymbol{E}''}{T}\Big)n\Big] = S'', \tag{8}$$

whose transformation rules are $D'' = JDJ^{\tau}$, $\boldsymbol{E}'' = J^{-\tau}\boldsymbol{E}$, and $S'' = S$. In this way, the transformed equation could keep form-invariant. Notably, Eq. (8) is generally an approximation of Eq. (7) because $\det J$ is position-dependent. In order to clearly understand this approximation, we rewrite its original transformed form using Eq. (7) as

$$\begin{aligned}&\frac{1}{\det J}\partial_t n - \nabla'\cdot\Big(\frac{JDJ^{\tau}}{\det J}\nabla' n\Big)\\ &\pm\nabla'\cdot\Big[\Big(\frac{JDJ^{\tau}}{\det J}\frac{J^{-\tau}\boldsymbol{E}}{T}\Big)n\Big] = \frac{S}{\det J}.\end{aligned} \tag{9}$$

Then we multiply $\det J$ to both sides of Eq. (9) and decompose it,

$$\begin{aligned}&\partial_t n - \nabla' \cdot (JDJ^{\tau}\nabla' n)\\ &\pm \nabla' \cdot \Big[\Big(\frac{JDJ^{\tau}J^{-\tau}\boldsymbol{E}}{T}\Big)n\Big] \pm \varDelta = S,\end{aligned} \tag{10}$$

where $\varDelta = \det J\nabla'(1/\det J)[JD\boldsymbol{E}n/T \mp JDJ^{\tau}\nabla' n]$. Comparing Eq. (10) with Eq. (8), the error caused by the approximation strictly depends on $\varDelta$. The error $\varDelta$ is closely related to $\det J$ and cannot be eliminated if $\det J \neq 1$. Thus, the values of diffusivity and electric field intensity become crucial to the effect of the theory. As a result, small values of diffusivity and electric field intensity help prevent the devices from seriously disturbing the background plasma. Thus, the quantities of D and $\boldsymbol{E}$ need to be small enough to avoid a large error. The following simulation results show that this approximation is feasible and reasonable.

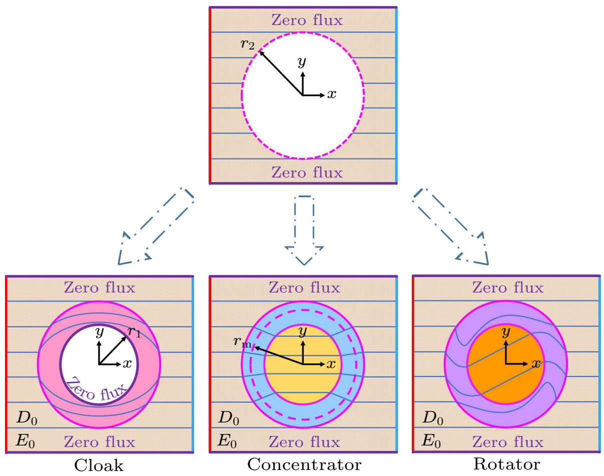

Fig. 1. Schematic of conceptual devices. Solid blue line represents the plasma flow. We set the side length of the background matrix as $l = 0.12\,\mathrm{m}$. Other parameters: $D_0 = 9.2 \times 10^{-7}\,\mathrm{m/s}$, $\boldsymbol{E}_0 = [1.04 \times 10^4, 0]\,\mathrm{V/m}$, $r_1 = 0.020\,\mathrm{m}$, $r_2 = 0.030\,\mathrm{m}$, $r_m = 0.025\,\mathrm{m}$, $\theta_0 = \pi/3$, and $T_0 = 2.0\,\mathrm{V}$.

We proposed three conceptional model devices to confirm the theory to realize cloaking, concentrating, and rotating transient plasma transport without (obviously) disturbing the plasma distribution in the background, as shown in Fig. 1. When the plasma is fed from the left-hand side, it remains unchanged on the right-hand side, as if there is no device in the middle. In particular, the cloak can hide objects in the central region. The concentrator can increase the density gradient in the core region, while the rotator can flexibly rotate the propagation direction of plasma flow.[22] Next, we introduce the cloak first.

