1. 定态薛定谔方程：

$$-\frac{\hbar^2}{2m}\nabla\psi(x\cdot y\cdot z)=E\psi(x\cdot y\cdot z)$$

束缚均为x方向，$y\cdot z$方向为平面波，得：

$$\psi(x\cdot y\cdot z)=\phi(x)\,e^{ik_y y}e^{ik_z z}$$

$$\frac{\partial^2\phi(x)}{\partial x^2}+K_x^2\phi(x)=0 \qquad \phi(x)=Ae^{ik_x x}+Be^{-ik_x x}$$

$$\phi(x)\big|_{x<0}=0 \Rightarrow A+B=0$$

$$\psi(x\cdot y\cdot z)=N\sin k_x x\, e^{ik_y y}e^{ik_z z}$$

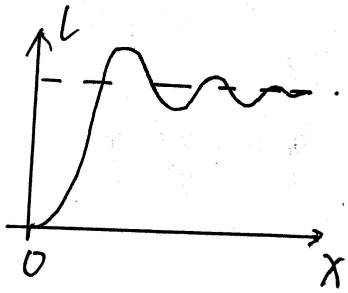
$$\rho(x\cdot y\cdot z)=\frac{e}{V}\sum_{k_x}\sum_{k_y}\sum_{k_z}|\psi|^2$$

$$=\frac{e}{V}\sum_{k_xk_yk_z}|\phi(x)|^2 \approx \frac{e}{V}\frac{2V}{(2\pi)^3}\int_0^{k_F}N^2\sin^2k_x x\,dk_x\int_{-k_F}^{k_F}dk_y\int_{-k_F}^{k_F}dk_z$$

$$=\frac{N^2e}{8\pi^3}\left(k_F-\frac{\sin 2k_F x}{2x}\right)(2k_F)^2$$

$K_F=(3\pi^2 n)^{\frac{1}{3}}$ 费米波矢

$\rho(x\cdot y\cdot z)$中含有$\frac{Ne^2}{4\pi^3}\frac{\sin 2k_F\cdot x}{2x}(2k_F)^2$，为振荡项．波矢为$2k_F$



$x>>0$时趋向于稳定．振荡周期为$\frac{\pi}{k_F}$

金属 $n=10^{22}\ cm^{-3}$ $\quad \frac{\pi}{k_F}\sim 4\ Å$

半导体 $n=10^{16}\ cm^{-3}$ $\quad \frac{\pi}{k_F}\sim 400\ Å$

2. (1) 在临界点附近时 $E_c+\Delta E = E_c + \frac{\hbar^2}{2}\left(\frac{k_x^2}{m_x}+\frac{k_y^2}{m_y}+\frac{k_z^2}{m_z}\right)$

$$\frac{2\Delta E}{\hbar^2} = \frac{k_x^2}{m_x}+\frac{k_y^2}{m_y}+\frac{k_z^2}{m_z}$$

k空间体积为 $\frac{4}{3}\pi\sqrt{m_x m_y m_z}\cdot\frac{2\Delta E}{\hbar^2}\sqrt{\frac{2\Delta E}{\hbar^2}}$

$\therefore D(E)\times\Delta E = \frac{V}{8\pi^2}\cdot\frac{4}{3}\pi\sqrt{m_x m_y m_z}\cdot\frac{2\Delta E}{\hbar^2}\sqrt{\frac{2\Delta E}{\hbar^2}}$

$D(E)\propto\sqrt{\Delta E}=\sqrt{E-E_c}$

(2) 由色散关系，等能面内的体积为： $E(k)=E_c$ 时 $\Rightarrow 0=\left(\frac{k_x^2}{m_x}+\frac{k_y^2}{m_y}-\frac{k_z^2}{m_z}\right)$

$k_z\geq 0 \quad \int_0^{k_B}\pi\sqrt{m_x m_y}\frac{k_z^2}{m_z}dk_z$

①' 当 $E(k)>E_c$ 时，设 $E(k)=E_c+\Delta E$ $\quad \frac{2\Delta E}{\hbar^2}+\frac{k_z^2}{m_z}=\frac{k_x^2}{m_x}+\frac{k_y^2}{m_y}$

体积为 $\int_0^{k_B}\pi\sqrt{m_x m_y}\left(\frac{k_z^2}{m_z}+\frac{2\Delta E}{\hbar^2}\right)dk_z$