To realize the plasma cloak, the coordinate transformation from a virtual space r_i to the physical space r'_i is set as[22]

$$r' = \frac{r_2 - r_1}{r_2} r + r_1, \qquad \theta' = \theta. \tag{11}$$

Here, r_1 and r_2 are the radii of the inner and outer boundaries of the cloak, respectively, as depicted in Fig. 1. This coordinate transformation can be physically explained as stretching the center dot into a circle with a radius of r_1 in the virtual space. Then we derive transformed parameters to achieve the cloaking of plasma flow using the aforementioned transformation rules.

Similarly, the coordinate transformations for realizing plasma concentrator and rotator can be written mathematically as[22]

$$\begin{aligned} r' &= \frac{r_1}{r_m} r, \quad r < r_m, \\ r' &= \frac{r_1 - r_m}{r_2 - r_m} r_2 + \frac{r_2 - r_1}{r_2 - r_m} r, \quad r_m < r < r_2, \\ \theta' &= \theta, \end{aligned} \tag{12}$$

$$\begin{aligned} r' &= r, \\ \theta' &= \theta + \theta_0, \quad r < r_1, \\ \theta' &= \theta + \theta_0 \frac{r - r_2}{r_1 - r_2}, \quad r_1 < r < r_2, \end{aligned} \tag{13}$$

where r_m ($r_1 < r_m < r_2$) and θ_0 represent the constant radius and angle, respectively. Equation (12) describes a physical picture that compresses a circle with a radius of r_m to a smaller circle with a radius of r_1 in the virtual space. In contrast, Eq. (13) shows that in the virtual space, a series of circles with different radii are twisted by different angles determined by the values of the corresponding radii. Therefore, we obtain transformed parameters to converge or rotate plasma flow. Notably, Eq. (8) for rotator is an accurate form instead of an approximation because of $\det J = 1$ in this case.

Results and Discussion. Now we can use COMSOL Multiphysics to perform finite element simulations. As reflected in Fig. 1, a periodic plasma source, set as $n = n_1 \cos\omega_0 t + n_0$, is applied to the left boundary (the red one) of the background matrix. Here $n_1 = 5.0 \times 10^{15}\,\mathrm{m^{-3}}$, $\omega_0 = 2\pi/10\,\mathrm{s^{-1}}$, and $n_0 = 1.0 \times 10^{17}\,\mathrm{m^{-3}}$. We set the opposite (right) side as an outflow boundary (the blue one). Both upper and lower sides (boundaries) are set with a zero-flux condition. Further, the zero-flux condition is also applied to the inner circle boundary of the cloak. The whole background matrix possesses a constant diffusivity D_0 and a uniform electric field $\boldsymbol{E}_0$. Then all the parameters can be designed according to the above transformation rules, and the simulation results of cloaking, concentrating, and rotating are shown in Figs. 2–4, respectively.

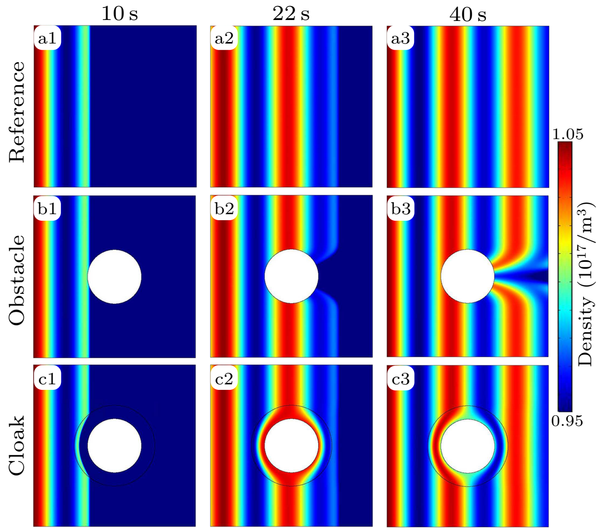

Fig. 2. Simulation results of the cloak at transient states. (a1)–(a3) Density profiles for pure background at 10, 22, and 40 s, respectively. (b1)–(b3) Density profiles for background with an obstacle at 10, 22, and 40 s, respectively. (c1)–(c3) Density profiles for background with the cloak at 10, 22, and 40 s, respectively.

Figure 2 depicts the transient simulation of plasma transport under three conditions: transporting in a pure background medium (set as the reference); a background medium with a bare obstacle; a background medium with an obstacle covered by the cloak. The columns from left to right are screenshots of distributions of the plasma density at 10, 22, and 40 s, respectively. The plasma streams forward in a wave-like pattern due to the boundary condition of harmonically oscillating density. Furthermore, the amplitude attenuation of the plasma flow is caused by diffusion; the decay rate is codetermined by the oscillation frequency, diffusivity, and electric field. As a result, we carefully selected suitable values to make the results more intuitive. The cloak designed with the transformation theory helps to cancel the scattering induced by the obstacle. Therefore, the density profiles of the background plasma remain nearly undisturbed, showing the validity of the theory.

Figure 3 presents the transient simulation results for the concentrator and rotator. The first row of snapshots shows the converging effect of the gradient of plasma density. Furthermore, a larger ratio (r_m/r_1) would bring a higher converging effect as a determinant of the converging effect. The maximum ratio is r_2/r_1. For the rotator, the rotation of plasma flow appears in Figs. 3(b1)–3(b3). Linearly deflecting concentric circles in the virtual space can account for the gradual deflection of the density profiles. The target rotation angle in the core region is determined by θ_0 in

Eq. (13). Particularly, det $J = 1$ for rotators helps to completely eliminate the disturbance to background plasma density.

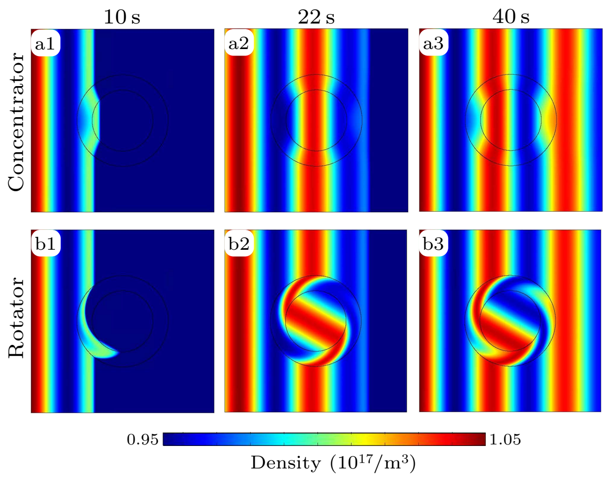

Fig. 3. Concentrator and rotator simulation results at transient states. (a1)–(a3) Density profiles for the concentrator at 10, 22, and 40 s, respectively. (b1)–(b3) Density profiles for the rotator at 10, 22, and 40 s, respectively.

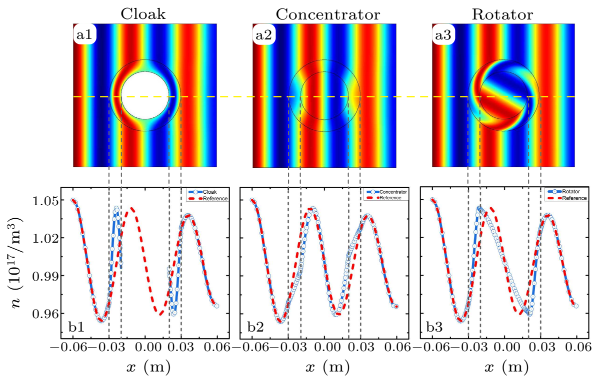

Fig. 4. (a1)–(a3) Color mapping of density profiles at 40 s with a cloak, concentrator, and rotator, respectively. (b1)–(b3) Comparisons of density profiles between pure background (reference) and those with a cloak, concentrator, and rotator, respectively. The gray dashed lines denote the position of the devices. Data are extracted along the yellow dashed line ($y = 0$) in (a1)–(a3).