$\therefore$ 改变量 $\int_0^{k_B}\frac{2\Delta E}{\hbar^2}dk_z$

$\Rightarrow D(E)\Delta E=\frac{V}{8\pi^3}\int_0^{k_B}\frac{2\Delta E}{\hbar^2}dk_z \Rightarrow D(E)\propto \text{const}$

② 当 $E(k)<E_c$ 时，设 $E(k)=E_c-\Delta E$ $\quad \frac{k_z^2}{m_z}-\frac{2\Delta E}{\hbar^2}=\frac{k_x^2}{m_x}+\frac{k_y^2}{m_y}$

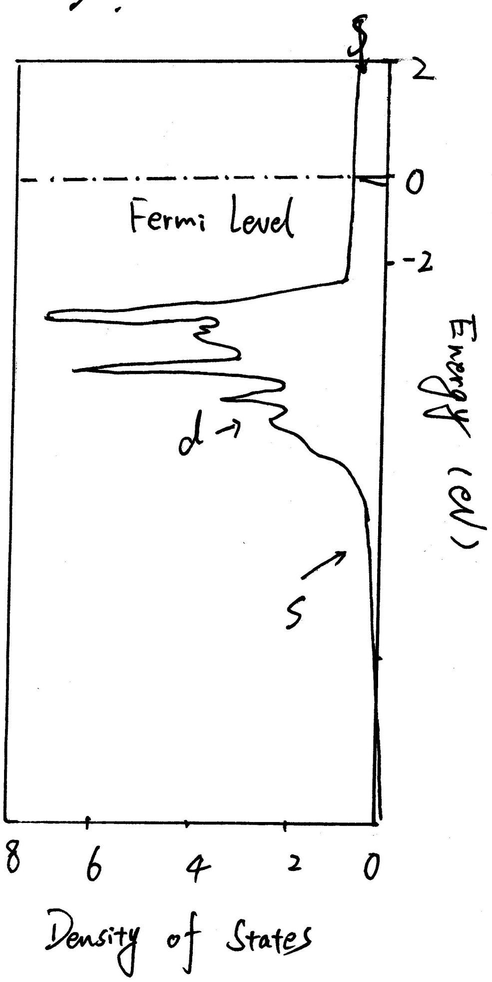
体积为 $\int_{\sqrt{\frac{2m_z\Delta E}{\hbar^2}}}^{k_B}\pi\sqrt{m_x m_y}\left(\frac{k_z^2}{m_z}-\frac{2\Delta E}{\hbar^2}\right)dk_z$

$\therefore$ 改变量 $\int_0^{\sqrt{\frac{2m_z\Delta E}{\hbar^2}}}\pi\sqrt{m_x m_y}\frac{k_z^2}{m_z}dk_z+\int_{\sqrt{\frac{2m_z\Delta E}{\hbar^2}}}^{k_B}\pi\sqrt{m_x m_y}\frac{2\Delta E}{\hbar^2}dk_z$

$=\pi\sqrt{m_x m_y}\left[\frac{1}{3m_z}\cdot\frac{2m_z\Delta E}{\hbar^2}\cdot\sqrt{\frac{2m_z\Delta E}{\hbar^2}}+\left(k_B-\sqrt{\frac{2m_z\Delta E}{\hbar^2}}\right)\frac{2\Delta E}{\hbar^2}\right]$

$\therefore D(E)\propto D_0-C(E_c-E)^{1/2}$

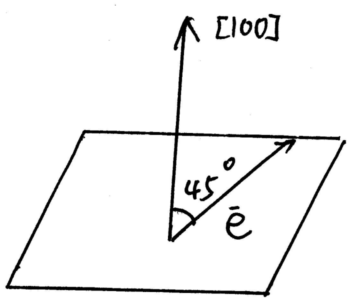
3.



right: Bandstructure $E(K)$ for copper along directions of high crystal symmetry.

如图所示，铜in d带位于费米面下2 eV以下，可见光能量在2.0 eV以上，很容易使电子从d带跃迁到s带，容易被吸收。2.0 eV以下in红光则更多被反射，使铜呈现颜色。

4.



① 电子在真空中in波矢

$$K = \sqrt{\frac{2m}{\hbar^2}(h\nu - \phi - E)}$$

$$= 0.512\sqrt{(40.8-4.5-2.2)}$$

$$= 2.99\ \mathring{A}^{-1}$$

② $K_{\parallel} = k\sin\theta = 2.114\ \mathring{A}^{-1}$

$K_{\perp} = k\cos\theta = 2.114\ \mathring{A}^{-1}$

电子在逃逸样品时，$K_{\parallel}$守恒，$K_{\perp}$不守恒。