To further explore the performance of the devices, we extracted the density values along a horizontal line (denoted by the yellow dashed lines in Fig. 4) from the results at 40 s and compared the density distribution of functional devices with that of reference as shown in Figs. 4(b1)–4(b3). Here, two regions should be remarked: One is the core region of the device, and the other is the background. All the red dashed lines in Figs. 4(b1)–(b3) indicate reference data, while the blue dotted lines represent the data of the cloak, concentrator, and rotator, respectively. In Fig. 4(b1), it is clear that the data are overlapped in the background, and the plasma is excluded well from the core region. Moreover, the relative difference in the plasma density in the background region is less than 0.15%. In Fig. 4(b2), the dotted line is indeed denser than the dashed line in the core region without being seriously dislocated in the background. The relative difference was less than 0.13%. In Fig. 4(b3), the relative difference is less than 0.01%, which is far smaller than the value of the cloak or concentrator. As mentioned above, the accurate transformation form of Eq. (8) may account for this nearly zero difference. Overall, the simulation can confirm the feasibility and reliability of the theory.

Next, we can foresee some potential applications of the devices designed using the transformation theory. For example, the cloak, whose core region is isolated, can be used to protect healthy tissue in plasma-curing infected wounds. In catalyst preparation, converged plasma flow, which usually has a denser density of the active particle clusters, is beneficial to the interaction between plasma and catalyst. Hence, the concentrator can be used to improve catalytic efficiency. In the aerospace industry, the concentrator possibly enhances the performance of plasma-assisted engines. Besides potential functions discussed above, separating or guiding[23] plasma could also be achieved by constructing appropriate coordinate transformation, applicable to control plasma etching or plasma depositing. Moreover, the transformation theory may also help design plasma metamaterial proposed to adjust electromagnetic waves.[24,25] Consequently, the proposed methodology based on the transformation theory does make sense. Furthermore, despite the difficulties of achieving the transformed diffusivities and electric fields, it is still possible to realize the same effect using other methods. There are numerous studies on customizing particle diffusivities. For example, a bilayer diffusive cloak can be fabricated from two homogeneous materials using the scattering cancellation method.[26] The effective medium theory[27] or the machine learning method[28] can also be used to realize complex diffusivity. Studies of the electrostatic cloak and magnetic cloak may provide useful inspiration for manipulating electric fields.[29,30]

Despite unavoidable challenges, many new mechanisms must be studied. Under more general conditions, the influence of magnetic field and gas-phase reaction in plasma should be considered. It is challenging to manipulate diffusivities, and electric fields since the complicated interactions between charged particles and electromagnetic fields are too hard to be controlled at will. Therefore, it is essential to introduce additional theories or methods, like particle-in-cell/Monte Carlo collision model[31] or nonequilibrium Green's function approach.[32] Furthermore, the temperature of plasmas is usually time-varied or space-varied at transient states, thus leading to different transformation rules. Advection may also happen in the plasma transport in some cases. We consider the advection term would make the regulation of plasmas more diverse. Moreover, the spatiotemporal modulation, a recent hot spot in heat diffusion,[33] may bring beneficial properties to plasma physics. In short, improving the transformation theory for plasmas merits more studies, attention, and effort.

In summary, we have employed a toy model, i.e., the diffusion-migration model, to describe plasma transport in this study. The feasibility of the transformation theory is demonstrated. As a result, it is found that the transformed diffusion-migration equation is strictly form-invariant at steady states but not at transient states. Nevertheless, we have shown that the transformed transient equation can be approximately form-invariant by setting small diffusivities. Then we designed three conceptual model devices, which function as a plasma cloak, concentrator, or rotator for transient plasma transport. Our results may broaden the approach to manipulating plasma flow, and they have potential applications in various fields, including medicine, the aerospace industry, etc.

Acknowledgments. We acknowledge financial support from the National Natural Science Foundation of China (Grant Nos. 11725521 and 12035004) and the Science and Technology Commission of Shanghai Municipality (Grant No. 20JC1414700).

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